Analysis of Active Constrained Layer Damping in composite sandwiched plates

Charles K. Kim
NASA-Goddard Space Flight Center, Code 542
Greenbelt, MD 20771

Chul H. Park
Wilcoxon Labs
Wilcoxon Research Inc.
Gaithersburg, MD 20878

Amr Baz
Mechanical Engineering Department
University of Maryland
College Park, MD 20742

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Abstract

This paper describes how a new class of composite sandwich plates is implemented to control vibration of the bending and torsional modes using built-in Active Constrained Layer Damping (ACLD). The paper also gives an insight into the fundamentals governing the dynamics and active/passive control of smart composite sandwiched plate structures to use as the potential basic building block of structures where effective vibration damping is essential to their successful operation.

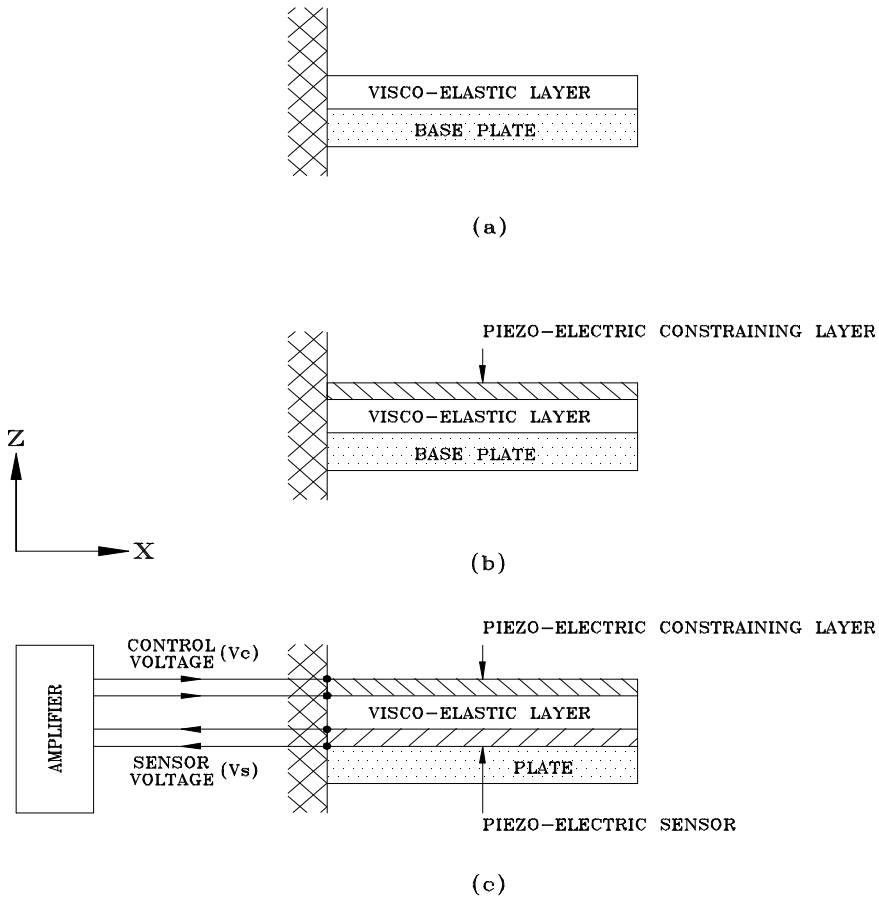
To improve predictions of the dynamics and controlled vibration of the composite sandwiched plate/ACLD, a powerful assumed displacement field for the finite element modeling is introduced. This assumed displacement field differs from the classical laminated theory and offers a definite advantage in finite element modeling as it gives a displacement distribution along the whole thickness of the laminates and requires fewer degrees of freedoms to represent the kinematical relationships for viscoelastic layers with piezoelectric layers in ACLD. Also, the predictions of the finite element model using this assumed displacement field have been validated by comparing of modal frequencies and damping loss factors with experiment and are found to be in close agreement.

Outline of Presentation

- Introduction
- Active Constrained Layer Damping (ACLD)
- Theoretical Development (Finite Element Modeling)
- Experimental Performance
- Comparison between Theory & Experiments
- Summary

