STRUCTURAL ANALYSIS OF INFLATED MEMBRANES WITH APPLICATIONS TO LARGE SCIENTIFIC BALLOONS

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Overview

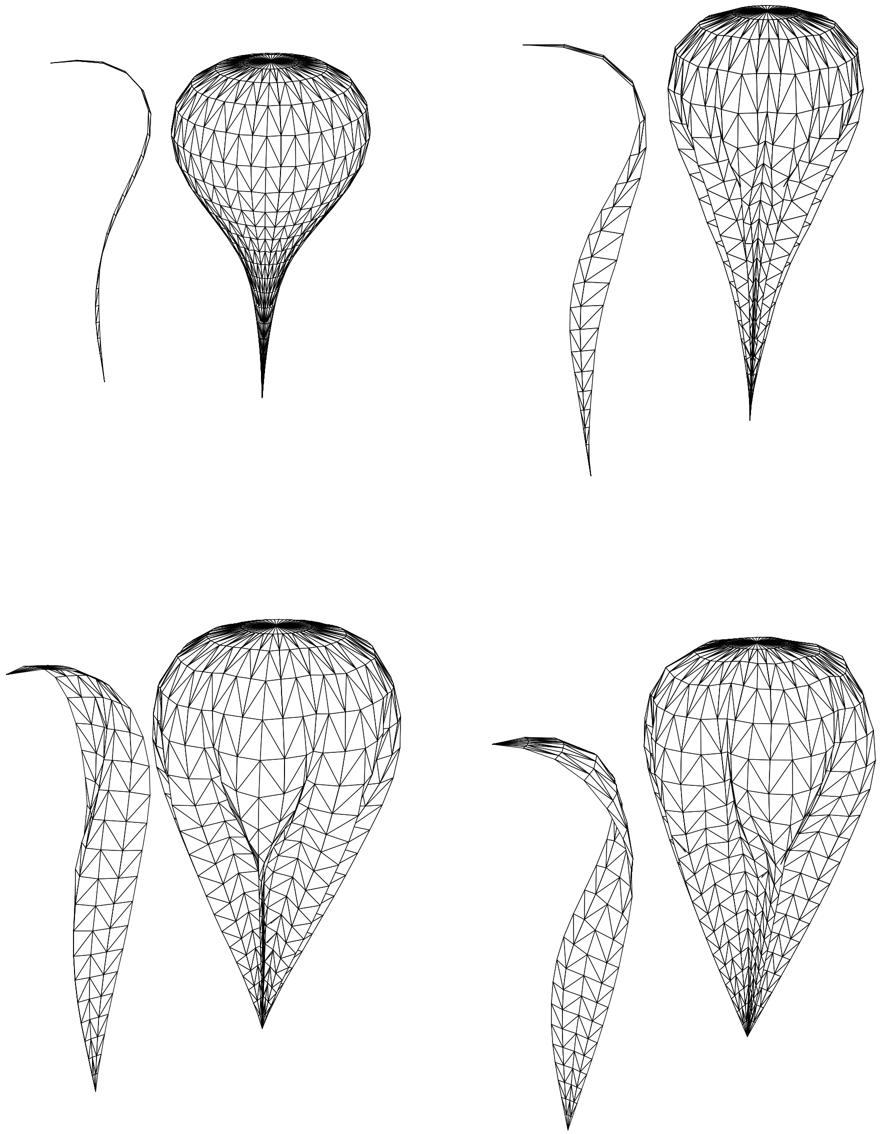
1. The balloon problem
2. Mathematical model for the analysis of partially inflated strained balloons
3. Analysis of pumpkin balloon

The Balloon Problem: Design and Analysis

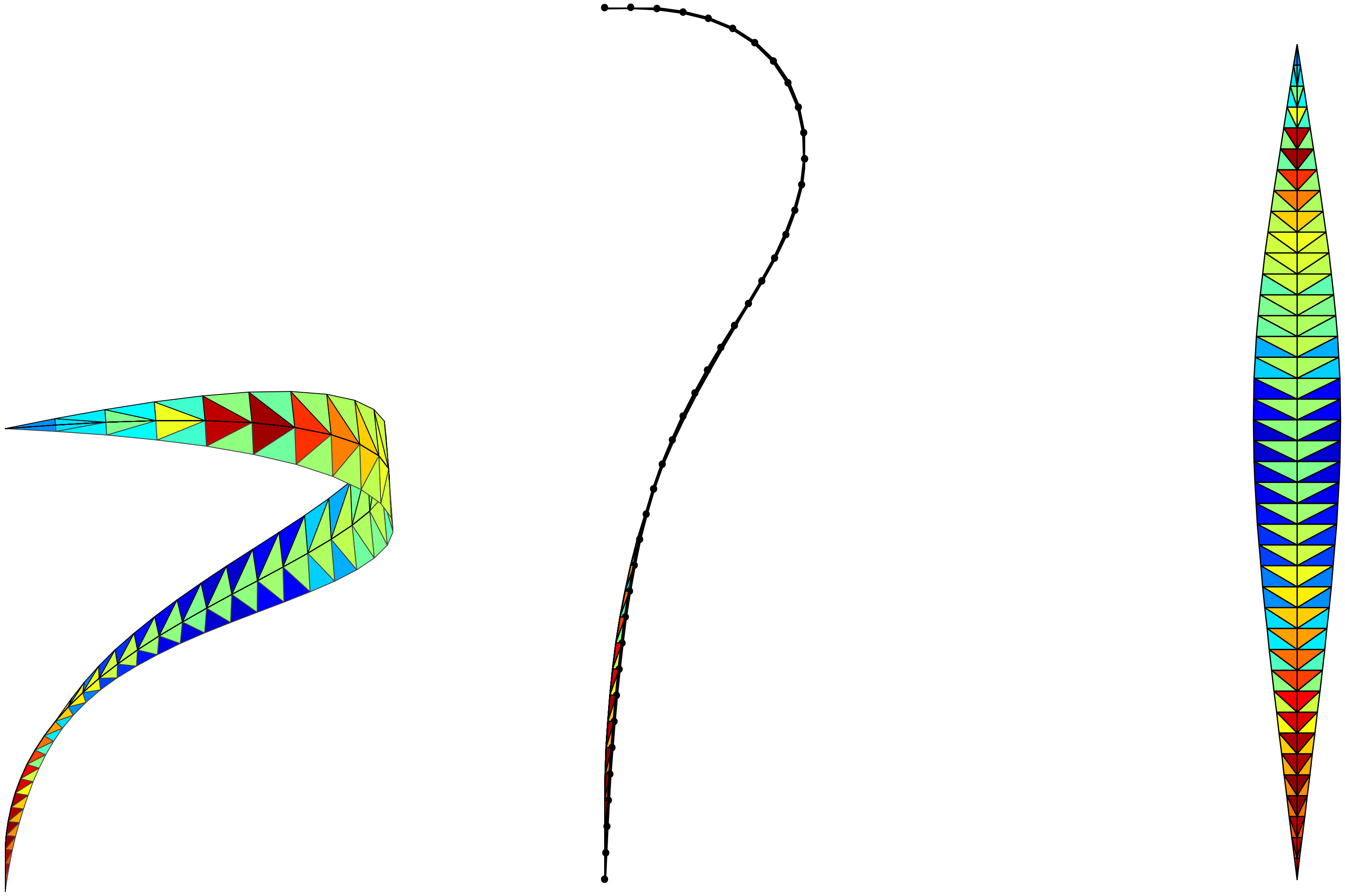
- Design - Determine the shape of a balloon to carry a payload of weight L at a constant altitude.
  - Typically, assume a statically determinate shape (consider balloon system weight and hydrostatic pressure).
  - Actual balloon is constructed from long tapered *flat* sheets of thin film that are sealed edge-to-edge. Load tendons are attached along each seam.

