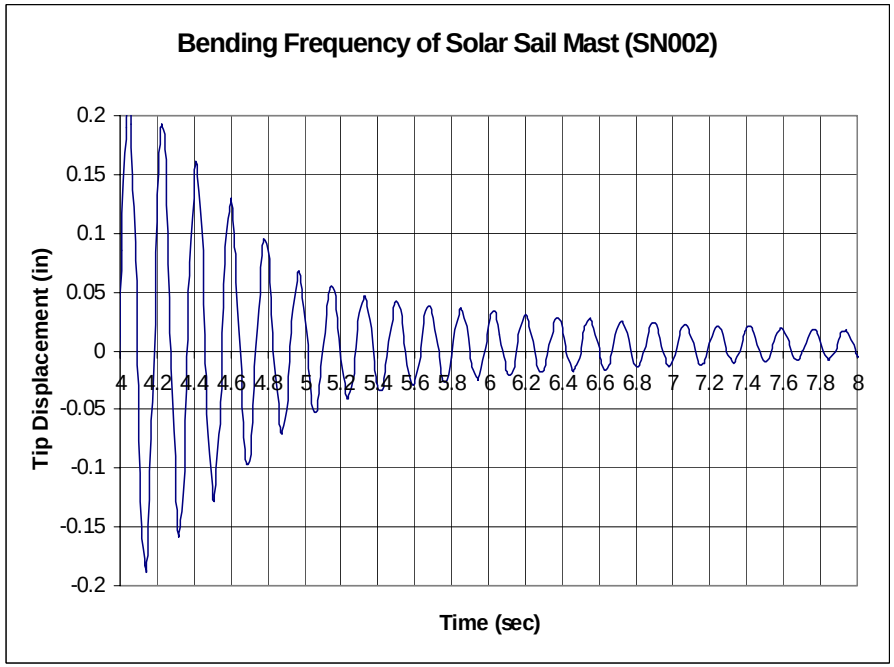
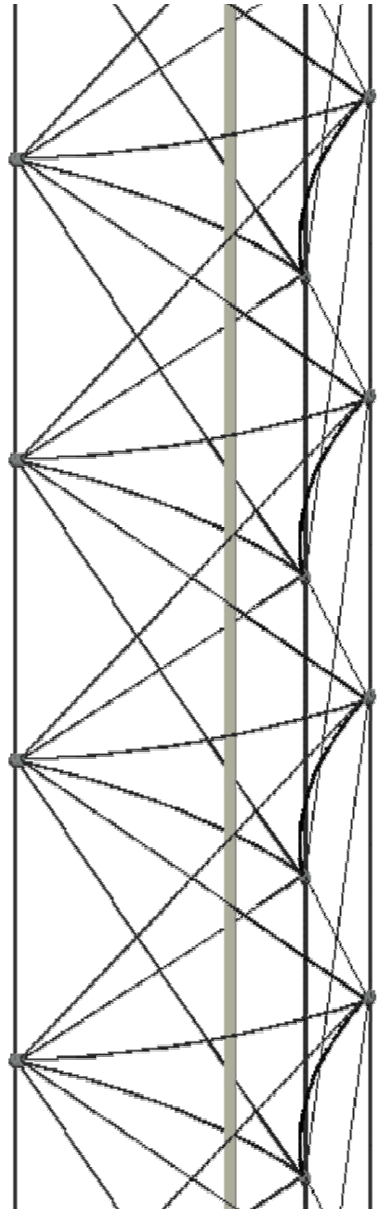
DTFM Modeling and Sensitivity Analysis for Long Masts

May 4, 2004

Current Status

- Completed formulations for DTFM modeling of long masts
- Initiated MATLAB programming for a multiple-bay mast dynamic analysis





Distributed Transfer Function Method

Why DTFM is unique?

--In the Laplace domain.

--Using Distributed Transfer Function instead of Shape Function.

Why DTFM is distinctively suitable for solar sails?

--DTFM decomposes the structure only at those points where multiple structural components are connected ⇒ minimum number of nodes, small matrices, & high computational efficiency.

--Closed form analytical solutions ⇒ reliable results.

