DTFM Modeling and Analysis Method for Gossamer Structures

Houfei Fang and Michael Lou
Jet Propulsion Laboratory
California Institute of Technology
Bingen Yang
University of Southern California

Introduction

What is DTFM?

--Distributed Transfer Function Method. An advanced method developed for structural analysis

Why DTFM is unique?

--In the Laplace domain

--Using Distributed Transfer Function instead of Shape Function

What are the advantages

--Exact and closed form solutions for 1-d components

--Generates very small matrices and is computationally efficient

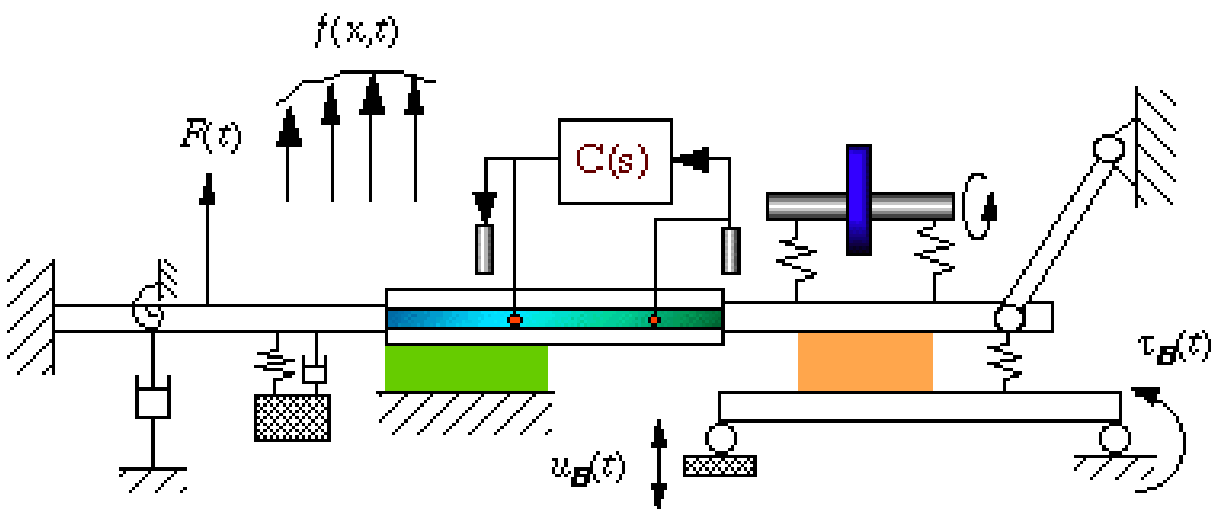
--Capable to handle properties which are functions of frequency

--Capable to treat very slim structures (e.g. long inflatable booms) with asymmetric cross-sections, non-uniform, and/or surface/material imperfections

Distributed Transfer Function Method (DTFM)

DTFM has been successfully developed to obtain exact frequency and time-domain solutions for one-dimensional (1-D) distributed systems involving:

- Multi-body Coupling
- Damping and gyroscopic forces
- Feedback control systems
- Structures with embedded sensors and actuators



Distributed Transfer Function Method (DTFM)

DTFM can obtain exact solutions for general 1-D structures:

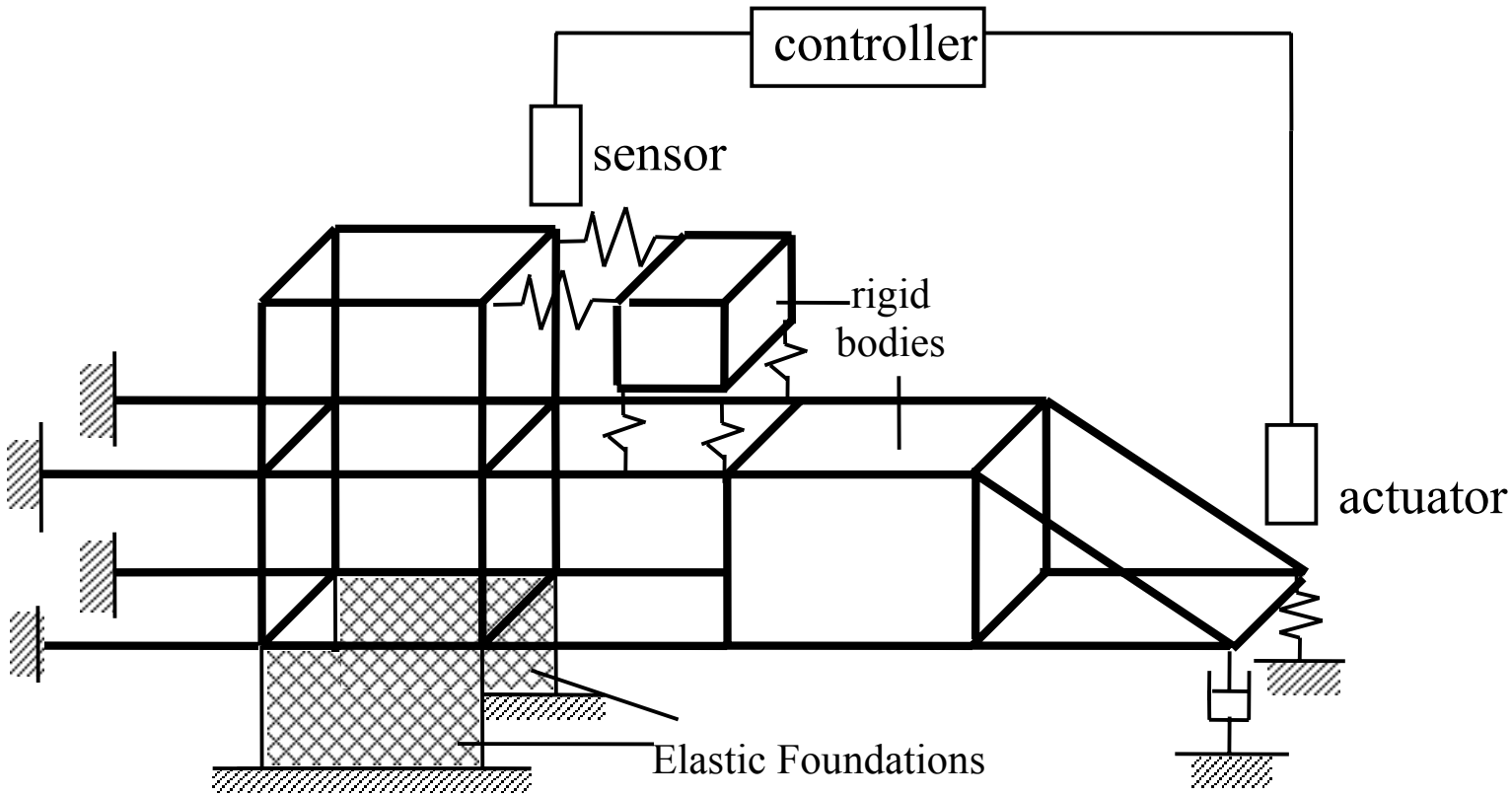
- Strips, bars, beams and beam-columns
- Rotating shafts
- Axially moving continua
- Pipes conveying fluids
- Flexible robots
- Beams with embedded constrained damping layers

For general 2-D structures and components:
A Strip Distributed Transfer Function Method (SDTFM) has been developed to obtain semi-exact solutions

