Structural Analysis & Synthesis Tools for Solar Sails

Michael Lou and Houfei Fang
Jet Propulsion Laboratory
California Institute of Technology

Bingen Yang
University of Southern California

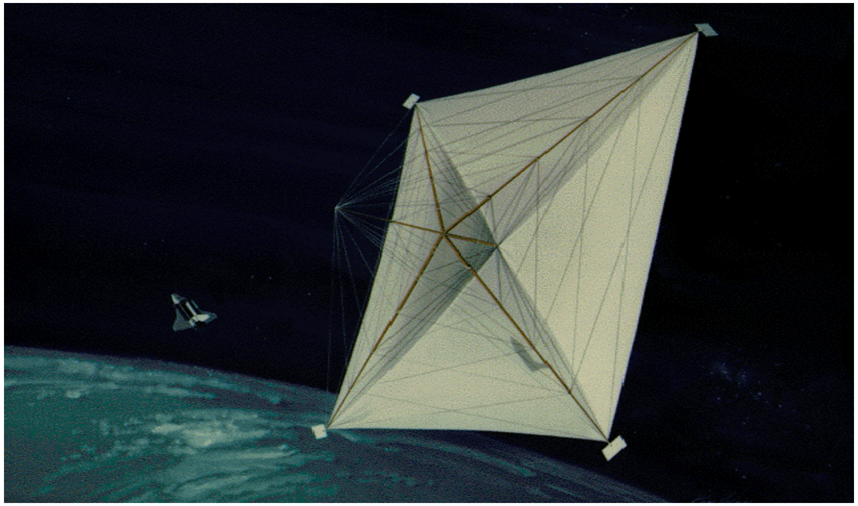
A Presentation at the NASA Workshop on Structural and Testing, GSFC, Greenbelt, MD., May 5 & 6, 2004.

OUTLINE

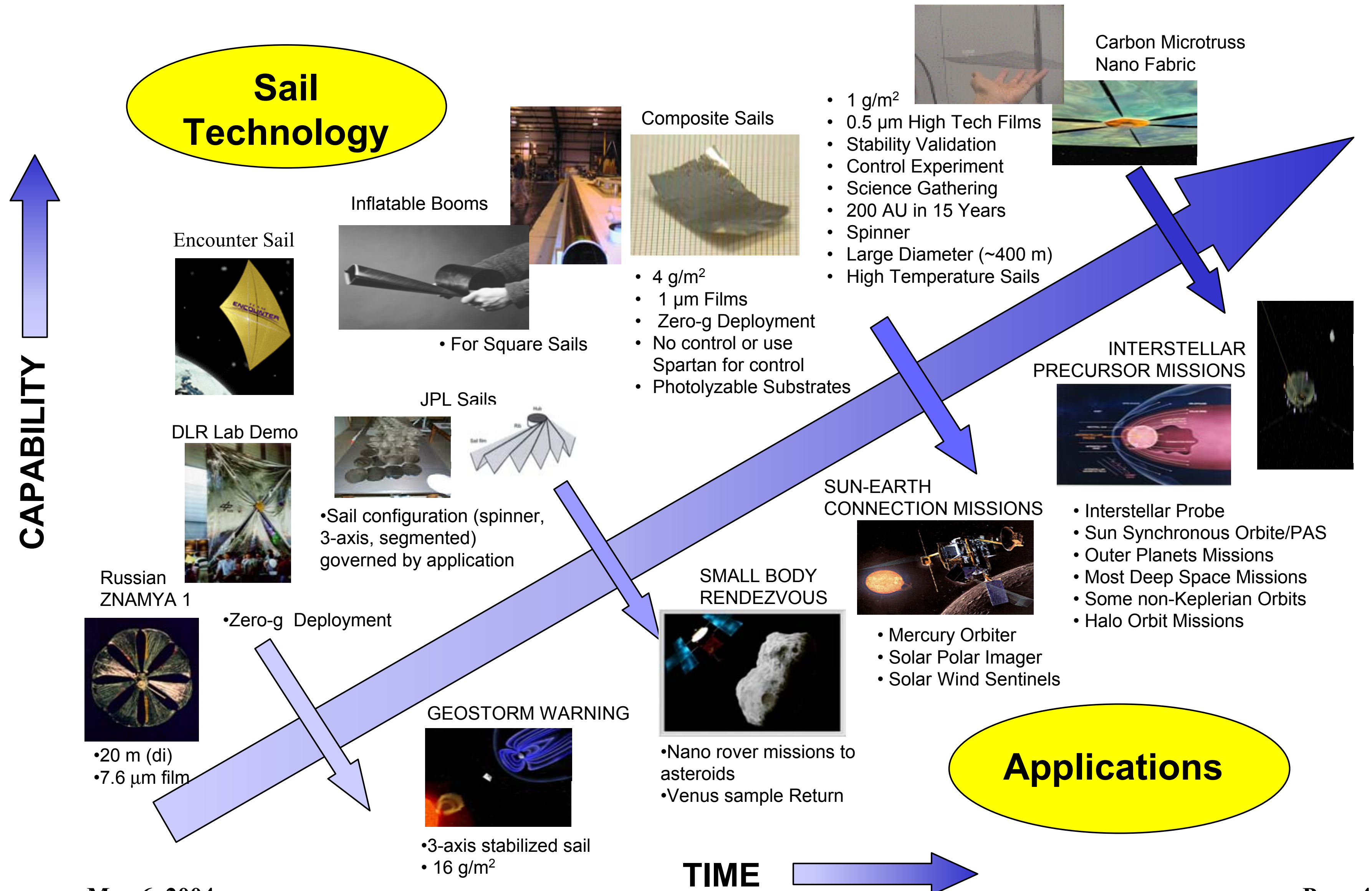
- Background/Motivation
- Issues with Solar Sail Booms Modeling
- Issues with Solar Sail Membrane Analysis
- Technical Objectives
- Technical Approach
  - DTFM Modeling of Booms
  - PVP Analysis of Wrinkled membranes
  - DTFM System Model Synthesis
- Status and Concluding Remarks