- Analysis - Estimate film stresses.
  - Model the balloon as an elastic membrane
  - Include elastic reinforcing load tendons
  - Consider launch, ascent, and float configurations.
  - Mathematical model for the analysis of strained/partially inflated balloons supported by NASA Awards: NAG5-697, 5292, 5353.

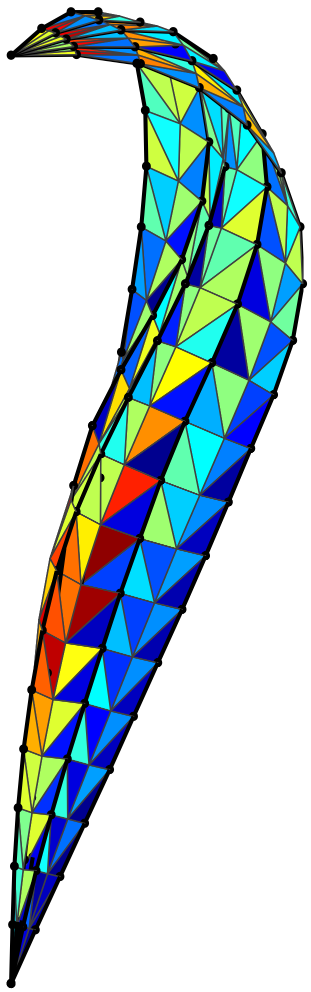
Partially Inflated Balloons (same loading)

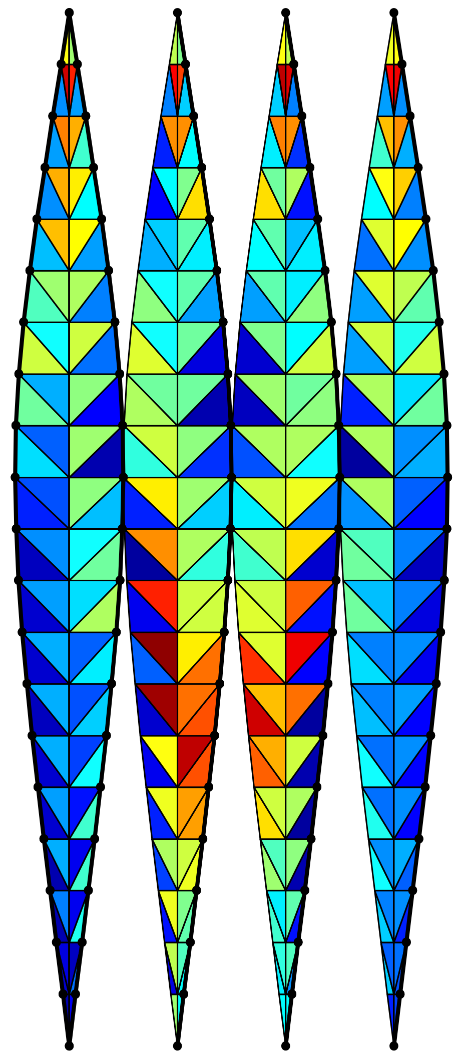


Partially Inflated Balloon (Single Gore)



Partially Inflated Balloon with Lobes

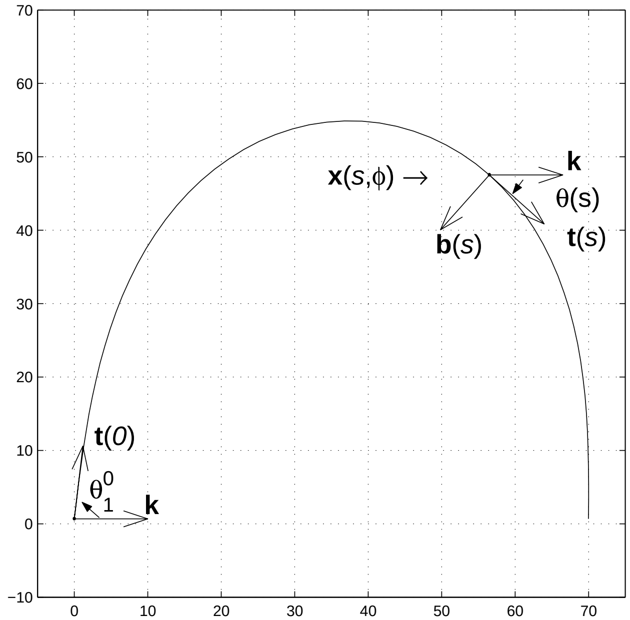




Design Related Considerations

Natural-Shape Equations ($\sigma_c = 0$)

Axisymmetric membrane theory:
UMN, 1950s; further balloon development by J. Smalley, 1960-70s.

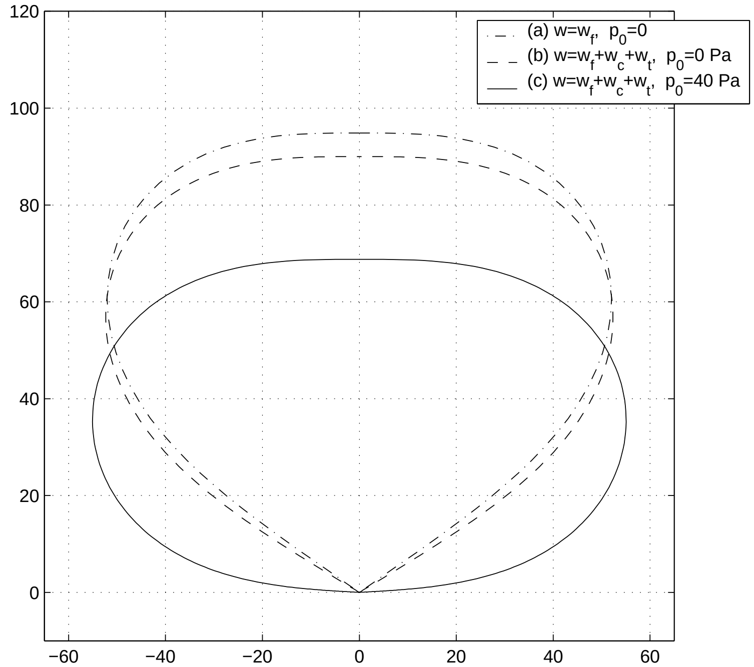


$$\vec{0} = \frac{\partial}{\partial s}(r\sigma_m \mathbf{t}) - \sigma_c \mathbf{e}_1(\phi) + r\mathbf{f}$$

$T(s) = 2\pi r(s)\sigma_m(s)$ - total meridional tension

$\mathbf{f}$ - $\underbrace{\text{hydrostatic pressure}}_{p = bz + p_0}$ and film/tendon weight

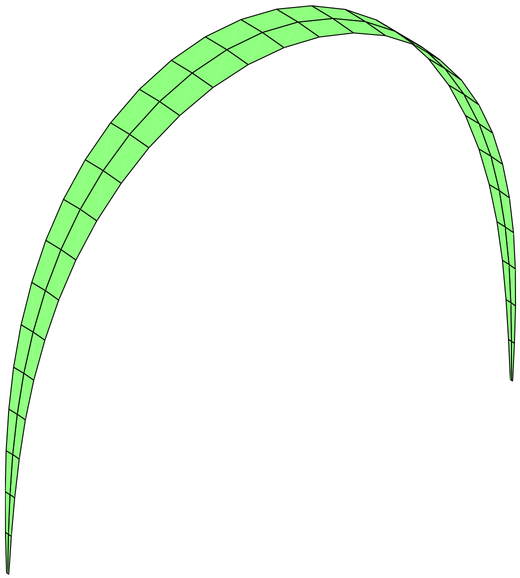
Natural-Shape Balloons
Zero Pressure and Super-Pressure Designs