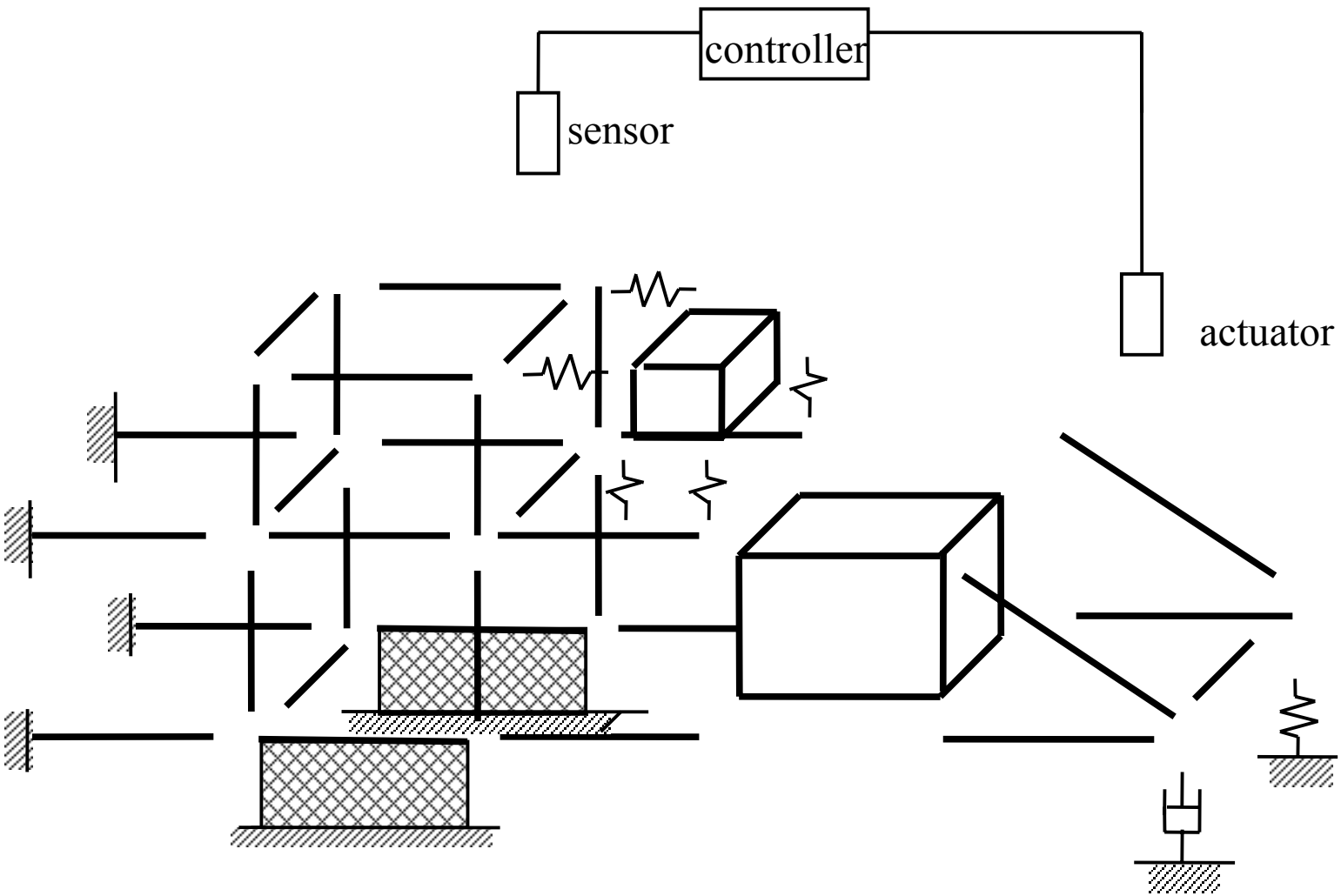
Structural Analysis Using the DTFM

1. Decomposition of a complex structure into components.
2. Generation of state space form for each component.
3. Generation of distributed transfer function for each component.
4. Generation of dynamic stiffness matrix for each component and assembly of components.
5. Static and dynamic solutions:
   - Natural Frequencies and mode shapes.
   - Buckling analyses of thin-walled structures.
   - Frequency Responses.
   - Static and Dynamic Stress Analyses.
   - Time Domain Responses.