Background/Motivation

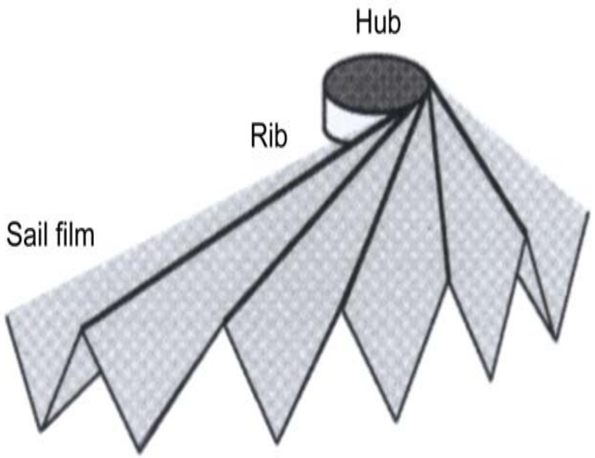
Many future space missions currently planned by NASA require large lightweight space deployable structures, including solar sails that have thin-film membrane apertures of hundreds of square meters.



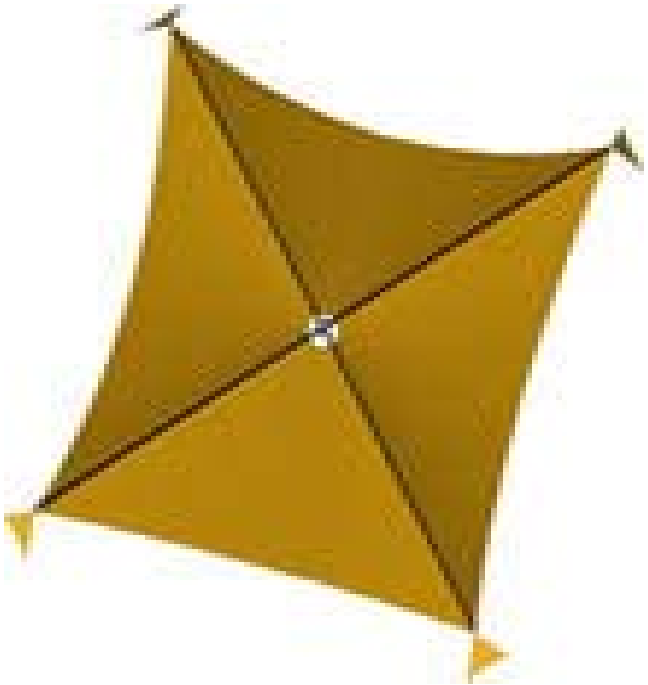
Structural Analysis & Synthesis Tools for Solar Sails



