Using Plate Elements for Modeling Fillets in Design, Optimization, and Dynamic Analysis

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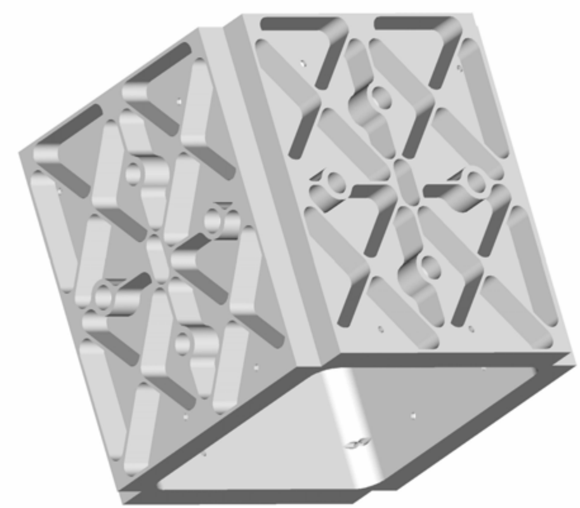
Motivation

- Many structures best modeled with plate elements
  - More accurate for "plate-like" structures
  - Significantly less computationally intensive – still important for optimization, dynamic analysis, small business
- Presence of fillets, however, requires solid modelling
  - Automeshing thin-walled structures with solids problematic
  - Manually meshing fillets with solids not trivial

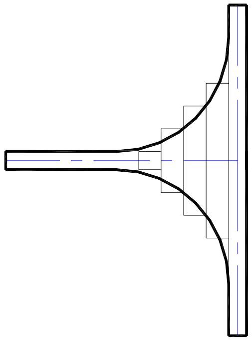


- Can ignore fillet frequently, but *not always*
  - Significantly underpredict stiffness for thin-walled structures

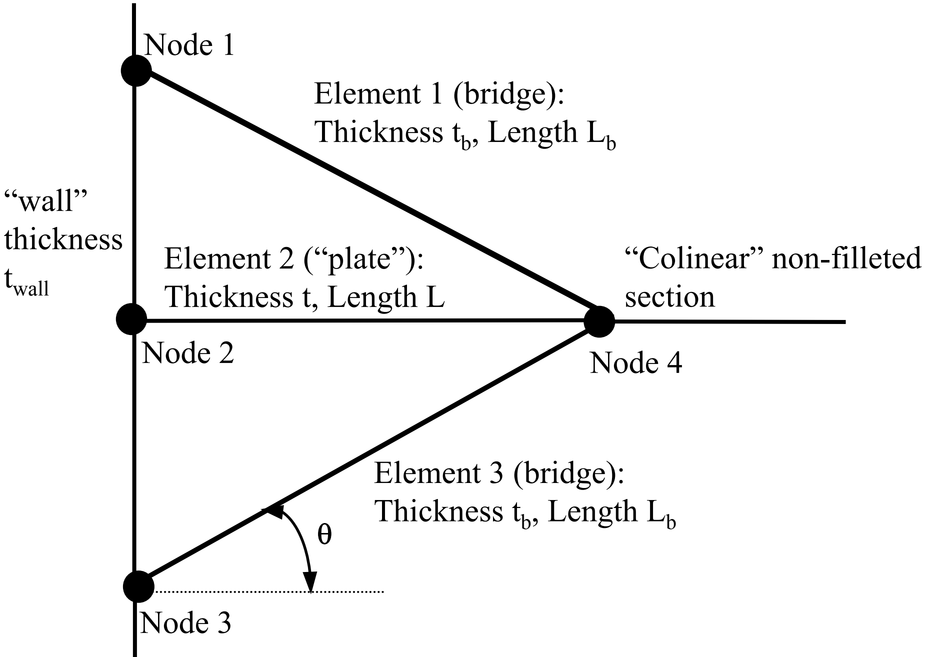
Solution & Methodology

- **Goal**: Generate simplified technique for modeling fillets for use in design, dynamic analysis.
- Literature search performed on analytical methods for natural frequencies of plates with variable cross section.

- Evaluated several options
  - NASTRAN "GENEL" explicit element
  - "stepped-up thickness" method



- Decided on "Bridge Element" method.