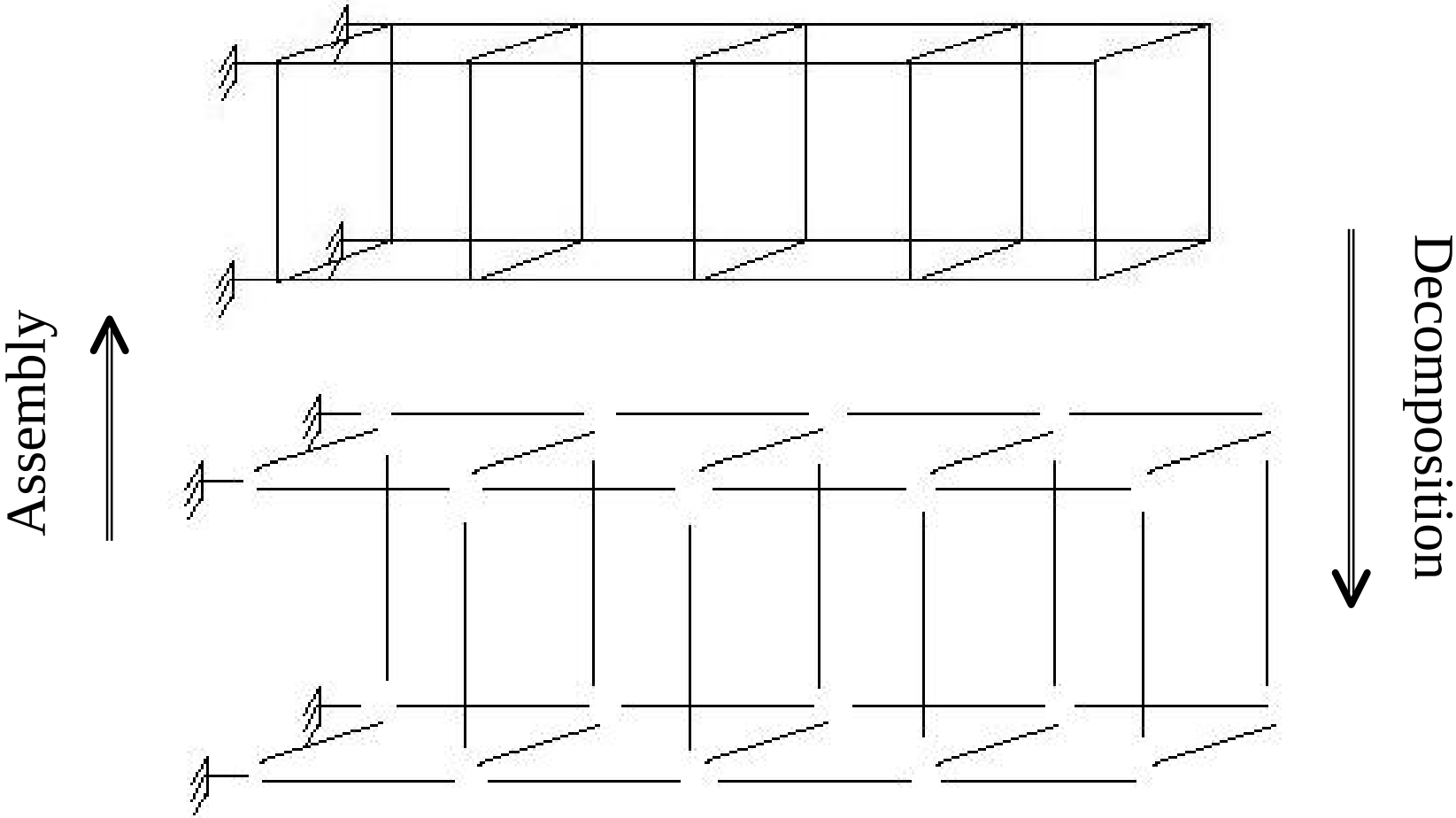
--Able to model local material and geometrical imperfections.

--Convenient in handling structural systems with passive and active damping, gyroscopic effects, embedded smart material layers as sensing and actuating devices, and feedback controllers.

Mast Analysis Using the DTFM

1. Decomposition of a mast into components.
2. Generation of state space form for each component.
3. Generation of distributed transfer function for each component.
4. Generation of dynamic stiffness matrix for each component and assembly of components.
5. Static and dynamic solutions:
   - Natural Frequencies and mode shapes.
   - Buckling analyses.
   - Frequency Responses.
   - Static and Dynamic Stress Analyses.
   - Time Domain Responses.

