



Solving Dynamic Problems Outside of NASTRAN

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PURPOSE

- **Describe method for solving dynamic problems using normal modes data**
- **Show how to export the necessary data from a NASTRAN normal modes run using DMAP**
- **Show that the calculation of dynamic response using modal data can be easily implemented outside of NASTRAN**
- **Provide examples of solving frequency response problems using a MATLAB like code called Octave**



Frequency Response - Equations

Basic equation of motion (ignoring damping)

$$[M]\{\ddot{X}\} + [K]\{X\} = F(\omega) \quad (1)$$

We can define a transform based on the system eigenvectors as

$$\{\dot{X}\} = [\phi]\{\dot{q}\} \quad \text{or} \quad \{X\} = [\phi]\{q\} \quad (2)$$

If we use the transform shown in (2) and pre-multiply by $[\phi]^T$, then we can write (1) as

$$[\phi]^T [M] [\phi] \{\ddot{q}\} + [\phi]^T [K] [\phi] \{q\} = [\phi]^T \{F(\omega)\} \quad (3)$$

If the eigenvectors are mass-normalized (which is typically the case), then (3) can be written as

$$[I]\{\ddot{q}\} + [\omega^2]\{q\} = [\phi]^T \{F(\omega)\} \quad (4)$$

If we include modal damping, then we can write the full dynamic equation of motion using generalized coordinates based on the system eigenvectors as

$$[I]\{\ddot{q}\} + [2\zeta\omega]\{\dot{q}\} + [\omega^2]\{q\} = \{\bar{F}(\omega)\} \quad (5)$$

where

$$\bar{F}(\omega) = \text{Generalized Force} = [\phi]^T \{F(\omega)\}$$

$$[I], [2\zeta\omega], [\omega^2] = \text{Diagonal matrices}$$

Each row of equation (5) represents the equation of a series of single DOF systems which can be written as

$$\ddot{q}_i + 2\zeta\omega_i \dot{q}_i + \omega_i^2 q_i = \bar{F}_i(\omega) \quad (6)$$

where $i = 1 \dots \text{Number of Modes}$

We can use standard methods for solving a second-order differential equation and write the solution to each of the single DOF systems given in equation (6) as

$$\ddot{q}_i = H_i(\omega) \bar{f}_i \quad (7)$$

where

$$H_i(\omega) = \frac{-(\omega/\omega_i)^2}{[1 - (\omega/\omega_i)^2 + i2\zeta(\omega/\omega_i)]}$$

$\bar{f}_i = \text{Amplitude of generalized force}$

We can solve equation (7) very easily using matrix methods to solve for the generalized acceleration response as

$$\begin{Bmatrix} \ddot{q}_1 \\ \vdots \\ \ddot{q}_n \end{Bmatrix} = \begin{bmatrix} \bar{f}_1 & & \\ & \ddots & \\ & & \bar{f}_n \end{bmatrix} \begin{bmatrix} H_1(\omega) \\ \vdots \\ H_n(\omega) \end{bmatrix} \quad (8)$$

We can use equation (8) to calculate the physical responses using the transform given in equation (2).

$$\{\ddot{X}\} = [\phi]\{\ddot{q}\} = [\phi] \begin{bmatrix} \bar{f}_1 & & \\ & \ddots & \\ & & \bar{f}_n \end{bmatrix} \begin{bmatrix} H_1(\omega) \\ \vdots \\ H_n(\omega) \end{bmatrix} \quad (9)$$

Equation (9) is fairly straightforward to implement in a simple program or even easier to write a small script for MATLAB/Octave. The information required from a NASTRAN normal modes run are the eigenvalues (ω^2) and the eigenvectors (ϕ) for the system that you wish to excite. This info can be output from NASTRAN using some fairly simple DMAP statements.