Theoretical Development - Bridge

- First recognize that "wide-beam" theory allow use of beam equations for plate deflection under plane strain:

$$E_{wide-beam} = \frac{E}{1-\nu^2}$$

- Generate Stiffness matrix partition for node 4 of bridge configuration:

$$[K]_{44} = \frac{1}{L}\left[\begin{array}{c|c} \frac{E_b t_b}{\sqrt{2}} + \frac{E_b t_b{}^3}{2L^2\sqrt{2}} + \frac{Et^3}{L^2} & -\frac{E_b t_b{}^3}{2L\sqrt{2}} - \frac{Et^3}{2L} \\ \hline sym & \frac{2E_b t_b{}^3}{3\sqrt{2}} + \frac{Et^3}{3} \end{array}\right]$$

- Equation for rotation at node 4 in form of coefficients of load P and moment M in terms of unknowns t_b and E_b:

$$\theta_4 = f(E, E_b, t, t_b, t_{wall}, r) * P + f(E, E_b, t, t_b, t_{wall}, r) * M$$

Theoretical Development – plane fillet

- For actual fillet, express rotation at tangent point as

$$\theta_f = \int \frac{(M + P * r)}{EI_f} dx + C$$

- Where

$$h_f = r - \sqrt{x(2r - x)} \qquad t_f = 2\left(\frac{t}{2} + h_f\right) \qquad I_f = \frac{bt_f^3}{12}$$

- Applying boundary condition, obtain nonlinear expression for rotation at tangent:

$$\theta_f = f(E, E_b, t, t_b, t_{wall}, r) * P + f(E, E_b, t, t_b, t_{wall}, r) * M$$

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Equations for E_b and t_b

- Equate the coefficients of P & M in each expression for θ_f

$$\frac{\frac{e\,t^3}{\left(r+\frac{\text{twall}}{2}\right)^2}+\frac{\text{eb}\,\text{tb}^3}{2\sqrt{2}\left(r+\frac{\text{twall}}{2}\right)^2}+\frac{\text{eb}\,\text{tb}}{\sqrt{2}}}{\left(\frac{e^2\,t^6}{12\left(r+\frac{\text{twall}}{2}\right)^4}+\frac{e\,\text{eb}\,\text{tb}^3\,t^3}{3\sqrt{2}\left(r+\frac{\text{twall}}{2}\right)^4}+\frac{e\,\text{eb}\,\text{tb}\,t^3}{3\sqrt{2}\left(r+\frac{\text{twall}}{2}\right)^2}+\frac{\text{eb}^2\,\text{tb}^6}{24\left(r+\frac{\text{twall}}{2}\right)^4}+\frac{\text{eb}^2\,\text{tb}^4}{3\left(r+\frac{\text{twall}}{2}\right)^2}\right)\left(r+\frac{\text{twall}}{2}\right)}=$$

$$\frac{6\left(8r^2\sqrt{4r+t}\,t^{3/2}+2r\sqrt{4r+t}\,t^{5/2}+12r^3\sqrt{4r+t}\sqrt{t}+3\pi r^2(2r+t)^2+6r^2(2r+t)^2\tan^{-1}\left(\frac{2r}{\sqrt{t}\sqrt{4r+t}}\right)\right)}{e\,t^{5/2}(2r+t)(4r+t)^{5/2}},$$

$$-\frac{-\frac{e\,t^3}{2\left(r+\frac{\text{twall}}{2}\right)}-\frac{\text{eb}\,\text{tb}^3}{2\sqrt{2}\left(r+\frac{\text{twall}}{2}\right)}}{\left(\frac{e^2\,t^6}{12\left(r+\frac{\text{twall}}{2}\right)^4}+\frac{e\,\text{eb}\,\text{tb}^3\,t^3}{3\sqrt{2}\left(r+\frac{\text{twall}}{2}\right)^4}+\frac{e\,\text{eb}\,\text{tb}\,t^3}{3\sqrt{2}\left(r+\frac{\text{twall}}{2}\right)^2}+\frac{\text{eb}^2\,\text{tb}^6}{24\left(r+\frac{\text{twall}}{2}\right)^4}+\frac{\text{eb}^2\,\text{tb}^4}{3\left(r+\frac{\text{twall}}{2}\right)^2}\right)\left(r+\frac{\text{twall}}{2}\right)}=$$

$$\frac{6\left(4(r^2)^{3/2}\sqrt{4r+t}\,t^{3/2}+4r^3\sqrt{4r+t}\,t^{3/2}+r^2\sqrt{4r+t}\,t^{5/2}+8r^4\sqrt{4r+t}\sqrt{t}+8r^3\sqrt{r^2}\sqrt{4r+t}\sqrt{t}\right)}{e\,t^{5/2}(2r+t)(4r+t)^{5/2}}$$