DTFM structural Analysis: Step 1--Decomposition



DTFM structural Analysis: Step 1--Decomposition



DTFM structural Analysis: Step 2--State Space Form

A set of governing equations for each individual component:

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j}\left(a_{ijk}+b_{ijk}\frac{\partial}{\partial t}+c_{ijk}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^k u_j(x,t)}{\partial x^k}=f_i(x,t)$$

$$x\in(0,L),\quad t\geq 0,\quad i=1,\cdots,n$$

↓ Example: a beam component

$$EI\frac{\partial^4 v}{\partial x^4}+\rho A\frac{\partial^2 v}{\partial t^2}=p$$

DTFM structural Analysis: Step 2--State Space Form

State space form: $$\frac{d}{dx}\eta(x,s) = F(s)\eta(x,s) + q(x,s)$$

Example: a beam component

$$F(s) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-\rho A s^2}{EI} & 0 & 0 & 0 \end{bmatrix} \quad \eta(x,s) = \begin{Bmatrix} \bar{v}(x,s) \\ \bar{v}'(x,s) \\ \bar{v}''(x,s) \\ \bar{v}'''(x,s) \end{Bmatrix} \quad q(x,s) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ p(x,s)/EI \end{Bmatrix}$$

DTFM structural Analysis: Step 3--DTF

A boundary value problem:

$$\frac{d}{dx}\eta(x,s) = F(s)\eta(x,s) + q(x,s) \qquad x \in (0,L)$$

$$M\eta(0,s) + N\eta(L,s) = r(s)$$

The solution is expressed as transfer functions:

$$\eta(x,s) = \int_0^L G(x,\zeta,s)q(\zeta,s)d\zeta + H(x,s)r(s) \qquad x \in (0,L)$$

$$G(x,\zeta,s) = \begin{cases} e^{F(s)x}(M + Ne^{F(s)L})^{-1}Me^{-F(s)\zeta} & \zeta \le x \\ -e^{F(s)x}(M + Ne^{F(s)L})^{-1}Ne^{F(s)(L-\zeta)} & \zeta \ge x \end{cases}$$

$$H(x,s) = e^{F(s)x}(M + Ne^{F(s)L})^{-1}$$

DTFM structural Analysis : Step 3--DTF

State space vector: $\eta(x,s) = \left[\alpha^T(x,s) \quad \varepsilon^T(x,s)\right]^T$

Displacement vector: $\alpha(x,s) = \left[\alpha_1{}^T(x,s) \quad \alpha_2{}^T(x,s) \quad \cdots \quad \alpha_n{}^T(x,s)\right]^T$

Strain vector: $\varepsilon(x,s) = \left[\varepsilon_1{}^T(x,s) \quad \varepsilon_2{}^T(x,s) \quad \cdots \quad \varepsilon_n{}^T(x,s)\right]^T$

Force vector: $\sigma(x,s) = \overline{E}\varepsilon(x,s)$

$\downarrow$ Example: a beam component

$$\alpha(x,s) = \begin{Bmatrix} \overline{v}(x,s) \\ \overline{v}'(x,s) \end{Bmatrix} \qquad \varepsilon(x,s) = \begin{Bmatrix} \overline{v}''(x,s) \\ \overline{v}'''(x,s) \end{Bmatrix}$$

$$\sigma(x,s) = \begin{Bmatrix} Q(x,s) \\ M_f(x,s) \end{Bmatrix} = \overline{E}\varepsilon(x,s) = \begin{bmatrix} 0 & EI \\ EI & 0 \end{bmatrix} \begin{Bmatrix} \overline{v}''(x,s) \\ \overline{v}'''(x,s) \end{Bmatrix}$$

DTFM structural Analysis : Step 4--Dynamic Stiffness Matrix

Force vectors at two ends of the component:

$$\begin{bmatrix} \sigma(0,s) \\ \sigma(L,s) \end{bmatrix} = \begin{bmatrix} \overline{E}H_{\sigma 0}(0,s) & \overline{E}H_{\sigma L}(0,s) \\ \overline{E}H_{\sigma 0}(L,s) & \overline{E}H_{\sigma L}(L,s) \end{bmatrix} \begin{bmatrix} \alpha(0,s) \\ \alpha(L,s) \end{bmatrix} + \begin{bmatrix} p(0,s) \\ p(L,s) \end{bmatrix}$$