Base Drive - Equations

Write the dynamic equation of motion (without damping) in terms of boundary (r) dof and internal (l) dofs.

$$\begin{bmatrix} M_{rr} & M_{lr}^T \\ M_{lr} & M_{ll} \end{bmatrix} \begin{Bmatrix} \ddot{u}_r \\ \ddot{u}_l \end{Bmatrix} + \begin{bmatrix} K_{rr} & K_{lr}^T \\ K_{lr} & K_{ll} \end{bmatrix} \begin{Bmatrix} u_r \\ u_l \end{Bmatrix} = \begin{Bmatrix} F(\omega)_r \\ 0 \end{Bmatrix} \quad (10)$$

The lower part of equation (10) can be re-written as

$$M_{lr} \ddot{u}_r + M_{ll} \ddot{u}_l + K_{lr} u_r + K_{ll} u_l = 0 \quad (11)$$

If we break up the response of the internal degrees of freedom into rigid body motion and elastic response, we can write u_l as

$$u_l = u_l^r + u_l^e = \phi_r u_r + u_l^e \quad (12)$$

where

ϕ_r = Rigid body vector. Motion of l dofs due to motion at boundary

We can now rewrite equation (11) using the relationship defined in (12).

$$M_{ll} \ddot{u}_l^e + K_{ll} u_l^e = -[K_{lr} + K_{ll} \phi_r] u_r - [M_{lr} + M_{ll} \phi_r] \ddot{u}_r \quad (13)$$

Now we can show from statics that

$$[K_{lr} + K_{ll} \phi_r] = 0 \quad (14)$$

And that the coupling terms of M_{lr} are typically small so that we can rewrite equation (13) as

$$M_{ll} \ddot{u}_l^e + K_{ll} u_l^e = -[M_{ll} \phi_r] \ddot{u}_r \quad (15)$$

Equation (15) relates the internal elastic response of the structure to the prescribed acceleration at the boundary.

We can use the same modal transform shown in equation (2) using the fixed base eigenvectors to decouple the equations of motion given in equation (15) to write

$$[\phi_l^T M_{ll} \phi_r] \{\ddot{q}\} + [\phi_l^T K_{ll} \phi_r] \{q\} = -[\phi_l^T M_{ll} \phi_r] \ddot{u}_r \quad (16)$$

or if we include damping

$$\{\ddot{q}\} + [2\zeta\omega] \{\dot{q}\} + [\omega^2] \{q\} = [mpf] \ddot{u}_r \quad (17)$$

where

$$[mpf] = \text{mass participation factors} = -[\phi_l^T M_{ll} \phi_r]$$

Once again, each row of equation (17) is the equation of a single dof system. Using standard methods, the solution to each single dof system can be written as

$$\ddot{q}_i = H_i(\omega) [mpf_i] \ddot{u} \quad (18)$$

where

\ddot{u} = input acceleration magnitude

mpf_i = mass participation factor for mode i

$H(\omega)_i$ = SDOF transfer function (see eq. 7)

We can use matrix methods to solve equation (18) for the physical elastic acceleration responses as

$$\begin{Bmatrix} \ddot{X}^e \end{Bmatrix} = [\phi_l] \{\ddot{q}\} = [\phi_l] \begin{bmatrix} gf_1 & & \\ & \ddots & \\ & & gf_n \end{bmatrix} \begin{bmatrix} H_1(\omega) \\ \vdots \\ H_n(\omega) \end{bmatrix} \quad (19)$$

where

$gf_i = [mpf_i] \{\ddot{u}_r\} = \text{generalized force based on acceleration}$

The total physical response is given by

$$\ddot{X}_{tot} = \ddot{X}_l^e + \ddot{X}_l^r = \ddot{X}_l^e + \phi_r \ddot{u}_r \quad (20)$$

Equations (19) and (20) write the total physical response of the structure in terms of the input acceleration at the boundary. Once again, this problem can be solved externally from NASTRAN. In addition to the eigenvalues (ω^2) and eigenvectors (ϕ), you will also need to extract the mass participation factors and the rigid body vector. All of these quantities are easily extracted from a normal modes run with simple DMAP.