- Solve for t_b/t and E_b/E using Mathematica™ 4.1

Results

- Surface fit for each generated to fit data for easy implementation:

$$\frac{E_b}{E} = \frac{0.9782601242110346}{x^2} - \frac{0.7149708253246347}{x^3} - \frac{1.9678245005077897}{x^4} + \frac{1.4899111209264795}{x^5} + \frac{0.045579223088219975}{y} + \frac{0.7522289111879881}{x\,y} + \frac{2.088608970319005}{x^2\,y} + \frac{3.898702480893012}{x^3\,y} - \frac{0.3476295889834863}{y^2} - \frac{1.5284136218083295}{x\,y^2} - \frac{1.5528800000806626}{x^2\,y^2} + \frac{0.784033948805702}{y^3} + \frac{0.8445177307299556}{x\,y^3} - \frac{0.6771296045396015}{y^4} + \frac{0.19714890087642176}{y^5} - \frac{0.08229688966201235}{x}$$

$$\frac{t_b}{t} = -0.000704997\,x^2 - 0.00390799\,yx + 0.316686\,x - 0.00362814\,y^2 + 0.200802\,y - 0.297332$$

where x=r/t and y=t_{wall}/t

- Matrix of exact results also generated for $E_b/E = f(r/t, t_w/t)$, $t_b/t = f(r/t, t_w/t)$.
- For accurate dynamic analysis, pseudo density of bridges also calculated:

$$\rho_b = \frac{t\,4\sqrt{2}}{r(4-\pi)}\,\rho_f$$

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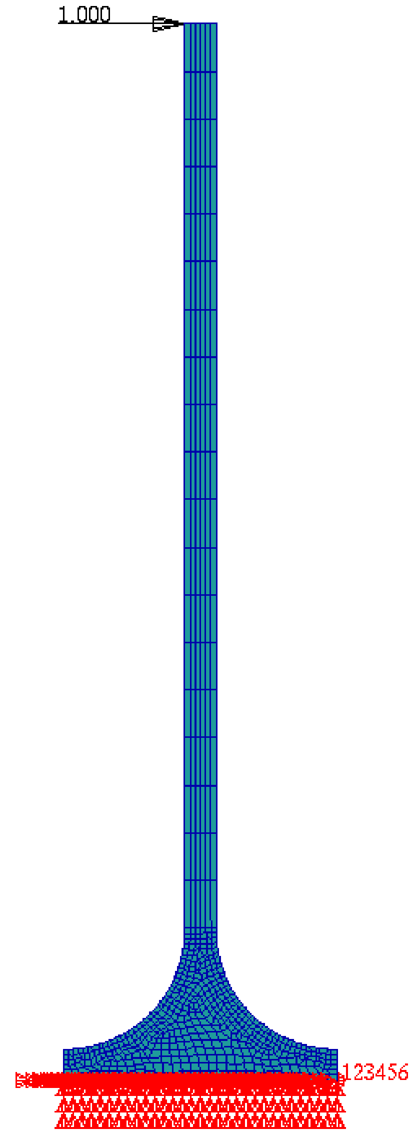
Validation – Sub-Component Level

- "Wide-beam" and plane-strain assumptions validated with f.e. models.
- Fillet tangent rotation equation using beam theory compared with f.e.; 7.8% error