Typical Solar Sail Designs



A Spin Sail

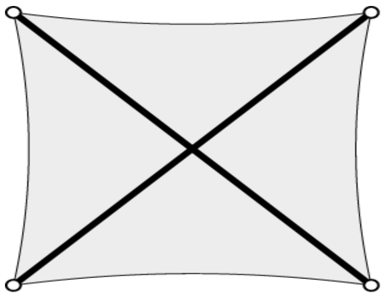


A Square Sail

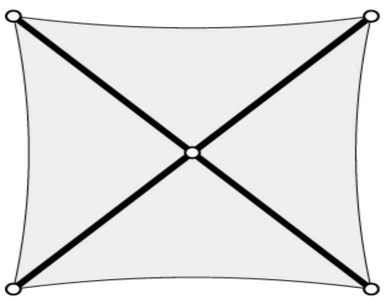
Background/Motivation (cont'd)

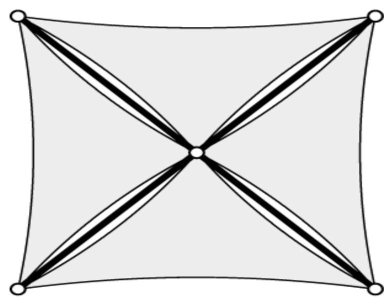
- Ground testing of these ultra-large and ultra-lightweight solar sails, due mainly to the effects of gravity and air damping, is very difficult or even impossible.
- Design verification and prediction of in-space performance of solar sail structures can only be achieved by modeling and analysis.
- Commonly used FEM structural analysis codes were found to have certain shortcomings for treating this class of structures.
- NASA's Space Transportation Technology Program has initiated efforts to develop improved computational capabilities.
- The same program has also funded AEC-Able and L'Garde to design, fabricate, and assemble two 20-meter class sails for ground demo and testing, including dynamic testing in GRC's Plum Brook 100-ft-diameter vacuum chamber.

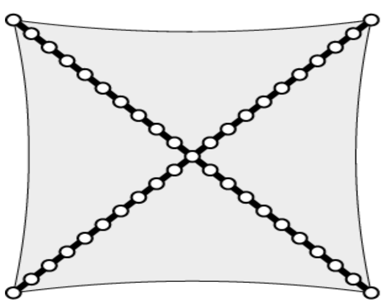
Structural Analysis & Synthesis Tools for Solar Sails

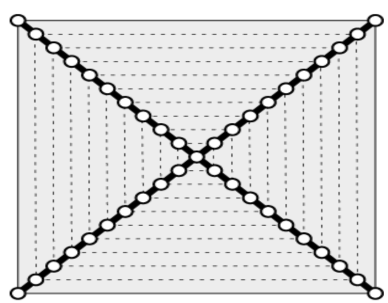

(a) Four-point suspension.

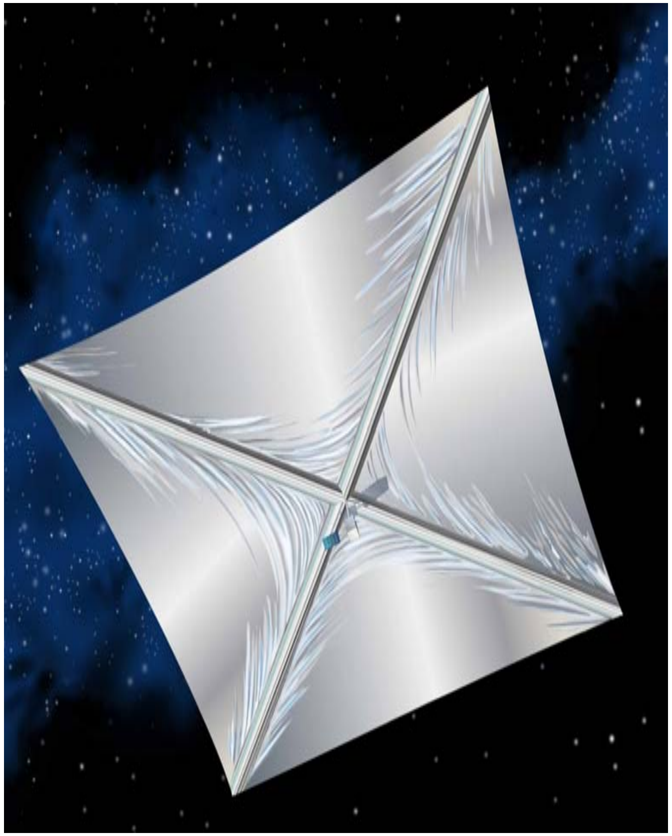
Legend:
- sail suspension from boom
- sail surface edge
- strip borders / stress direction
- boom


(b) Five-point suspension.