Software Used to Run Example Problems

- **MSC/NASTRAN Solution 3 and DMAP** alter to write out frequencies, mode shapes, rigid body vectors, and mass participation factors
- **C program “nas2mat”** which converts NASTRAN binary output4 data to binary MATLAB data
- **Octave** - Open source program for numerical computation and matrix math. It is MATLAB compatible and freely available at <http://www.octave.org> . Runs under UNIX and Windows OS.
- **Octave script files** for each test case

[illegible]



General Octave Scripts for Frequency Response

```
Function resp=ssr(phig,freq,lam,zeta,gf);  
% resp=ssr(phig,freq,lam,zeta,gf)  
%  
% by S. Gordon, July 1999  
  
H=sdoftf(lam,zeta,freq);  
gf1=diag(gf);  
mp=H*gf1;  
resp=mp*phig.';
```

```
function H=sdoftf(lambda,zeta,fr)  
% y=sdoftf(lambda,zeta,fr)  
%  
% by S. Gordon, July 1999  
  
omega=sqrt(lambda);  
fn=omega/(2*pi);  
freq=(fr(1):fr(2):fr(3));  
rat1=freq.'*(1./fn).';  
rat2=rat1.*rat1;  
H=1-rat2+i*2*zeta*rat1;  
H=-rat2./H;
```



Example NASTRAN Data Deck

```
ASSIGN OUTPUT4='bar_modes_lam.nas',STATUS=unknown,UNIT=21
ASSIGN OUTPUT4='bar_modes_mpf.nas',STATUS=unknown,UNIT=22
ID Bar,Modes
SOL 3
TIME 3
DIAG 8,14
INCLUDE 'modal_data.dmp'
CEND
    ECHO = NONE
    METHOD = 1
    SPC = 1
    DISP(PLOT)=ALL
$
BEGIN BULK
PARAM,AUTOSPC,YES
PARAM,WTMASS,.00259
PARAM,GRDPNT,0
$
EIGRL          1          10
SPC            1          1 123456      0.
$ FEMAP Property 1 : Round Bar
PBAR          1          1 3.14027  0.7854  0.7854 1.56947
$ FEMAP Material 1 : 6061-T651 Al
MAT1          19900000.      0.332.539E-4
    70.
$ Bulk data Follows
.
.
.
ENDDATA
```