DTFM Mast Analysis: Step 1--Decomposition



DTFM Mast Analysis: Step 2--State Space Form

A set of governing equations for each individual component:

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j}\left(a_{ijk}+b_{ijk}\frac{\partial}{\partial t}+c_{ijk}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^k u_j(x,t)}{\partial x^k}=f_i(x,t)$$

$$x\in(0,L),\quad t\geq 0,\quad i=1,\cdots,n$$

↓ Example: a beam component

$$EI\frac{\partial^4 v}{\partial x^4}+\rho A\frac{\partial^2 v}{\partial t^2}=p$$

DTFM Mast Analysis: Step 2--State Space Form

State space form: $$\frac{d}{dx}\eta(x,s) = F(s)\eta(x,s) + q(x,s)$$

Example: a beam component

$$F(s) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-\rho A s^2}{EI} & 0 & 0 & 0 \end{bmatrix} \quad \eta(x,s) = \begin{Bmatrix} \bar{v}(x,s) \\ \bar{v}'(x,s) \\ \bar{v}''(x,s) \\ \bar{v}'''(x,s) \end{Bmatrix} \quad q(x,s) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ p(x,s)/EI \end{Bmatrix}$$

DTFM Mast Analysis: Step 3--DTF

A boundary value problem:

$$\frac{d}{dx}\eta(x,s) = F(s)\eta(x,s) + q(x,s) \qquad x \in (0,L)$$

$$M\eta(0,s) + N\eta(L,s) = r(s)$$

The solution is expressed as transfer functions:

$$\eta(x,s) = \int_0^L G(x,\zeta,s)q(\zeta,s)d\zeta + H(x,s)r(s) \qquad x \in (0,L)$$

$$G(x,\zeta,s) = \begin{cases} e^{F(s)x}(M + Ne^{F(s)L})^{-1}Me^{-F(s)\zeta} & \zeta \le x \\ -e^{F(s)x}(M + Ne^{F(s)L})^{-1}Ne^{F(s)(L-\zeta)} & \zeta \ge x \end{cases}$$

$$H(x,s) = e^{F(s)x}(M + Ne^{F(s)L})^{-1}$$

DTFM Mast Analysis : Step 3--DTF

State space vector: $\eta(x,s) = \left[\alpha^T(x,s) \quad \varepsilon^T(x,s)\right]^T$

Displacement vector: $\alpha(x,s) = \left[\alpha_1{}^T(x,s) \quad \alpha_2{}^T(x,s) \quad \cdots \quad \alpha_n{}^T(x,s)\right]^T$

Strain vector: $\varepsilon(x,s) = \left[\varepsilon_1{}^T(x,s) \quad \varepsilon_2{}^T(x,s) \quad \cdots \quad \varepsilon_n{}^T(x,s)\right]^T$

Force vector: $\sigma(x,s) = \overline{E}\varepsilon(x,s)$

$\downarrow$ Example: a beam component

$$\alpha(x,s) = \begin{Bmatrix} \overline{v}(x,s) \\ \overline{v}'(x,s) \end{Bmatrix} \qquad \varepsilon(x,s) = \begin{Bmatrix} \overline{v}''(x,s) \\ \overline{v}'''(x,s) \end{Bmatrix}$$

$$\sigma(x,s) = \begin{Bmatrix} Q(x,s) \\ M_f(x,s) \end{Bmatrix} = \overline{E}\varepsilon(x,s) = \begin{bmatrix} 0 & EI \\ EI & 0 \end{bmatrix} \begin{Bmatrix} \overline{v}''(x,s) \\ \overline{v}'''(x,s) \end{Bmatrix}$$

DTFM Mast Analysis : Step 4--Dynamic Stiffness Matrix

Force vectors at two ends of the component:

$$\begin{bmatrix} \sigma(0,s) \\ \sigma(L,s) \end{bmatrix} = \begin{bmatrix} \overline{E}H_{\sigma 0}(0,s) & \overline{E}H_{\sigma L}(0,s) \\ \overline{E}H_{\sigma 0}(L,s) & \overline{E}H_{\sigma L}(L,s) \end{bmatrix} \begin{bmatrix} \alpha(0,s) \\ \alpha(L,s) \end{bmatrix} + \begin{bmatrix} p(0,s) \\ p(L,s) \end{bmatrix}$$