Passive and active layer damping

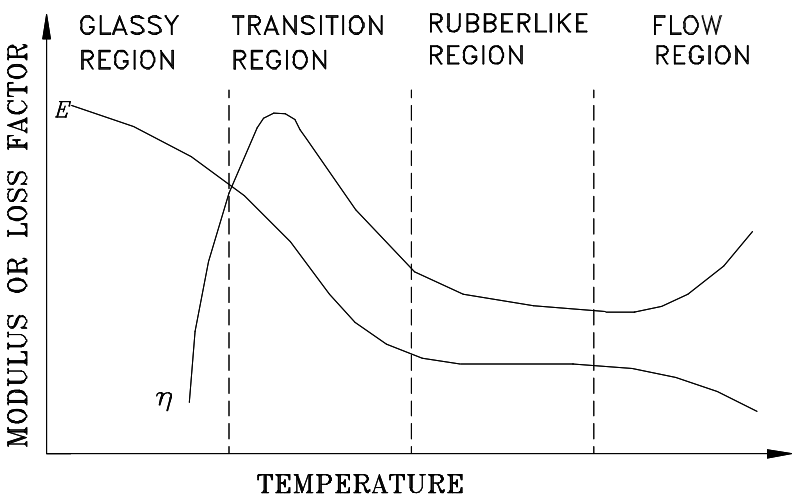
(a)-Passive: unconstrained
(b)-Passive: constrained
(c)-Active: constrained



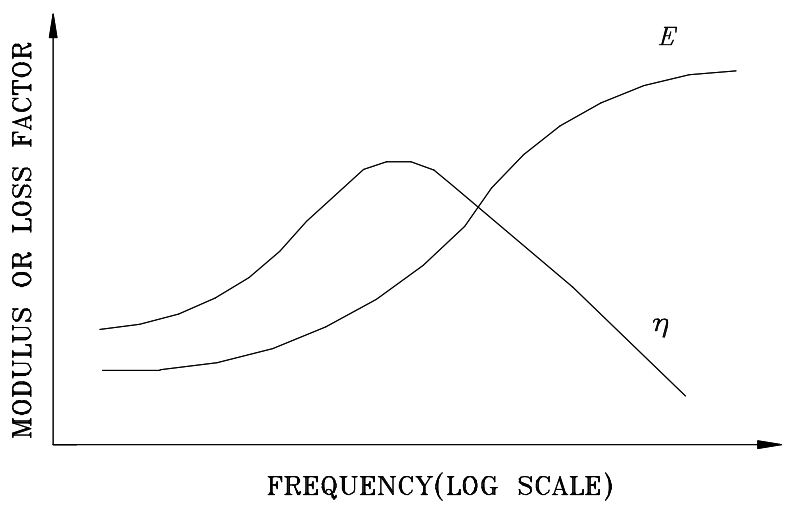
Viscoelastic material

$$E(f) = a_0 + a_1 f + a_2 f^2 + a_3 f^3 + a_4 f^4$$

$$h(f) = b_0 + b_1 f + b_2 f^2 + b_3 f^3 + b_4 f^4$$

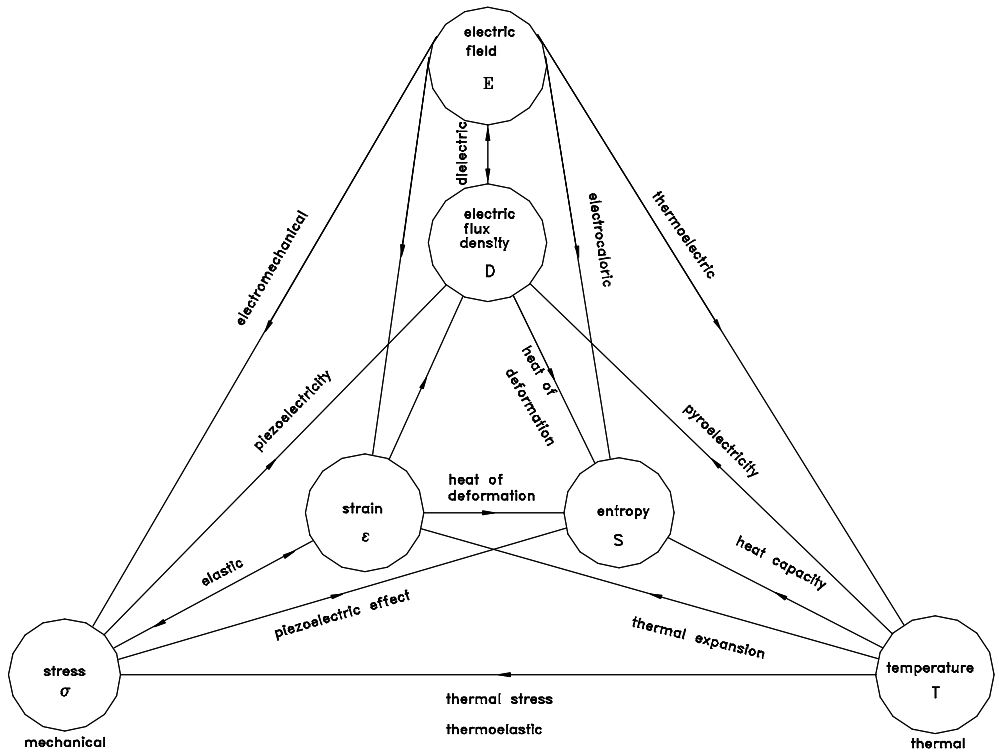


Variation of storage modulus, and loss factor, with temperature
[Nashif et al. 1985]