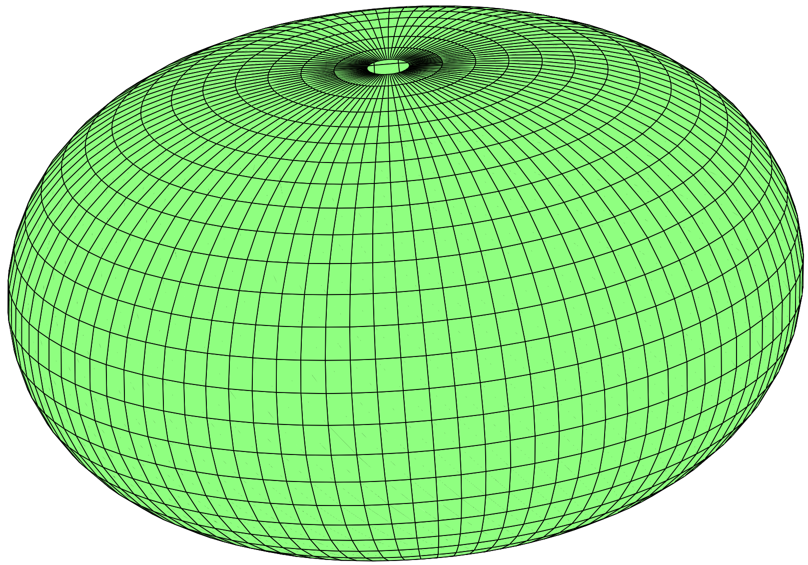
$z = 0$

- Zero-pressure balloons ($p_0 = 0$).
  Typical missions are several days.
  Open at base and need ballast to maintain constant altitude.
- Super-pressure balloon ($p_0 >> bz_{\max} > 0$).
  Add sufficient pressure so that day/night volume changes are reduced.

Super-Pressure Natural-Shape Balloon



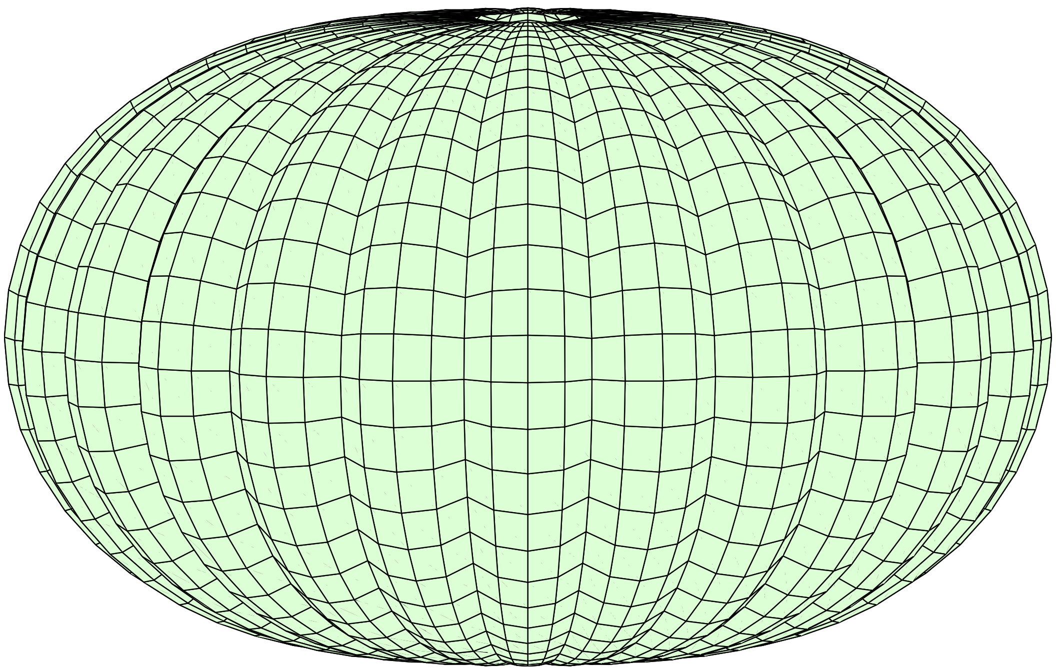
A developable (ruled) surface



“Manufactured” design

- While the natural-shape design is axisymmetric, manufactured design consists of piecewise ruled surfaces.
- ZP-balloons can handle the film stresses that are normally encountered.
- With a natural-shape superpressure design, available thin films are not strong enough to contain the pressure, or too heavy, or too expensive.
- Solution: A pumpkin shape with very strong tendons.

The Pumpkin Balloon



- Curvature in the hoop direction transfers load from film to the tendons.
- Increased tendon stiffness can be achived by tendon shortening
  (there is a film/tendon mismatch!).

Background on the Pumpkin Balloon

- ○ J. Smalley coined the term *pumpkin balloon*. Extensibility of the film is used to achieve the pumpkin gore shape (early 1970s).

- ○ CNES built several small pumpkin balloons, cutting half-gore panels with extra material (mid-late 1970s)

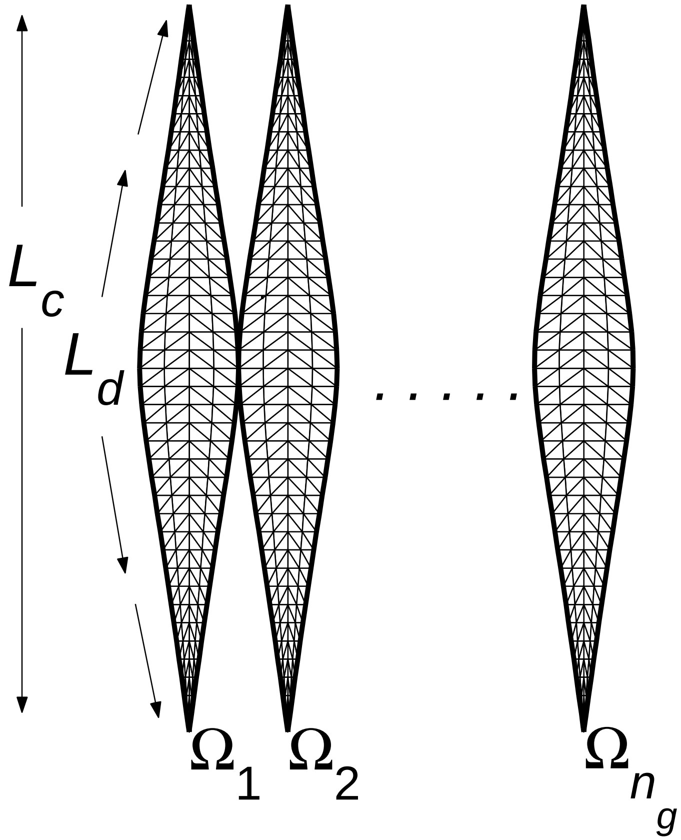
- ○ Sewing techniques to gather material at gore seams (N. Yajima, Japan, 1998, see Adv. in Space Res., 2000).

- ○ NASA/ULDB - structural lack-of-fit (shorten tendons) + material properties (W. Schur, PSL/WFF, 1998, see AIAA-99-1526).

- ● There are several versions of the pumpkin balloon. We will analyze a NASA ULDB pumpkin design flown in 2001.

Strain Analysis

The Natural (unstrained) State of a Complete Balloon



$n_g = 290$ for the ULDB we consider here.

Observations and EM-Model Assumptions

- Linear stress-strain constitutive law
- Isotropic material (E-Youngs modulus, ν-Poisson's ratio)
- Constant strain model ($T \in S_{Ref} \longleftrightarrow T \in S$)

- Wrinkling via energy relaxation (Pipkin) - facets are *taut, slack, wrinkled*
- Energy relaxation allows a tension field solution
- Folds can be used to describe distribution of excess material.
- Load tendons behave like sticky linearly elastic strings

- Shapes are characterized by *large deformations* but *small strains*.