Dynamic stiffness matrix

Transformed from distributed external forces

Systematically assembles dynamic stiffness matrices of each component

Dynamic stiffness matrix of the whole system

$$\boldsymbol{K}(s) \times \boldsymbol{U}(s) = \boldsymbol{P}(s)$$

DTFM Mast Analysis: Step 5--Static and Dynamic Solutions

Resonant frequencies of the structure:

$$\det\left[\boldsymbol{K}(s_i)\right] = 0 \qquad s_i = \sqrt{-1} \times \omega_i$$

Mode shapes--nontrivial solutions:

$$\boldsymbol{K}(s_i) \times \boldsymbol{U}(s_i) = 0$$

Frequency responses:

$$\boldsymbol{U}(s) = \boldsymbol{K}^{-1}(s) \times \boldsymbol{P}(s)$$

Static analysis:

$$\boldsymbol{K}(0) \times \boldsymbol{U}(0) = \boldsymbol{P}(0)$$

Time domain responses:

Inverse Laplace transform

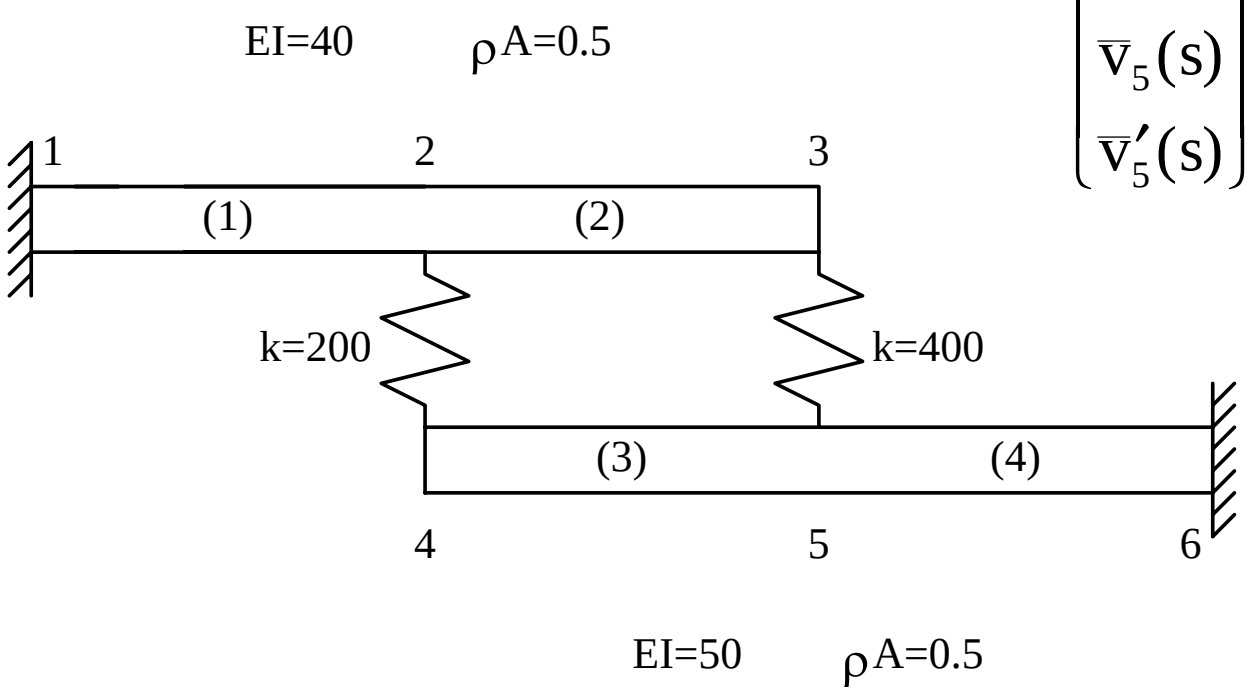
Examples of DTFM Analyses

(1) Two elastically coupled beams

(2) Sensitivity Analysis of a Light-Weight Gossamer Boom

Example (1)--Two elastically Coupled Beams

$$K(s) * \begin{Bmatrix} \bar{v}_2(s) \\ \bar{v}_2'(s) \\ \bar{v}_3(s) \\ \bar{v}_4(s) \\ \bar{v}_5(s) \\ \bar{v}_5'(s) \end{Bmatrix} = \begin{Bmatrix} \tilde{Q}_2(s) \\ \tilde{M}_{f2}(s) \\ \tilde{Q}_3(s) \\ \tilde{Q}_4(s) \\ \tilde{Q}_5(s) \\ \tilde{M}_{f5}(s) \end{Bmatrix}$$