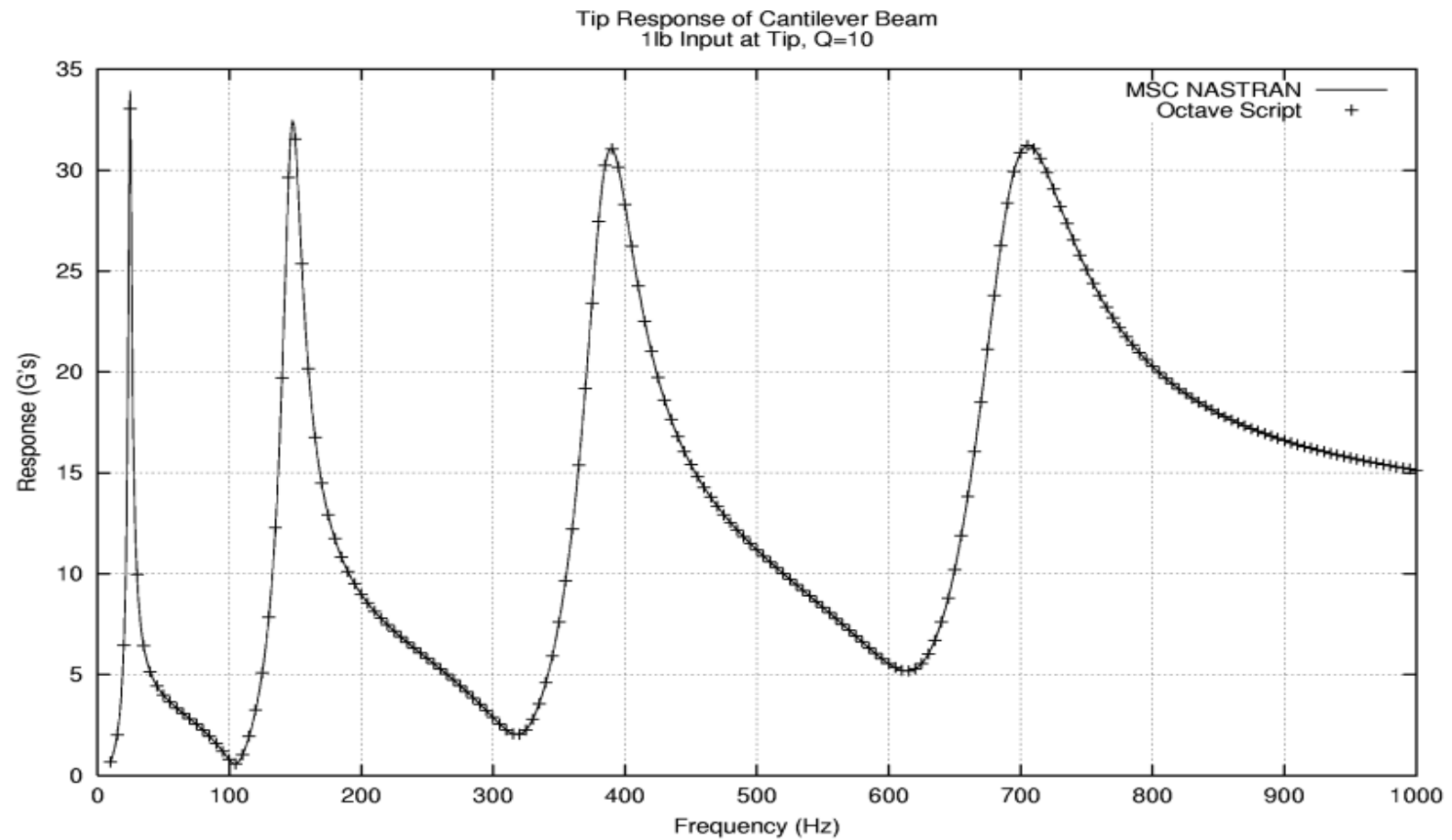

Test Case 1

```
%start
%
% Get Modal Data
load "bar_modes_lam.mat"
lam=RLAM;
phig=UGV1;
[nr,nc]=size(phig);
%
% Convert to Cycles Per Second (CPS)
fn=1./(2*pi)*sqrt(lam);
%
% Define Input Parameters
%10 to 1000 Hz in 1Hz incr.
freq=[10,1,1000];
% Use 5 percent damping (Q=10)
zeta=.05force=zeros(nr,1);
%Input 1lb force at dof 92
force(92)=1.0;
%
gf=phig.'*force; %Calculate generalize force
%
resp=ssr(phig,freq,lam,zeta,gf);
%
% Save Results
fv=(freq(1):freq(2):freq(3));
save -mat-binary test_case1_res.mat fv resp
```