- Hydrostatic pressure is shape dependent

Variational Principle for a Strained Balloon

Problem *

For $S \in C$,

Minimize: $E_T(S) = E_P + E_f + E_t + S_t + S_f$

Subject to: $V = V_0$

S	balloon shape
C	set of allowable shapes
E_T	Total energy
V	Volume
E_P	hydrostatic pressure potential
E_f	gravitational potential energy due to film weight
E_t	gravitational potential energy due to tendon weight
S_t	strain energy of tendons
S_f	strain energy of film

Problem * is discretized and solved by *EMsolver* - developed for balloon applications, written in Matlab (uses `fmincon` - find minimum of a nonlinear multivariable function with linear and/or nonlinear constraints).

Aspects of EM-model have been implemented in Ken Brakke's *Surface Evolver*.

Energy Terms

Hydrostatic Pressure: $E_P = -\int_V p\, dV = -\int_S (\frac{1}{2}bz^2 + p_0 z)\vec{k} \cdot d\vec{S},$

Film Weight: $E_f = \int_S w_f z\, dA$

Tendon Weight: $E_t = \sum_{i=1}^{n_g} \int_0^{\ell_d} w_t^i z\, ds$

Tendon Strain: $S_t = \sum_{i=1}^{n_g} \int_0^{\ell_d} W_c^*(\dot{\gamma}_i)\, ds, \quad W_c(\dot{\gamma}_i) = \frac{1}{8}K_t(|\dot{\gamma}_i|^2 - 1).$

Film Strain: $S_f = \int_\Omega W_f(\mathbf{G})dA, \quad W_f(\mathbf{G}) = \frac{1}{2}\mathbf{S} : \mathbf{G};$

Strains: $\mathbf{G} = \frac{1}{2}(\mathbf{C} - \mathbf{I})$ - Green, $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ - Cauchy; $\mathbf{F}$ - Def. Grad.

Second Piola-Kirchoff stress tensor

$$\mathbf{S}(\mathbf{G}) = \frac{tE}{1-\nu^2}\left(\mathbf{G} + \nu \mathrm{Cof}(\mathbf{G}^T)\right).$$

Fine wrinkling: replace W_f by its relaxation W_f^*, allowing a *Tension Field*

Energy Relaxation $\Longrightarrow$ Tension Field

In Pipkin's approach decompose M into three disjoint regions:

S - *Slack* region: Cauchy-Green strains are both negative, $\delta_1 < 0$, $\delta_2 < 0$;
T - *Tense* region: both principal stress resultants are positive, $\mu_1 > 0$, $\mu_2 > 0$;
U - *Wrinkled* region ($\mathsf{U} = \mathsf{M} \setminus \mathsf{S} \cup \mathsf{T}$).

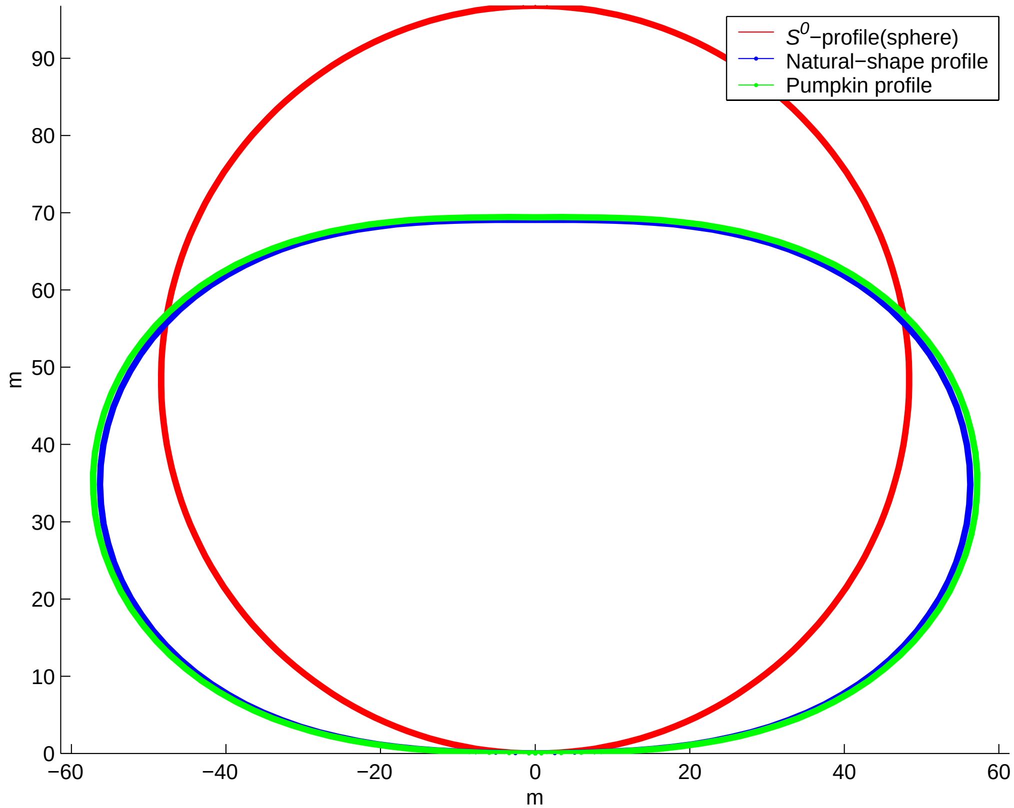
Classify each $T_l \in \Omega$

$$W_f^*(\delta_1, \delta_2; t, \nu, E) = \begin{cases} 0, \;\; \delta_1 < 0 \text{ and } \delta_2 < 0, \\ \frac{1}{2} t E \delta_2^2, \;\; \mu_1 \leq 0 \text{ and } \delta_2 \geq 0, \\ \frac{1}{2} t E \delta_1^2, \;\; \mu_2 \leq 0 \text{ and } \delta_1 \geq 0, \\ \dfrac{tE}{2(1-\nu^2)} (\delta_1^2 + \delta_2^2 + 2\nu\delta_1\delta_2), \\ \qquad\qquad\qquad \mu_1 \geq 0 \text{ and } \mu_2 \geq 0. \end{cases}$$

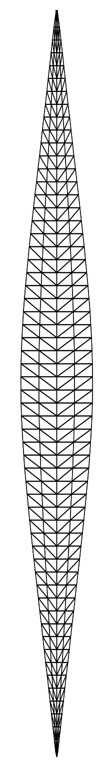
*See FB and Collier, AIAA J, Vol 39, No. 9, Sept 2001, 1662-1672.