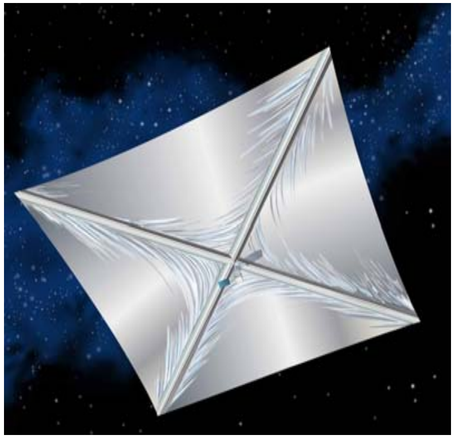

(c) Separate quadrants.


(d) Continuous connection.


(e) Stripped architecture.

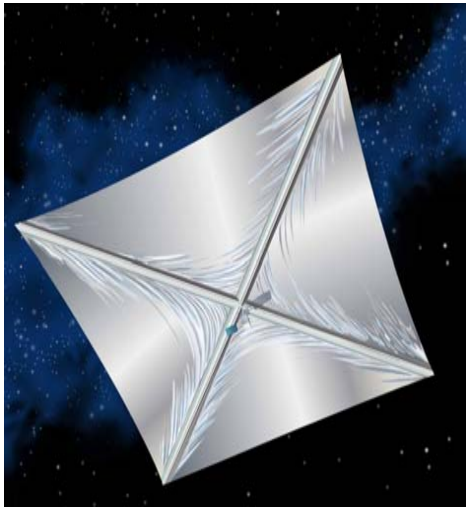


Issues for Solar Sail Booms



- Structural supports of square solar sails are typically very long booms that subjected mainly to axial compressions.
- Buckling capability of a long boom is highly sensitive to both the overall straightness, and the presence of any local material and/or geometrical imperfections.
- Investigation of the effects of local imperfections by the traditional FEM methods will require an extremely large number of elements to capture the details of the imperfections - Computationally inefficient and may exceed the computational capabilities readily available to most engineers.

Issues for Solar Sail Thin-Film Membranes



- The membrane aperture of a square sail needs to be tensioned by the booms to maintain flatness.
- A thin-film membrane subjected to in-plane tensioning will typically have three distinct regions: the taut, wrinkled (rippled), and slack areas.
- Wrinkling is a local buckling phenomenon introduced by uneven tensioning or shearing of the membrane.
- A slack area in a membrane has zero stiffness and will cause numerical instabilities for most, if not all, existing FEM structural analysis codes.
- No user-friendly analysis codes are currently available for effective treatment of wrinkled membranes.

Technical Approach (cont'd)

- Employ two innovative solution methods:
  - Distributed Transfer Function Method (DTFM)
  - Parametric Variational Principle (PVP) Method
- Focus on overcoming the identified shortcomings of commonly used FEM structural analysis codes (NASTRAN, ANSYS, ABAQUS, etc) in treating long booms and tensioned membranes. These shortcomings include low modeling efficiency and convergence problems.
- Include the treatment of:
  - Geometrical and material imperfections of booms
  - Forming and structural effects of wrinkles
- Emphasize on computational efficiency and user-friendliness – Leads to the selection of "MatLab" programming.

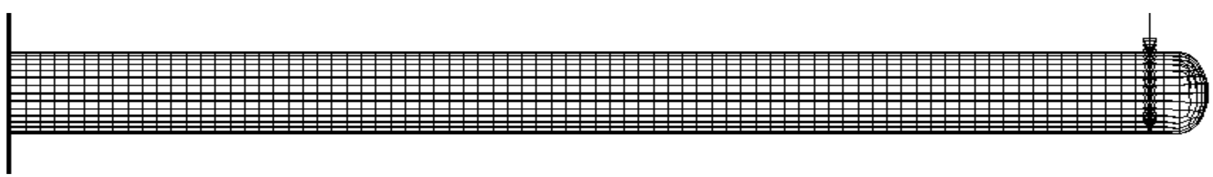
Distributed Transfer Function Method (DTFM)