Example (1)--Two Elastically Coupled Beams

Mode number	DTFM 6*6 matrix	FEM 18 Elements	FEM 34 Elements	FEM 66 Elements
1	16.3	16.3	16.3	16.3
2	41.0	41.1	41.0	41.0
3	54.6	53.1	54.2	54.5
4	79.2	77.8	78.9	79.1
5	144.7	138.3	143.1	144.3
6	157.0	150.5	155.4	156.6
7	273.9	258.1	269.9	272.9
8	305.2	288.2	289.9	304.1
9	448.7	415.4	440.4	446.6
10	500.5	463.9	491.2	498.1
11	669.1	601.7	653.7	665.3
12	747.5	672.7	730.5	743.3

Example (2)--Sensitivity Analysis of a Light-Weight Gossamer Boom

Buckling analysis of a boom: $$\frac{d^2}{dx^2}\left(EI\frac{d^2}{dx^2}w(x)\right)+P\frac{d^2}{dx^2}w(x)=0$$

EI is not a constant along the boom:

Divided the boom into a number of sections and each sections is considered to be uniform—Stepwise uniform

Transfer functions are expressed as :

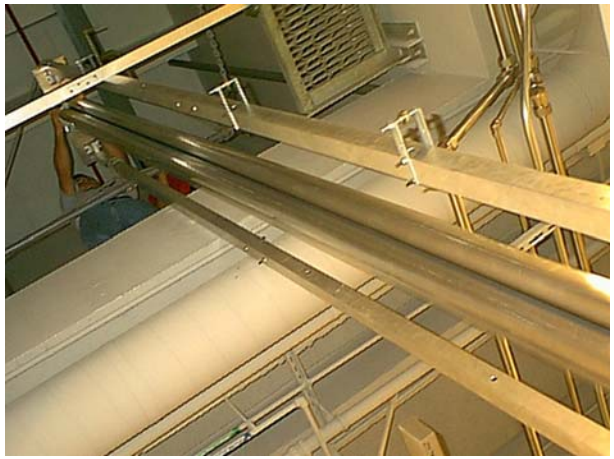
$$G(x,\xi)=\begin{cases} H(x)M\Phi^{-1}(\xi), & \xi<x \\ -H(x)N\Phi(L)\Phi^{-1}(\xi), & \xi>x \end{cases}$$

$$H(x)=\Phi(x)(M+N\Phi(L))^{-1}$$

$$\Phi(x,s)\approx\hat{\Phi}(x,s)=e^{F_{k+1}(x-x_k)}T_k(s)e^{F_k(x_k-x_{k-1})}\ldots T_2(s)e^{F_2(x_2-x_1)}T_1(s)e^{F_1(x_1)} \qquad x\in(x_k,x_{k+1})$$

$$T_k=\begin{bmatrix} I & 0 \\ 0 & E_{k+1}^{-1}E_k \end{bmatrix}\in C^{n\times n}$$

Example (2)--Sensitivity Analysis of a Light-Weight Gossamer Boom



Length of the inflatable boom: 197 inches
Bending stiffness EI_0: 656673 lb*in^2

$$EI = EI_0(1 + \varepsilon \times \sin(\frac{x\pi}{L}))$$

Buckling force as the function of bending stiffness deviation ε

ε	0%	± 2%	± 4%	± 6%	± 8%	± 10%
Pcr (+ %)	167.0	169.7	172.7	175.4	178.2	181.1
Pcr (- %)	167.0	164.2	161.2	158.5	155.6	152.8