Strained Pumpkin Balloon

(joint work with W. Schur PSL/WFF)



Red - Initial profile (sphere)
Blue - natural shape profile
Green - strained center profile



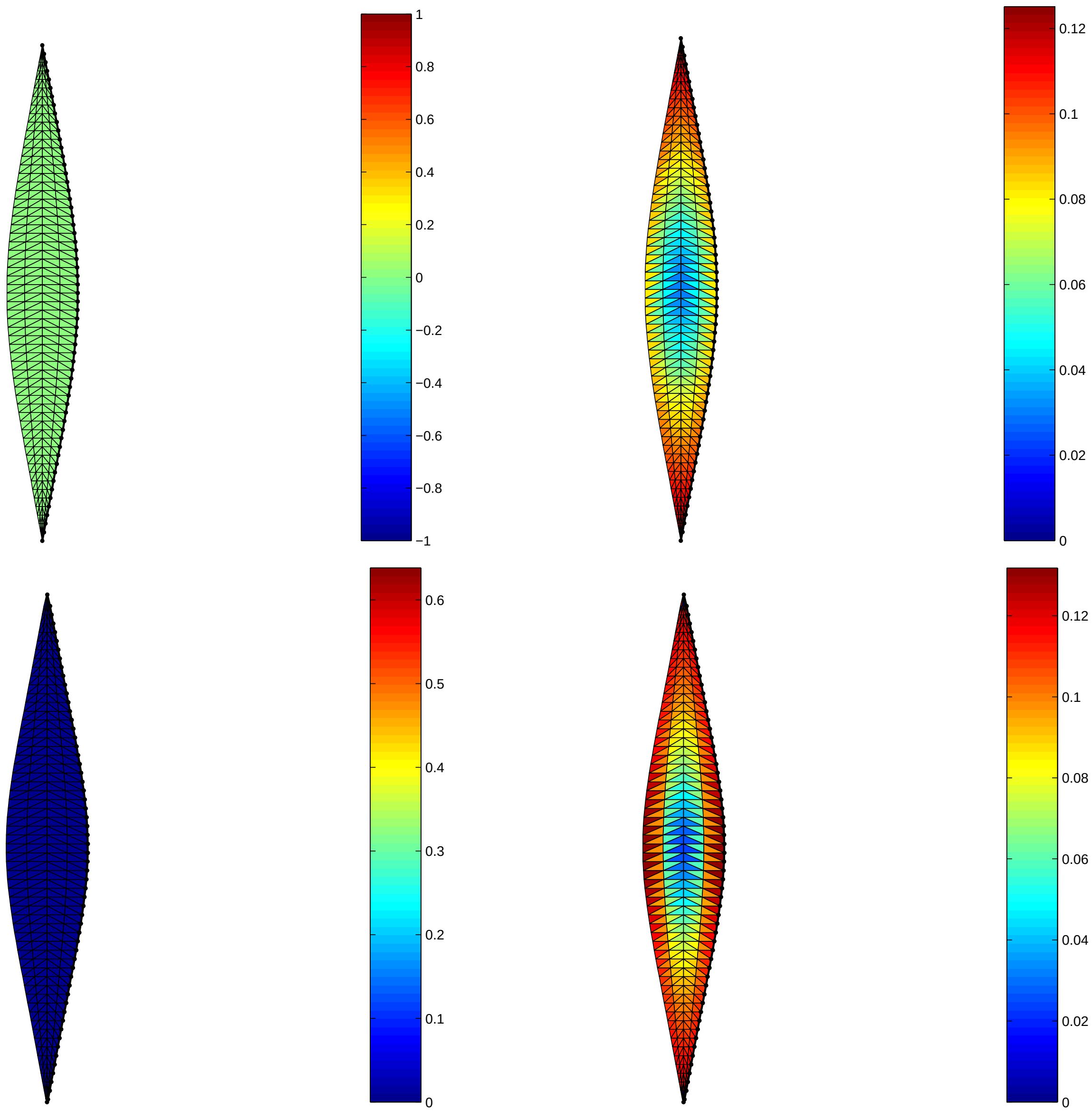
Unstrained Flat
Reference Configuration
(rescaled for display)

Wrinkling Summary

2.9% slack tendons | 2.2% short tendons

Natural

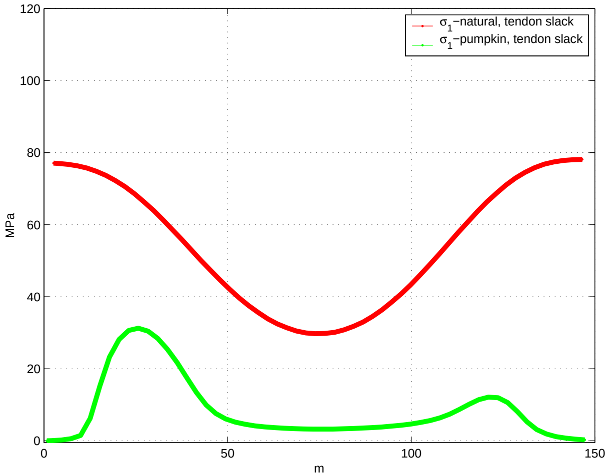
Pumpkin

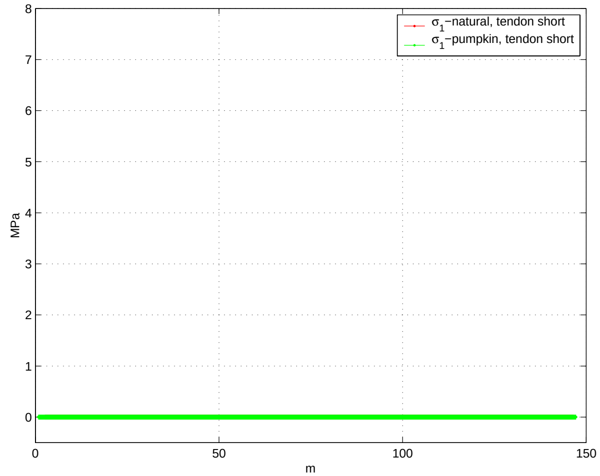


Principal Stresses: Superpresure Natural vs. Pumpkin

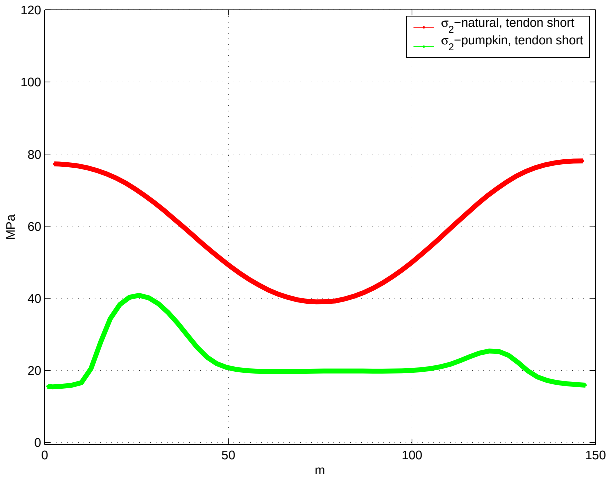
(MPa) | 2.9% slack tendons | (c) 2.2% short tendons

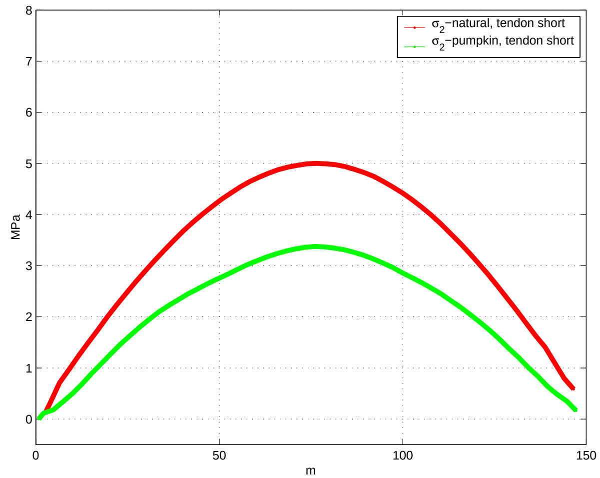
Meridional





Hoop





Stress Analysis Summary

$t = 38\mu$m (1.5 mil)		Max Stress (stress resultant)	
Tendon		Slack 2.9%	Shorten 2.0%
Natural	Meridional	78 MPa (17 lbf/in)	0 MPa (0 lbf/in)
	Hoop	78 MPa (17 lbf/in)	5.25 MPa (1.41 lbf/in)
Pumpkin	Meridional	28 MPa (6.09 lbf/in)	0 MPa (0 lbf/in)
	Hoop	40 MPa (8.70 lbf/in)	4.25 MPa (0.92 lbf/in)

Conclusions

- Pumpkin design *(shape + tendon shortening)* offers a significant reduction in maximum stresses compared to natural-shape superpressure design.