Dynamic stiffness matrix

Transformed from distributed external forces

Systematically assemble all component dynamic stiffness matrices

↓

Dynamic stiffness matrix of the whole system

↓

$$\boldsymbol{K}(s) \times \boldsymbol{U}(s) = \boldsymbol{P}(s)$$

DTFM structural Analysis: Step 5--Static and Dynamic Solutions

Natural frequencies of the structure

$$\det\left[\boldsymbol{K}(\mathrm{s_i})\right] = 0 \qquad \mathrm{s_i} = \sqrt{-1} \times \omega_\mathrm{i}$$

Mode shapes--nontrivial solutions

$$\boldsymbol{K}(\mathrm{s_i}) \times \boldsymbol{U}(\mathrm{s_i}) = 0$$

Frequency responses

$$\boldsymbol{U}(\mathrm{s}) = \boldsymbol{K}^{-1}(\mathrm{s}) \times \boldsymbol{P}(\mathrm{s})$$

Static analysis

$$\boldsymbol{K}(0) \times \boldsymbol{U}(0) = \boldsymbol{P}(0)$$

Time domain responses

Inverse Laplace transform

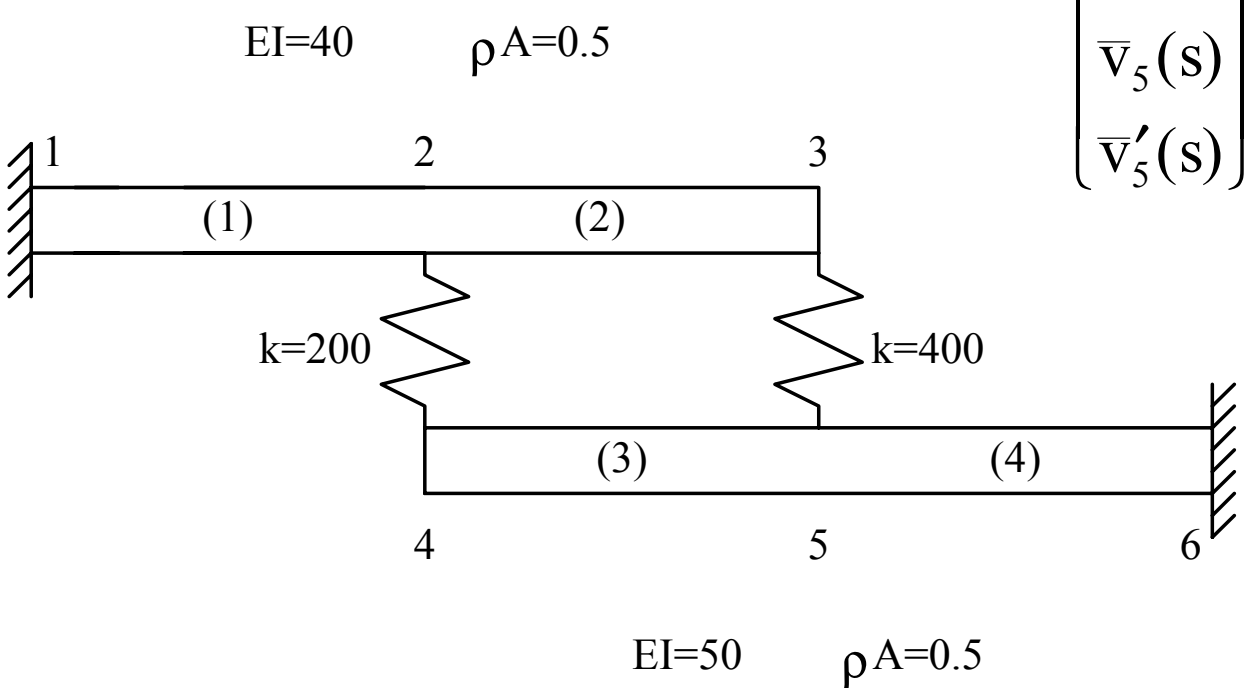
Examples of DTFM Analyses

(1) Two elastically coupled beams

(2) Buckling analysis of a thin-walled column

Example (1)--two elastically coupled beams