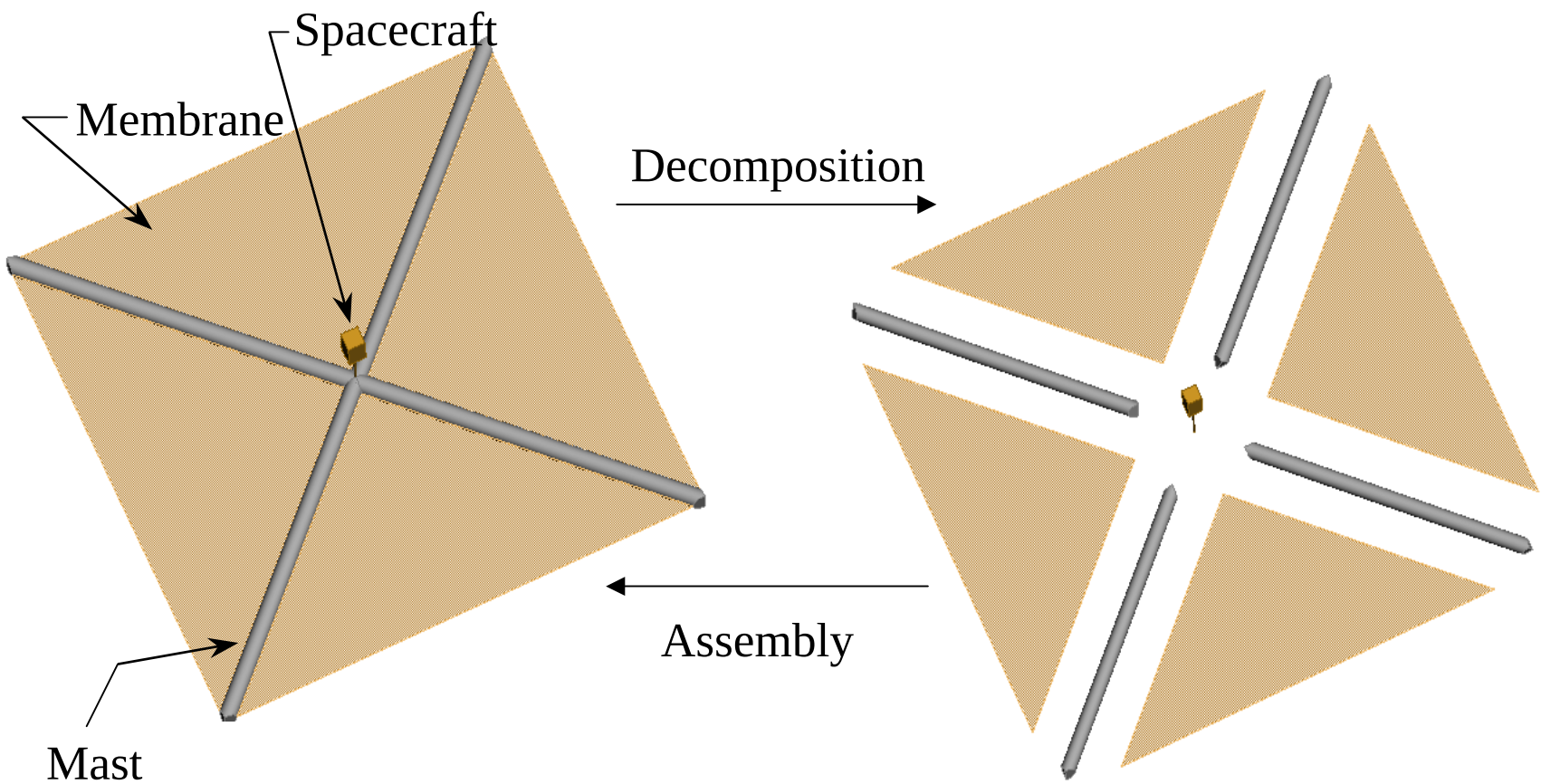
Ration of buckling force changing as the function of ε

ε	0%	± 2%	± 4%	± 6%	± 8%	± 10%
Pcr/Pcr_0	1.0000	1.017	1.034	1.051	1.067	1.085
Pcr/Pcr_0	1.0000	0.983	0.966	0.949	0.932	0.915

DTFM Synthesis for Solar Sails

May 4, 2004

Decomposition of a Solar Sail



Dynamic Stiffness Matrix Synthesis

Dynamic stiffness matrices of masts—ready.

Dynamic stiffness matrix of the spacecraft—lumped mass, ready.

Steps needed to get dynamic stiffness matrices of membranes:

1) PVP membrane analysis $\Longrightarrow$ $M\ddot{x} + Kx = f$
2) Laplace transform $\Longrightarrow$ $\left(Ms^2 + K\right)\hat{x} = \hat{f}$

Solar sail synthesis:

1) Displacement compatibility
2) Force balance