- The variational formulation and optimization based solution process of EMsolver provides an analytical tool that is readily adaptable to other membrane and gossamer structures.

Appendices

- (2002) Comparison of EMsolver predictions with measurements.
- Benchmark comparisons with ABAQUS
  - (1998) Zero pressure natural shape; EMsolver with virtual fold.
  - (2001 -) Spherical balloon with rope constraints; EMsolver with strain energy relaxation.

Compare EMsolver Predictions with Measurements

Joint work - Willi Schur (PSL/WFF); Tech. supp. - Roy Tolbert (NASA/WFF)

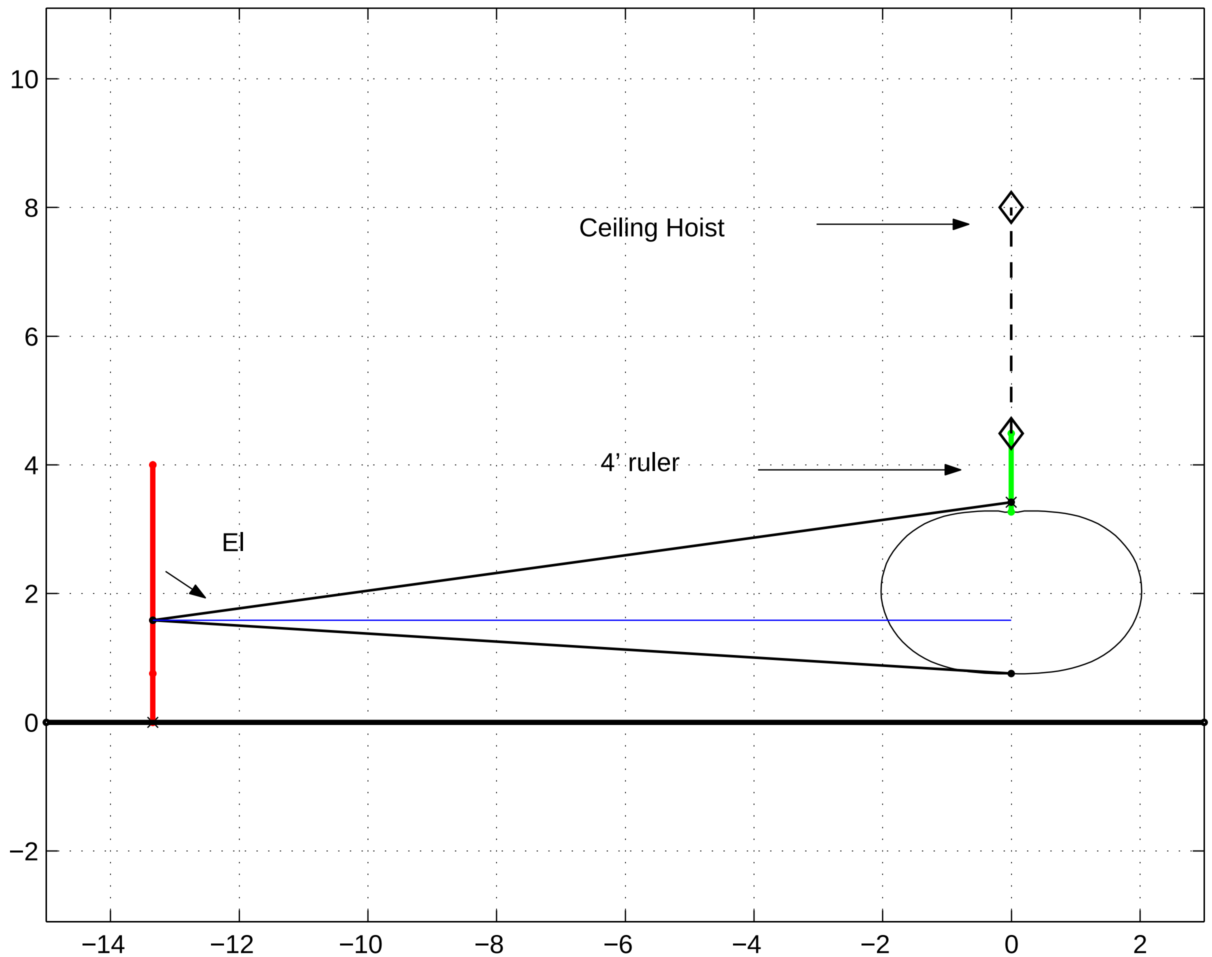
	Measured M	Predicted P	Absolute Error $\lvert M-P \rvert$	Relative Error $\lvert M-P \rvert / \lvert M \rvert$
Diameter	4.0606	4.034	0.0266	0.0064
Z(Diam)	1.2846	1.239	0.0456	0.0354
Height	2.4102	2.449	0.0388	0.0160

Set-up for test vehicle inflations: Elevation (el) and azimuth (az) were recorded.

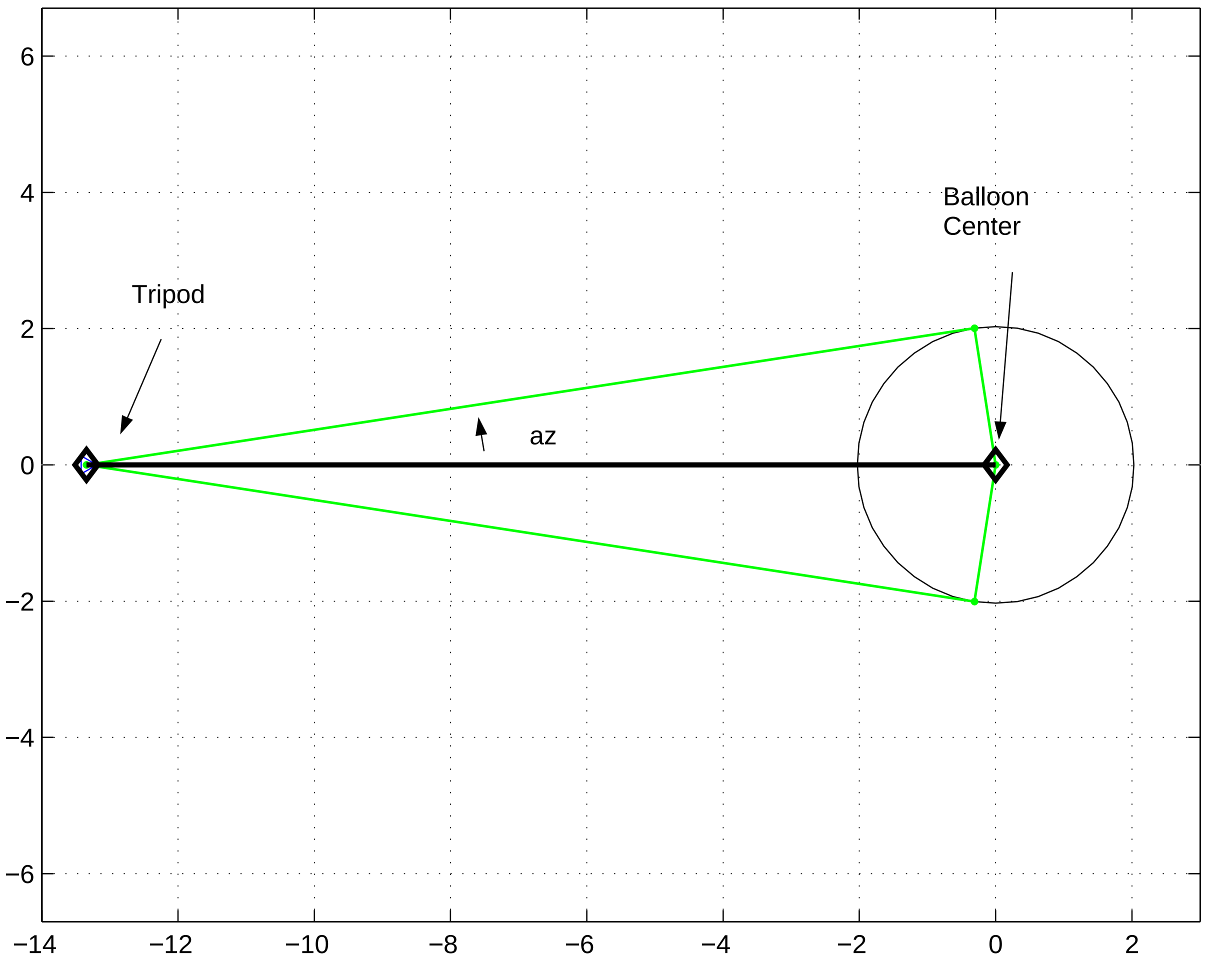
(a) Side view - elevation measurements; a 4 ft ruler was attached to an overhead hoist and lowered until it was just touching the top of the balloon.