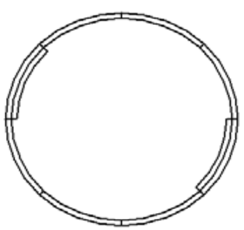
- The DTFM is as versatile as the FEM methods in modeling structures of complex configuration and boundary conditions.
- A major difference is that the FEM uses shape functions to analytically represent the elements and the DTFM uses distributed transfer functions to represent the components.
- The higher-level analytical formation makes the DTFM computationally more efficient and numerically stable.
- In synthesizing the system model of a multi-component structural system, the DTFM can use exact, closed-form representations for 1-D components and semi-analytical solutions for 2-D components.

Distributed Transfer Function Method

Shortcomings of using FEM to model A Long Boom

1) Tens of thousands elements are needed due to:
    - Accuracy requirement
    - Aspect ratio (a/b) limitation
2) Time-domain solutions ⇒ require small time steps for convergence ⇒ excessive computation time



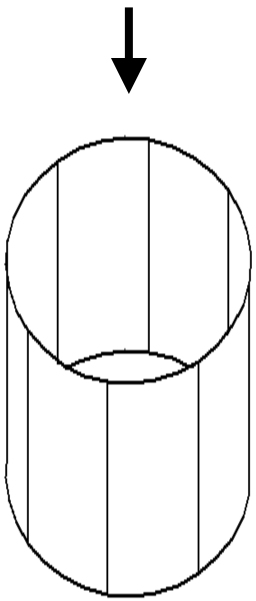


b

a

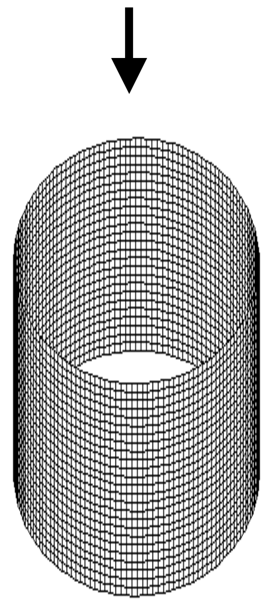
Comparison Between DTFM and FEM Modeling of A Spring-Tape-Reinforced (STR) Boom

Buckling Force



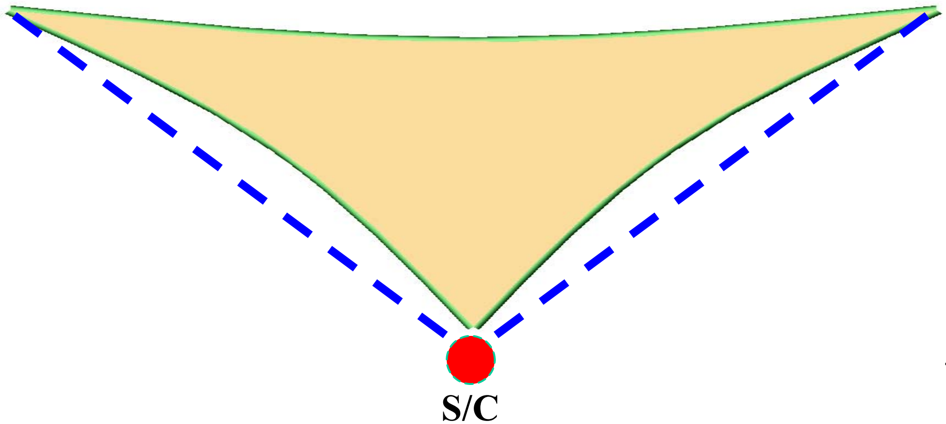
Strip Number	Buckling Force
2	381.28
4	381.28
6	381.28
8	381.28