$$K(s) * \begin{Bmatrix} \bar{v}_2(s) \\ \bar{v}_2'(s) \\ \bar{v}_3(s) \\ \bar{v}_4(s) \\ \bar{v}_5(s) \\ \bar{v}_5'(s) \end{Bmatrix} = \begin{Bmatrix} \tilde{Q}_2(s) \\ \tilde{M}_{f2}(s) \\ \tilde{Q}_3(s) \\ \tilde{Q}_4(s) \\ \tilde{Q}_5(s) \\ \tilde{M}_{f5}(s) \end{Bmatrix}$$



Example (1)--two elastically coupled beams

Mode number	DTFM 6*6 matrix	FEM 18 Elements	FEM 34 Elements	FEM 66 Elements
1	16.3	16.3	16.3	16.3
2	41.0	41.1	41.0	41.0
3	54.6	53.1	54.2	54.5
4	79.2	77.8	78.9	79.1
5	144.7	138.3	143.1	144.3
6	157.0	150.5	155.4	156.6
7	273.9	258.1	269.9	272.9
8	305.2	288.2	289.9	304.1
9	448.7	415.4	440.4	446.6
10	500.5	463.9	491.2	498.1
11	669.1	601.7	653.7	665.3
12	747.5	672.7	730.5	743.3

Gossamer Structures

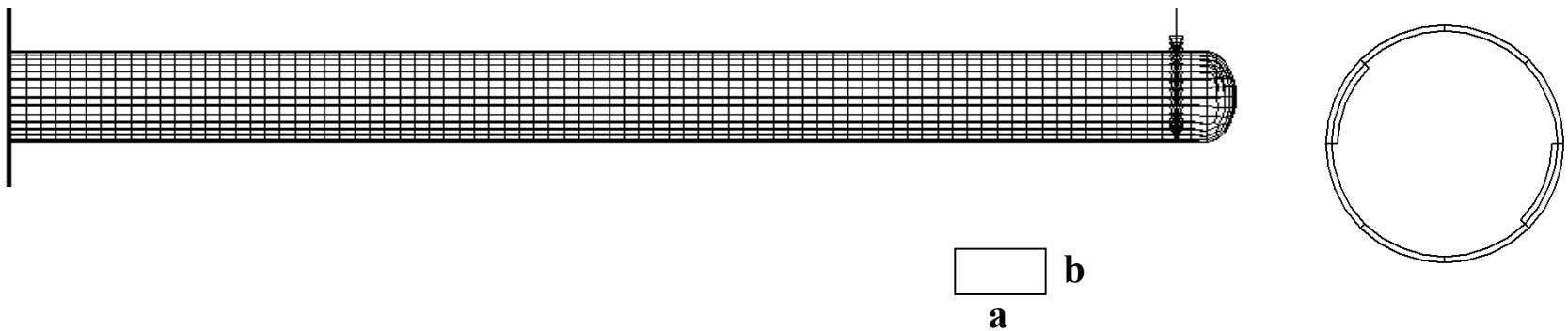
Gossamer structures:

»Mostly composed of highly flexible, long tubular components and pre-tensioned thin-film membranes.

»Offer order-of-magnitude reductions in mass and launch volume

»Revolutionize the architecture and design of space flight systems with large in-orbit configurations.