(b) Overhead view - azimuthal measurements, since it was difficult to locate the line of sight tangency point for az, the az-measurements are probably not as accurate as the el-measurements.

(a) Side view



(b) Top view



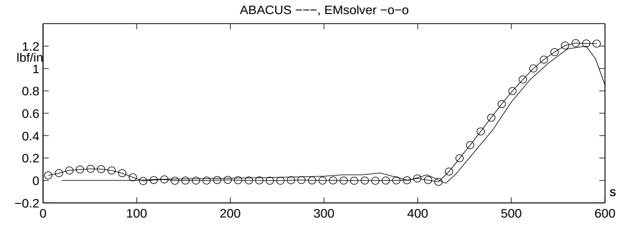
Benchmarks: ABAQUS and EMsolver

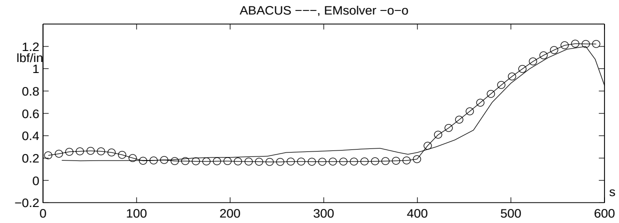
1998 Zero-pressure natural shape balloon. Analyzed single gore.
Joint work with W. Schur (PSL/WFF) for NASA Balloon Office

2001-present Spherical balloon with mooring ropes and rigid end caps.
Joint work with Laura Cadonati (Princeton/MIT) for The Borexino Project
(a solar neutrino particle detector experiment)

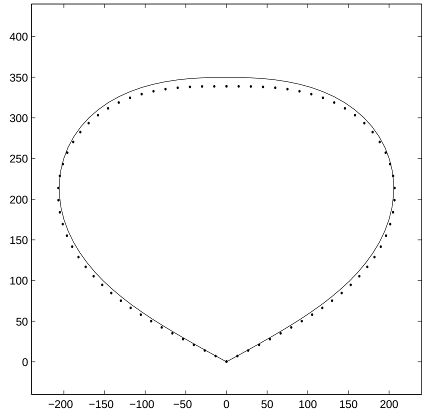
Comparison of
EMsolver (virtual fold, K. Brakke)
and
ABAQUS (tension field, W. Schur)

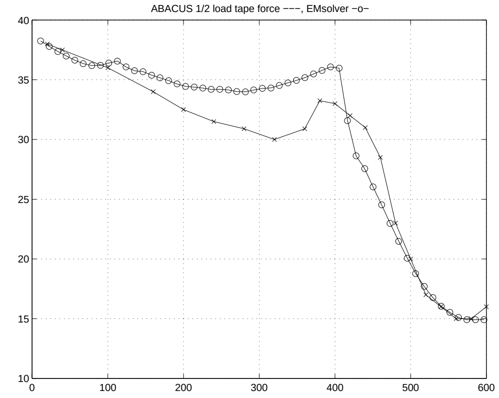
ZP-natural shape
Joint work with W. Schur (1998)



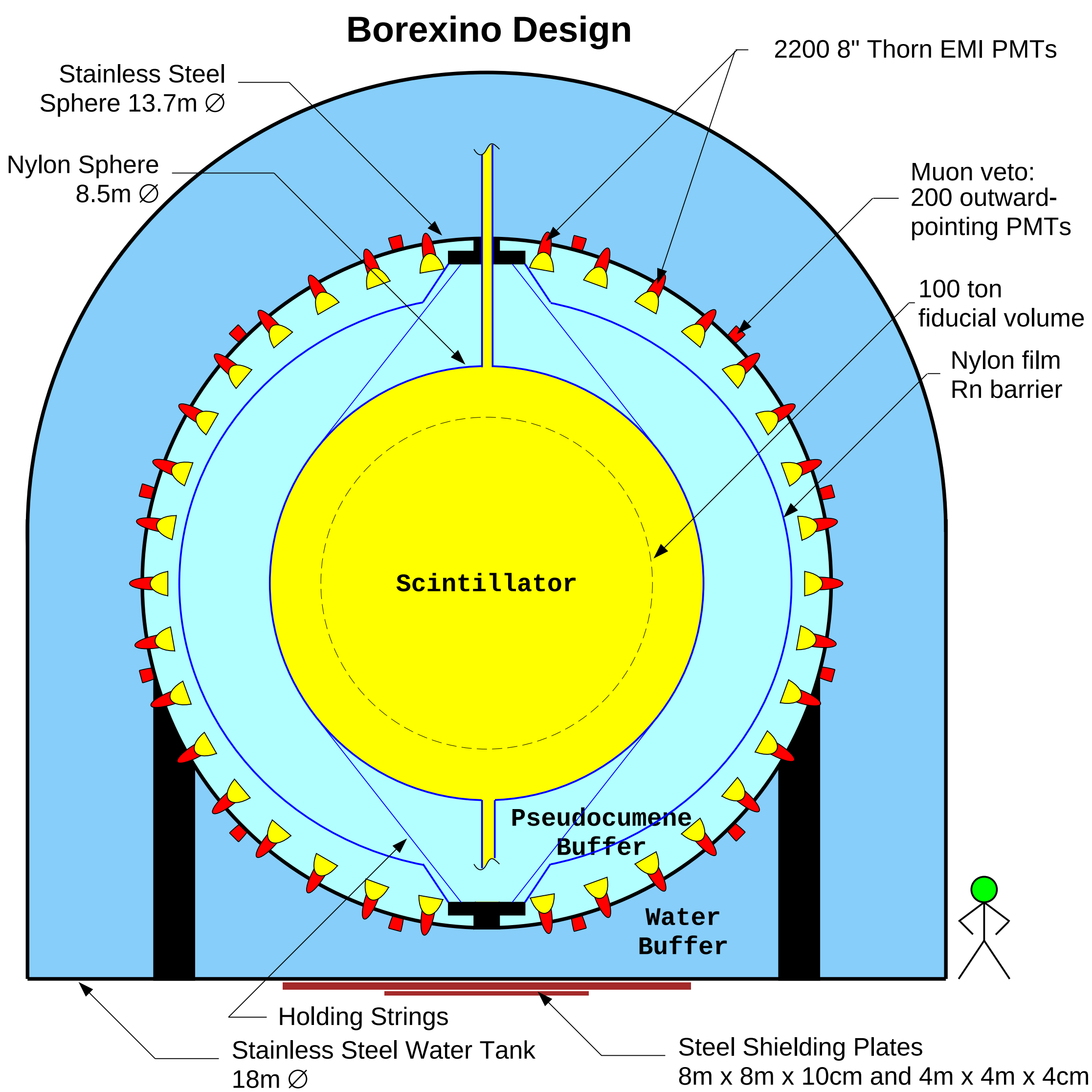


Parameters	
159 gores	Gore length 182 m
$b = 0.05429\text{N/m}^3$	$\nu = 0.82$
$E = 124$ MPa	$E_t = 26.24$ kN
$m_f = 18.7$ g/m^2	$m_t = 0.0313$ g/m
$V = 832515\text{m}^3$	(zero-slackness)