Results - Test Case 1

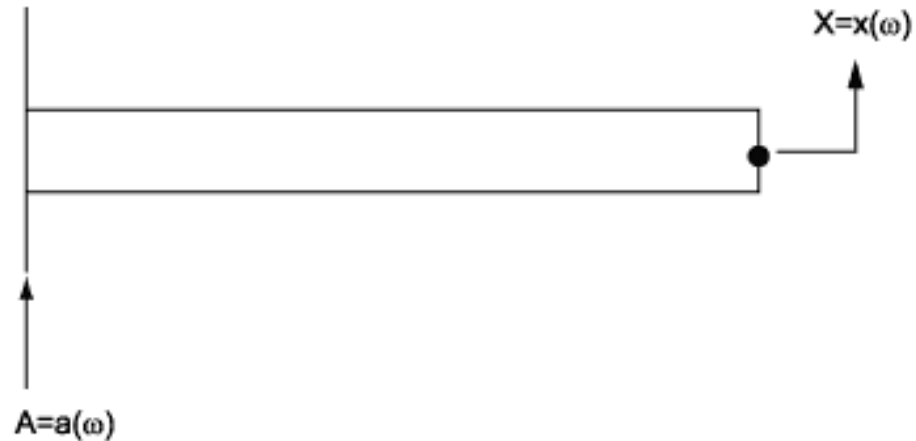




Test Case 2

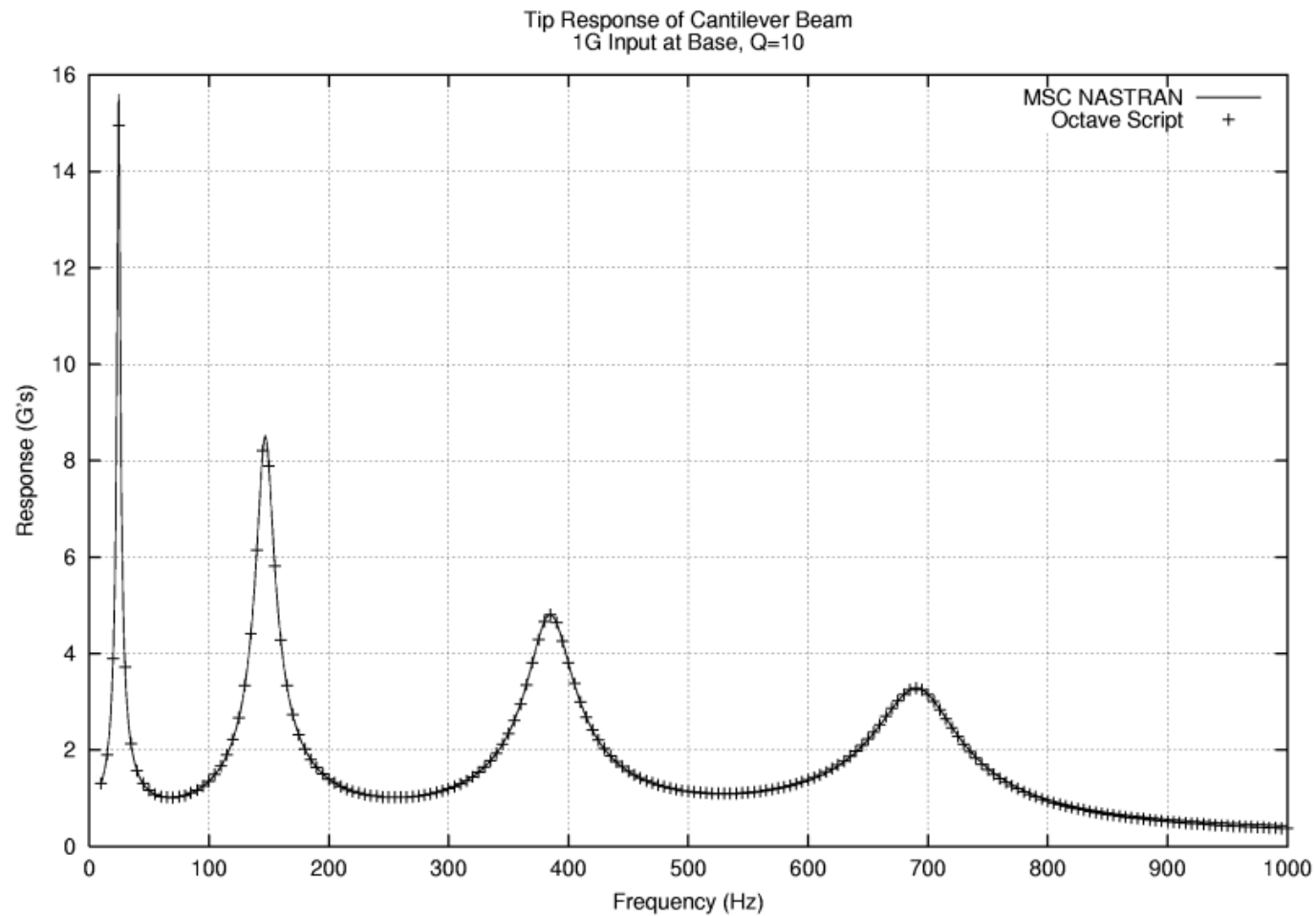
```
%start
% Load frequencies and eigenvectors
load bar_modes_lam.mat
lam=RLAM;
phig=UGV1;
clear RLAM UGV1;
%
% Get mass participation info
load bar_modes_mpf.mat
mpf=MPF;
rbvec=RBGL1;
clear MPF RBGL1;
%
mhb=mpf.';
%
%Convert eigenvalues to Hz
fn=1./(2*pi)*sqrt(lam);
%Input from 10 to 1000 Hz @ 1Hz Incr.
freq=[10,1,1000];
zeta=.05; %Use 5 percent damping (Q=10)
%
% Define Input Force
force=zeros(6,1);
force(2)=1.0;
gf=-1.0*mhb*force;
```

```
%
% Solve for Flexible Response
resp=ssr(phig,freq,lam,zeta,gf);
%
% Solve for Rigid Body Response
[r,c]=size(resp);
rb=rbvec*force;
rbr=diag(rb)*ones(c,r);
%
%Total response is sum of flexible + rigid body
total=rbr+resp.';
%
% Save Output
fv=(freq(1):freq(2):freq(3));
save -mat-binary test_case2_res.mat fv total
```