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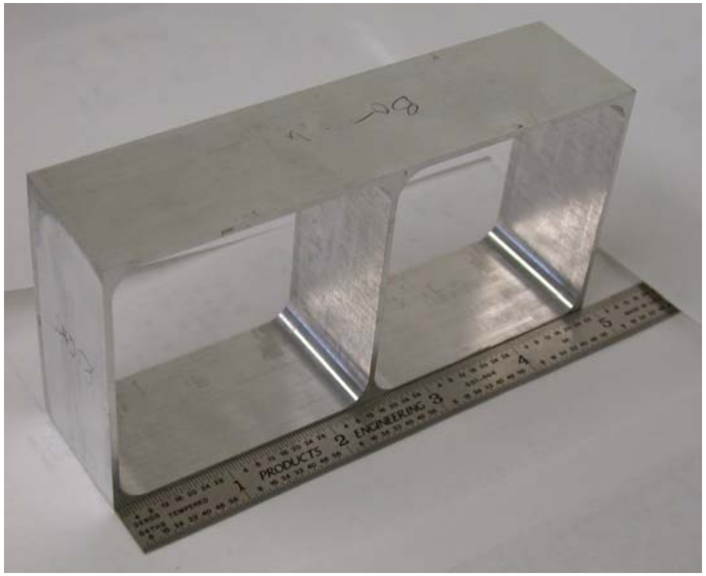
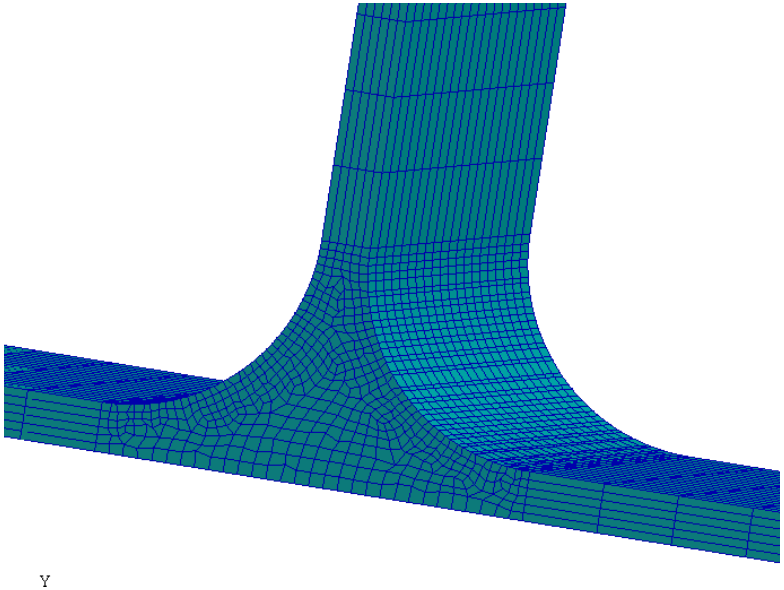
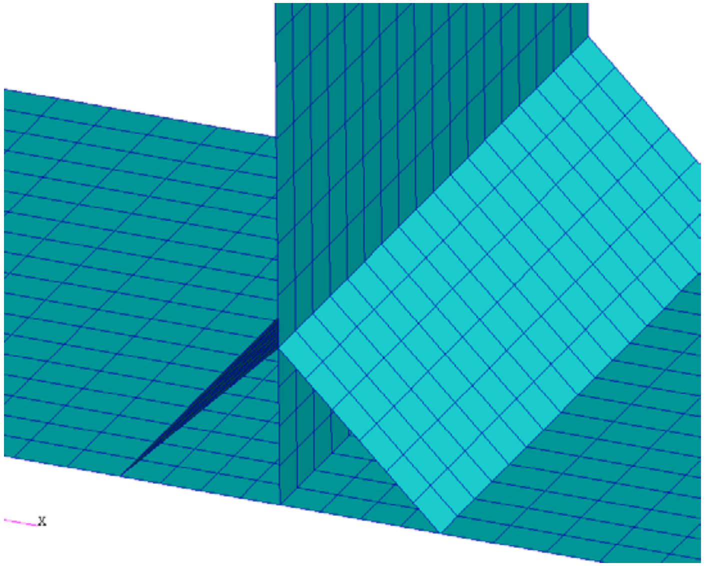
Validation – Structural Level

- Representative thin-walled structure fabricated, modeled, modal tested.
- Dense solid model, plate model with "bridge" fillets, plate model ignoring fillets altogether.







Excellent Correlation, 90% DOF Reduction

- Mode shapes consistent
- Error of “bridge” plate model same as solid model
- Error of plate model ignoring fillets substantial.
- Solid model - 257,600 dofs
- Plate “bridge” model 22,325 dofs

free-free mode number	test freq's (baseline)	solid model	frequency error	plate element model with "bridge" fillets	error	plate element model without fillets	error
7	484.0	487.6	0.8%	494.6	2.2%	449.1	-7.2%
8	540.0	534.3	-1.1%	541.0	0.2%	467.6	-13.4%
9	632.0	627.4	-0.7%	633.8	0.3%	547.1	-12.8%
10	1012.0	999.7	-1.2%	998.7	-1.3%	1000.7	-1.1%
11	1412.0	1403.4	-0.6%	1408.8	-0.2%	1282.2	-9.2%
12	1900.0	1903.6	0.2%	1921.8	1.1%	1687.7	-11.2%
13	2320.0	2290.8	-1.3%	2322.4	0.1%	1992.3	-14.1%
14	2810.0	2792.6	-0.6%	2806.6	-0.1%	2291.4	-18.5%

Conclusions & Future Work

- Methodology developed that uses plate instead of solid elements for modeling structures with 90º fillets
- Technique uses plate “bridges” with pseudo Young’s Modulus, thickness, density.
- Verified on typical filleted structure
  - accuracy better than or equal to a high-fidelity solid model
  - 90% reduction in dof’s.
- Application of method for parametric design studies, optimization, dynamic analysis, preliminary stress analysis
- Future work: extend theory to fillet angles other than 90°, multi-faceted intersections.

Reference

- Brown, A. M., Seugling, R.M. "Using Plate Finite Elements for Modeling Fillets in Design, Optimization, and Dynamic Analysis," NASA TP-2003-212340, March, 2003
- Found on http://trs.nis.nasa.gov/