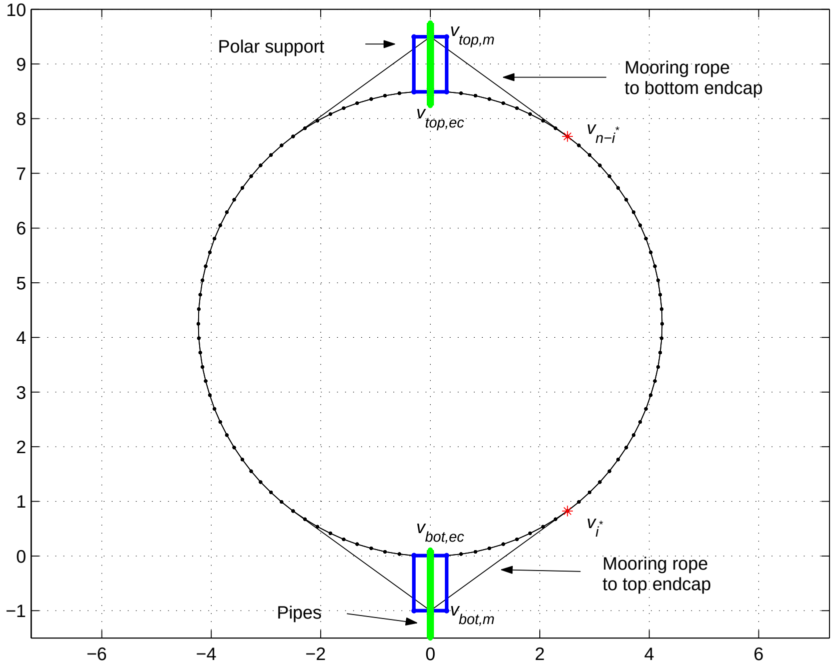


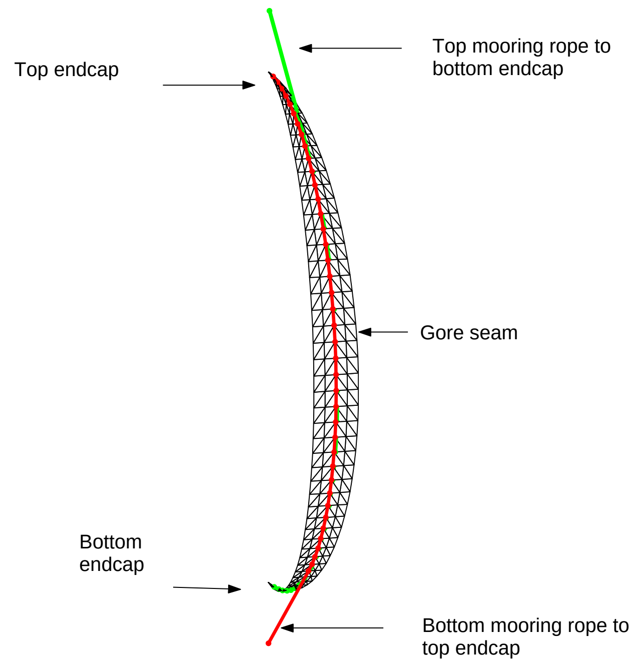


Borexino Containment Vessel (joint work with L. Cadonati Princeton/MIT)

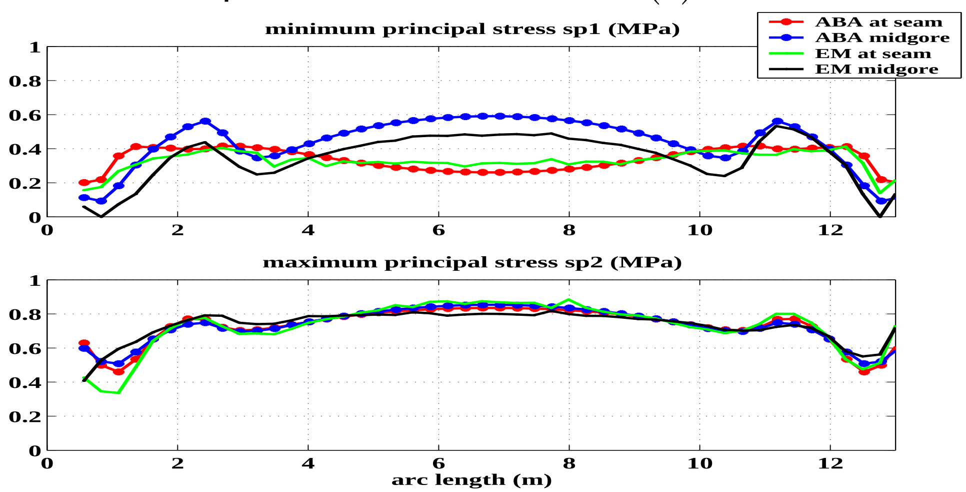


Borexino (continued)



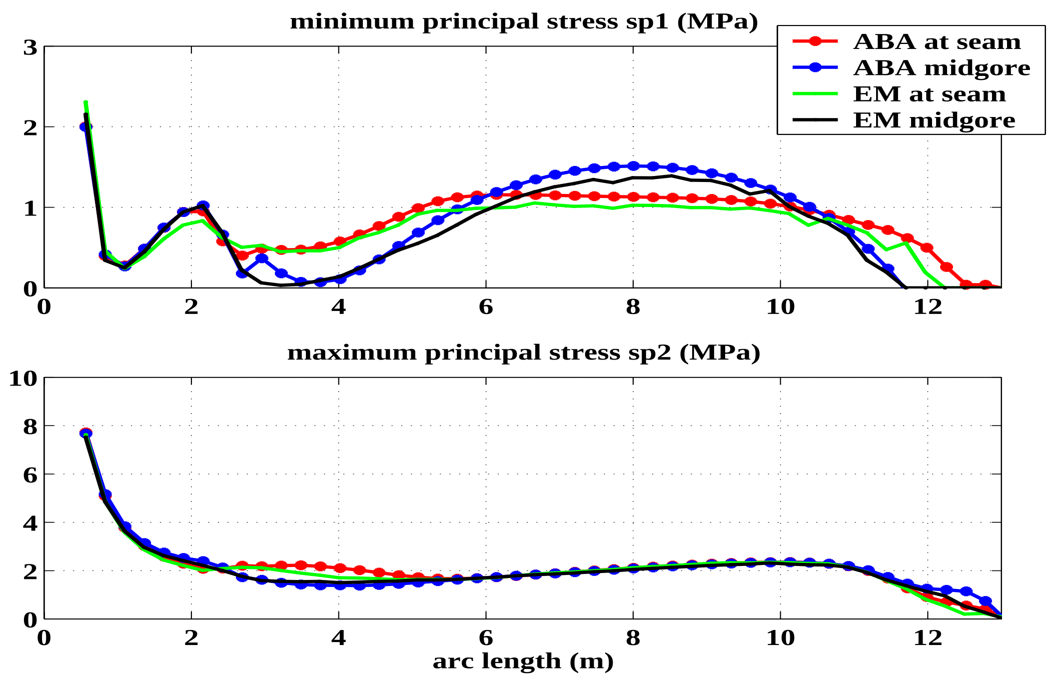


Principal Stress Resultants, $P(z) = 50$ Pa



Principal Stress Resultants

Open System: $P(0) = 96$ Pa, $P(2R) = 170$ Pa



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