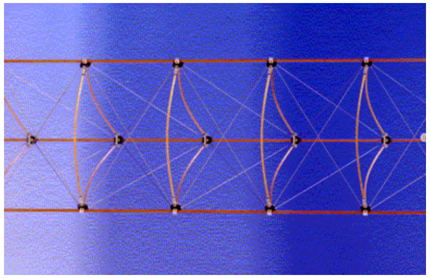
Buckling Force



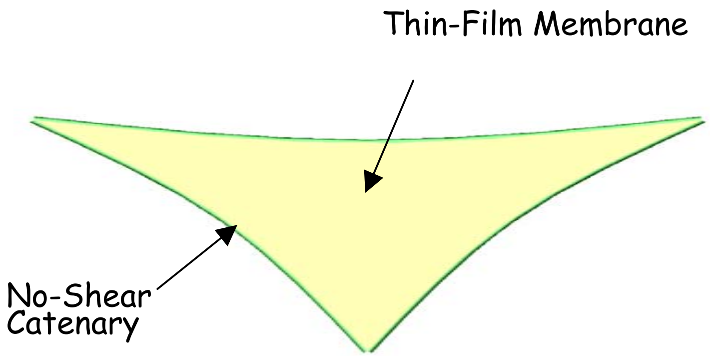
Element Number	Buckling Force
54	1269.6
218	393.3
864	386.8
3456	381.4

AEC-Able/SRS Sail Design
(1/4 Shown)





Multi-Bay Coilable Boom



Thin-Film Membrane Aperture w/No-Shear Catenary

Wrinkling Analysis Using the PVP Method

Structural analysis of a thin-film membrane involves three steps:

1. Develop a membrane model that does not have any out-of-plane stiffness.
2. Pre-tension the membrane to get the out-of-plane stiffness, namely, the differential stiffness - A function of the in-plane stress distribution.
3. Applying external loads, such as solar pressure, to the pre-tensioned membrane.

A particle point in the membrane is in one the following states defined by the principal stresses:

Taut state (S1): $\sigma_1 > 0, \quad \sigma_2 > 0$

Wrinkled state (S2): $\sigma_1 > 0, \quad \sigma_2 = 0$

Slack state (S3): $\sigma_1 = 0, \quad \sigma_2 = 0$

Wrinkling Analysis Using the PVP Method (Cont'd)

The PVP solution approach defines the material property constitutive law for a membrane as:

$$\tilde{E} = \frac{E}{1+\lambda_1}, \quad \tilde{\mu} = \mu + \lambda_2$$

λ_1 and λ_2 are nonnegative parameters

Three states in the membrane are defined as:

Taut state (S1): $\lambda_1 = 0, \quad \lambda_2 = 0$
Wrinkled state (S2): $\lambda_1 = 0, \quad \lambda_2 > 0$
Slack state (S3): $\lambda_1 > 0, \quad \lambda_2 > 0$

Wrinkling Analysis Using the PVP Method (Cont'd)

The potential energy functional of the membrane is then derived as:

$$\Pi_\lambda\left(\{u\}\right)=\int_\Omega \frac{1}{2}\{\varepsilon\}^T\left[\tilde{D}(\lambda_1,\lambda_2\right]\{\varepsilon\}h\,d\Omega-\int_{S_\sigma}\left\{\overline{T}\right\}^T\{u\}\,dS$$

The original wrinkling problem of the wrinkled membrane is equivalent to a nonlinear complementary problem (NCP) described as:

$$\Gamma\left(\{\lambda\}\right)\cdot\{\lambda\}=0$$

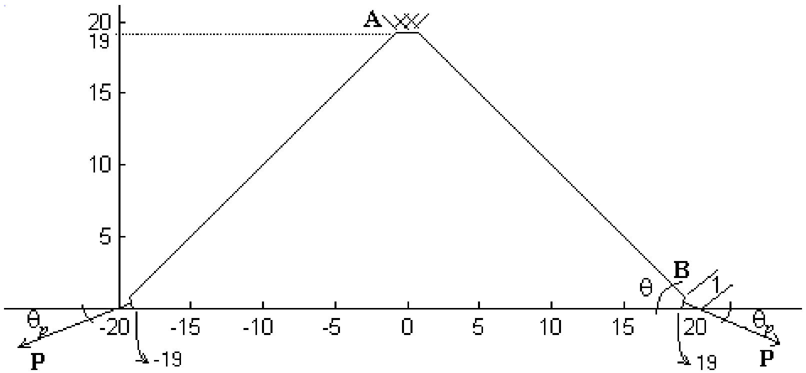
This NCP can be solved by the Smoothing Newton Method.