Disadvantages of FEM for Gossamer Structures



Major shortcomings of general finite element analysis:

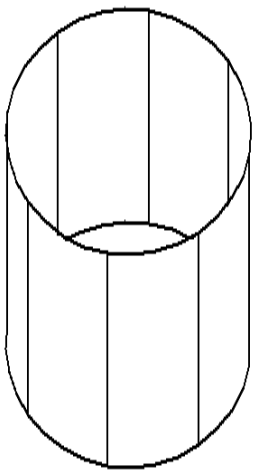
1) Tens of thousands elements are needed due to:
   - ➪ Accuracy requirements
   - ➪ Aspect ratio (a/b) limitations
2) Time-domain solutions ➪ require small time steps for convergence ➪ excessive computation time
3) Unable to investigate the effect of surface imperfection

Example (2)--Buckling Analysis of a Thin-Walled Column

(Strip-Discretization vs. Finite-Element Discretization)

SDTFM

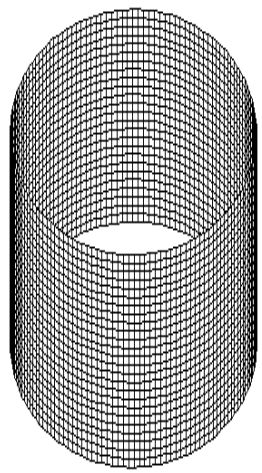
Applied Load



Number of Strips	Buckling Load
2	381.28
4	381.28
6	381.28
8	381.28

FEM

Applied Load

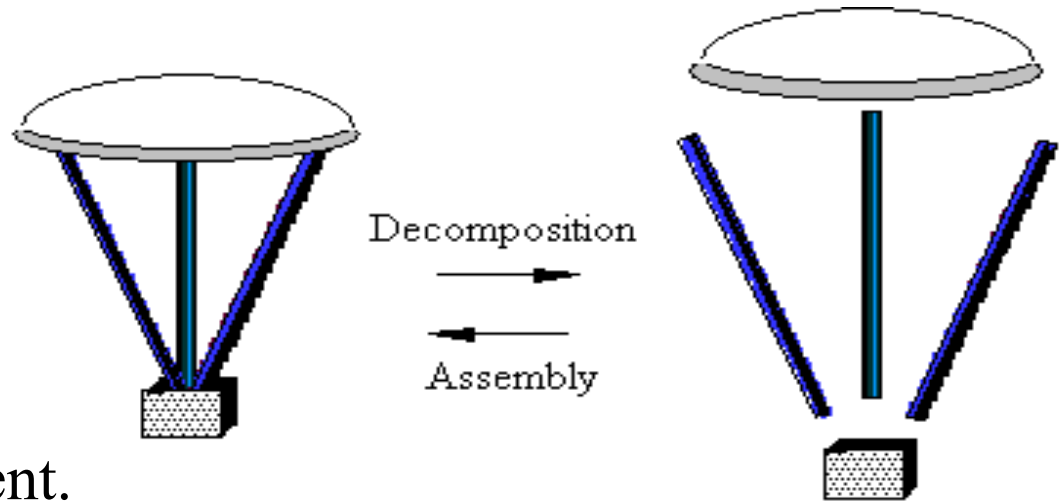


No. of Elements	Buckling Load
54	1269.6
218	393.3
864	386.8
3456	381.4

DTFM for Gossamer Structures

» Using a couple of super components instead of numerous tiny elements.

» Dealing with very small matrices.



» Computationally efficient.

» Capable of handling no-uniform long booms.

» Useful in studying surface and material imperfections.

» Easy to incorporate with control systems (Laplace domain).

» Capable of handling properties which are functions of frequency.

» Able to handle spinning space structures (gyroscopic forces).

Future Tasks

Goal--test-correlated modeling/analysis methods and user-friendly computer software that can be directly employed for the development of flight gossamer systems

- To complete the development of DTFM-based approach for solving structural problems related to gossamer structures.
- To develop analysis capabilities for studying design perturbations, geometric and material imperfections, long booms of non-uniform and non-axisymmetry cross-sections.
- To develop synthesis and assembly processes for modeling and analyzing general 2-dimensional and 3-dimensional gossamer structures formed by multiple long booms and membranes.
- To incorporate the developed DTFM into a selected general-purpose finite-element code to be user-friendly to all engineers.

THE END

Houfei Fang, Michael Lou, and Bingen Yang