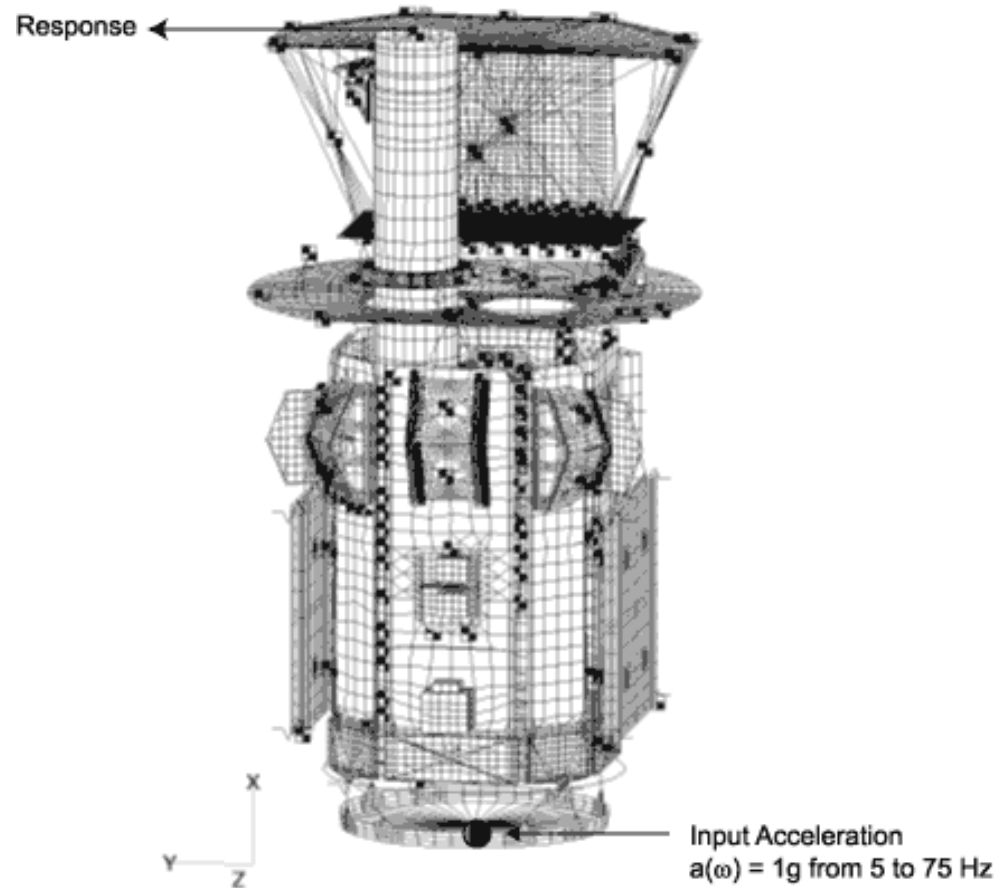
Results - Test Case 2





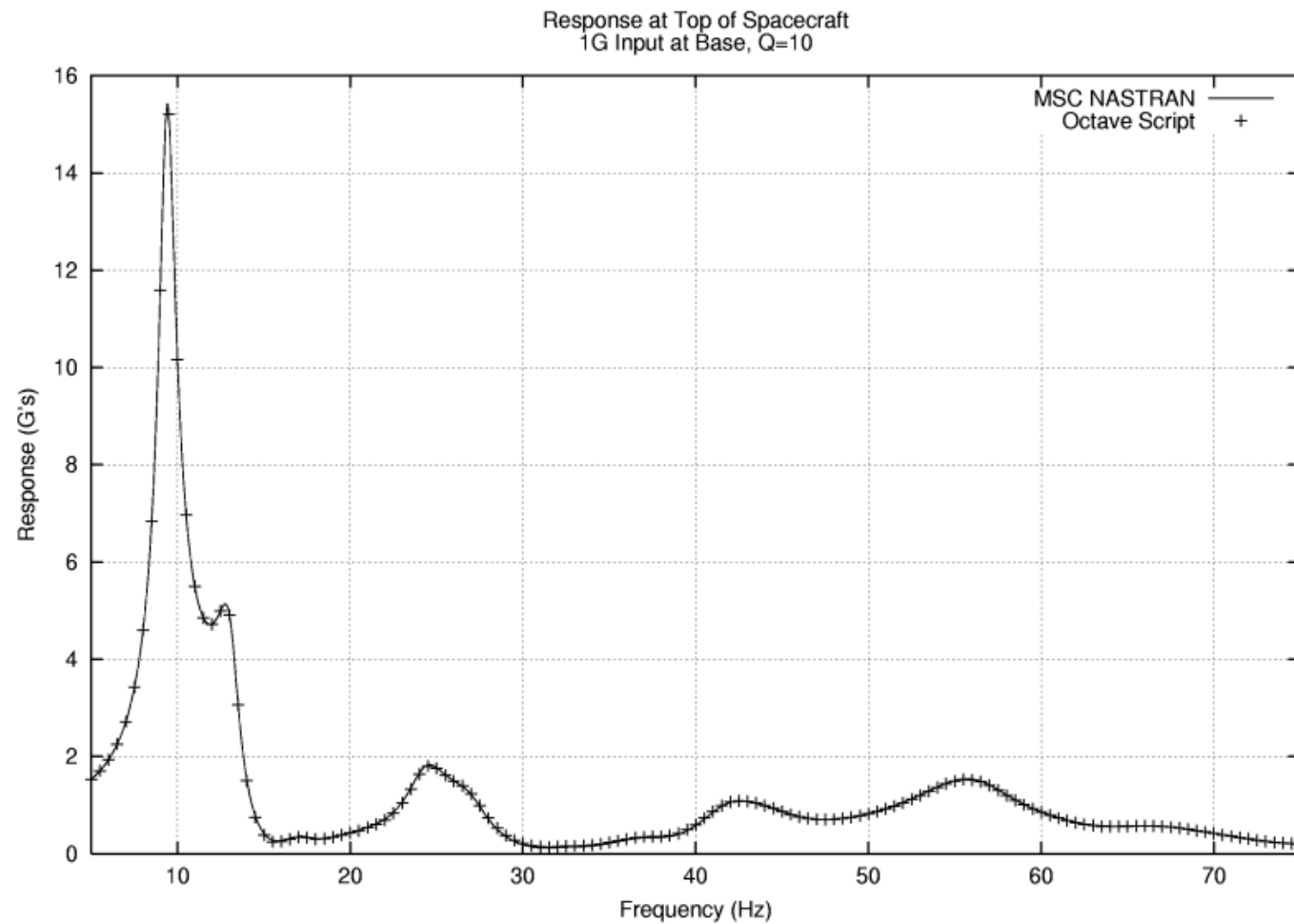
Test Case 3

```
%start
load swift20_obs_lam.mat
lam=RLAM;
phig=UGV1;
clear RLAM UGV1;
%
load swift20_obs_mpf.mat
mpf=MPF;
rbvec=RBGL1;
clear MPF RBGL1;
%
mhb=mpf.';
fn=1./(2*pi)*sqrt(lam);
[nr,nc]=size(fn);
%
freq=[5,0.1,75.];
zeta=.05;
%
force=zeros(6,1);
force(2)=1.0;
gf=-1.0*mhb*force;
% Solve for Flexible Response
resp=ssr(phig,freq,lam,zeta,gf);
% Solve for Rigid Body Response
[r,c]=size(resp);
rb=rbvec*force;
rbr=diag(rb)*ones(c,r);
total=rbr+resp.';
%
%Write Out Results
fv=(freq(1):freq(2):freq(3));
save -mat4-binary test_case3_res.mat fv total;
```





Results - Test Case 3





Summary/Conclusions

- **Once normal modes have been solved for in NASTRAN, it is fairly simple to extract necessary data and run frequency response analysis outside of NASTRAN**
- **Fairly straightforward matrix manipulation required to solve frequency response problem. Can easily be run in MATLAB, Octave or coded up in favorite programming language**
- **Examples shown here are for frequency response but similar approach can be used to solve transient or random vibration problem**
- **Significant processing power required to solve eigenvalue problem for large structures but once this is done dynamics solutions can be solved quickly on any CPU**
- **Methods applicable to models reduced using Craig-Bampton approach**