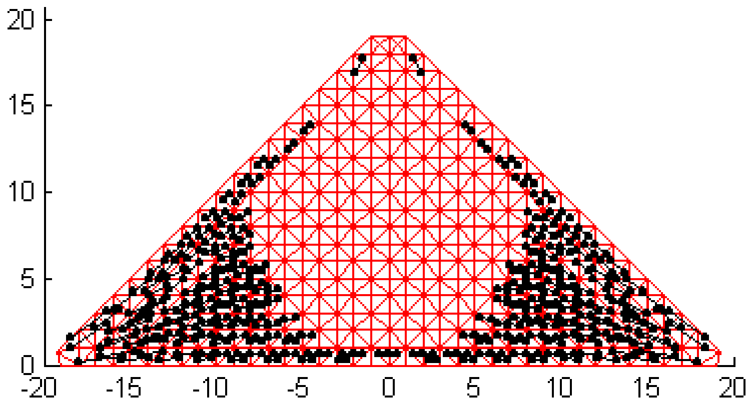
Wrinkling Analysis Using the PVP Method (Cont'd)

The PVP method for wrinkling analysis takes the following three steps:

1. Solve the NCP for viable parameters $\{\lambda\}$.
2. With $\{\lambda\}$ obtained, compute the membrane displacements.
3. With $\{\lambda\}$ and membrane displacements, compute the in-plane stress distribution of the membrane, and determine the wrinkled and slack regions.

Wrinkling Analysis Using the PVP Method (Cont'd)





- Solar sail membranes are extremely large and thin with very low in-plane stresses - Easy to wrinkle and to have many slack regions.
- The iteration-based methods used to treat solar sail membranes has the tendency to experience numerical instability and fail to converge.
- The PVP solution method has been proven computationally more efficient and without problems of numerical instability.

Wrinkling Analysis Using the PVP Method (Cont'd)

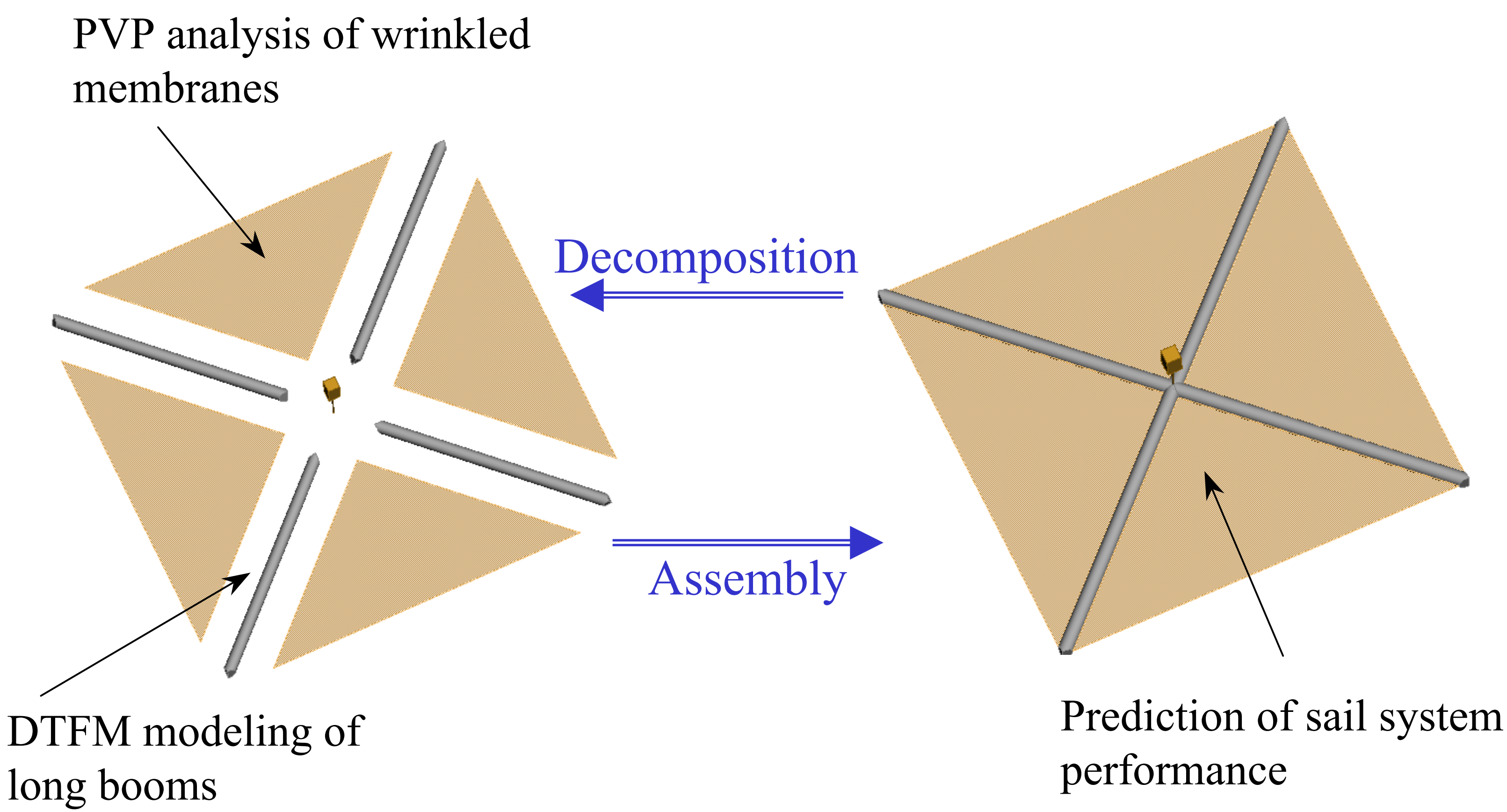
To calculate the out-of-plane deformation of a wrinkled membrane, we incorporate the in-plane stresses in the compatible equation of strain components, which leads to:

$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} = \phi(\sigma_x,\, \sigma_y,\, \tau_{xy})$$

w is the transverse (out-of-plane) displacement of the membrane.

This is a nonlinear Monge-Ampere equation. The solution of this equation exists and is unique.

Synthesis of A Sail System Model



Synthesis of A System Model (Cont'd)

Step one--Decomposition:

The sail is decomposed into components - booms, apertures, and S/C bus.

Step two--Component Representation:

The components are analytically represented by its distributed transfer functions in terms of unknown displacement parameters at the nodal points (interfacing points). This results in a nodal displacement representation of the component:

$$\{q_e\} = [K_e]\{u_e\}$$

Step 3--Assembly:

The components are re-assembled by imposing force balances and displacement compatibility at the interfacing points, leading to a global dynamic equilibrium equation:

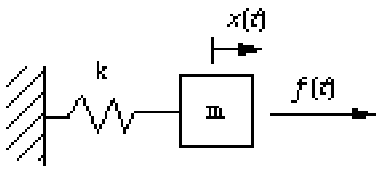
$$\{q\} = \left[K_g\right]\{u\}$$

Technical Approach - Deployment Analysis

- The deployment of a solar sail will be controlled by the deployment of its supporting booms.
- A deploying boom will be represented by a series of beam elements.
- These beam elements will have the same stiffness as the boom during the deployment process. (Determined by testing boom samples)
- A kinematical analysis model of the motions of these beams can then be assembled to analytically simulate the deployment of the boom.
- The unfolding membrane aperture is assumed to have only limited effects on sail deployment. If needed, it can be analytically represented by inertia and resistance forces. (Determined by testing).

Transfer Function Solutions
(Time Domain vs. Laplace Domain)

Step 1. Obtain governing equation by first laws



$$m\ddot{x} + kx = f$$

Step 2. Take Laplace transform of the equation

$$ms^2 X(s) + kX(s) = F(s)$$

$$X(s) = \int_0^\infty x(t)e^{-st}dt \quad \text{with} \quad d/dt \leftrightarrow s$$

Differential equation reduced to algebraic equation

Step 3. Derive dynamic stiffness and transfer function

$$K_{ds}(s)X(s) = F(s)$$

where dynamic stiffness $K_{ds}(s) = ms^2 + k$

$$X(s) = G(s)F(s)$$

where transfer function

$$G(s) = 1/K_{ds}(s) = 1/(ms^2 + k)$$

Step 4. Obtain solutions

Static analysis: $s = 0$ => $X = G(0)\,F = F/k$

Vibration analysis: $K_{ds}(s) = ms^2 + k = 0$

=> $\omega_n = \sqrt{k/m}$

Frequency response: $f(t) = a\sin\omega t$

$$x(t) = a|G(j\omega)|\sin(\omega t + \phi)$$

Time-domain response: inverse Laplace transform

Concluding Remarks

- Solar sails form a special class of ultra-large, ultra-lightweight space deployable structures that has uniquely modeling and analysis challenges.
- NASA has recognized the need for computational capabilities that can effectively and efficiently address these challenges.
- JPL is tasked by NASA to develop, based on the proven DTFM and PVP resolution methods, a set of structural modeling and analysis tools for solar sails.
- The developed analysis tools will be verified by vacuum chamber tests conducted on two 20-meter square sails.