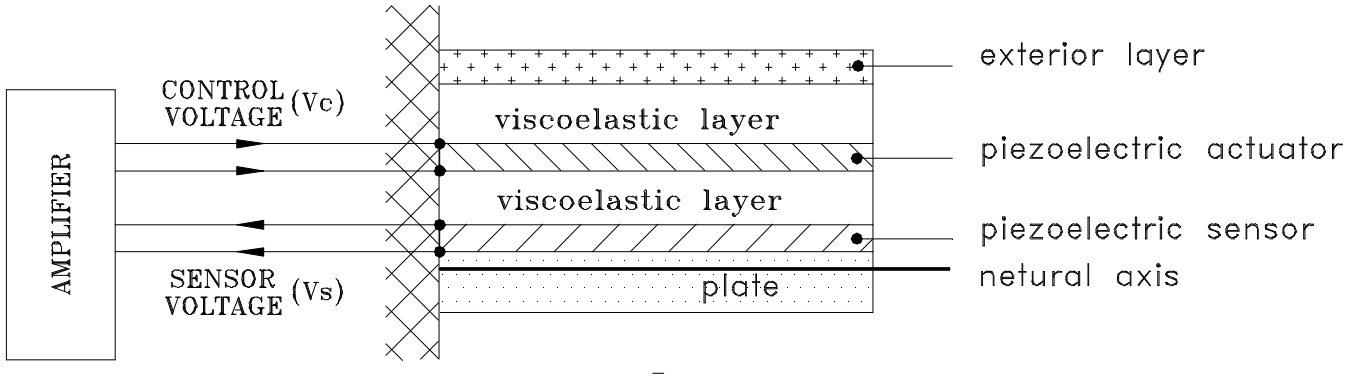
Variation of storage modulus, and loss factor, with frequency
[Nashif et al. 1985]

Interaction processes between the electrical, mechanical, and thermal system



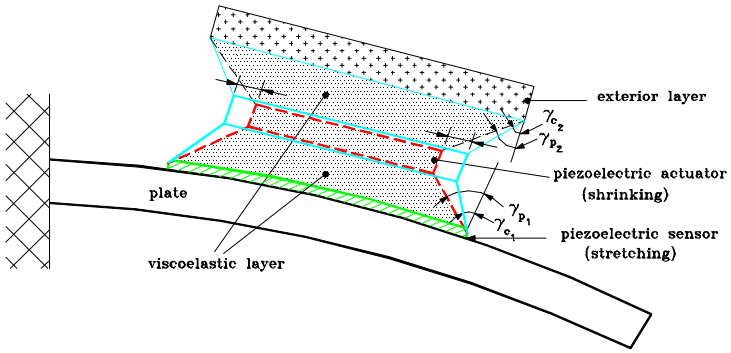
[Ikeda Takuro; Fundamental of Piezoelectricity,1990]

Active constrained layer damping

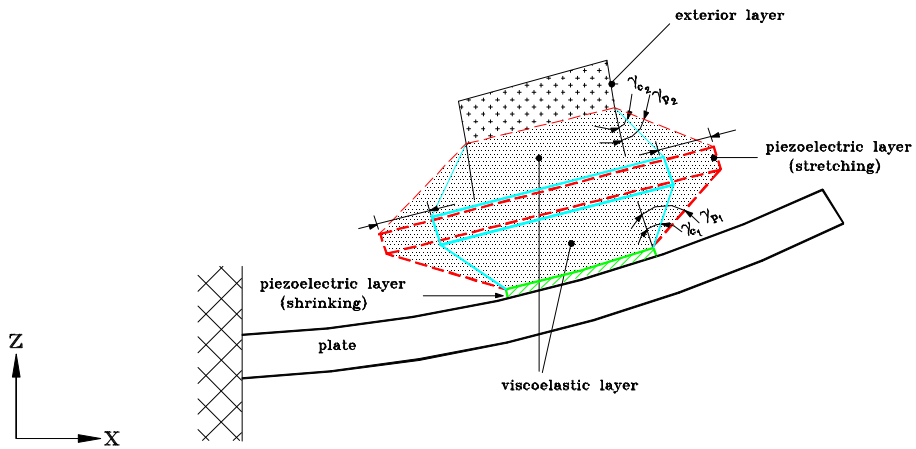


- **Combines active & passive damping control**
- **Enhances energy dissipation characteristics**

Operating principle of sandwiched plate/ACLD system.



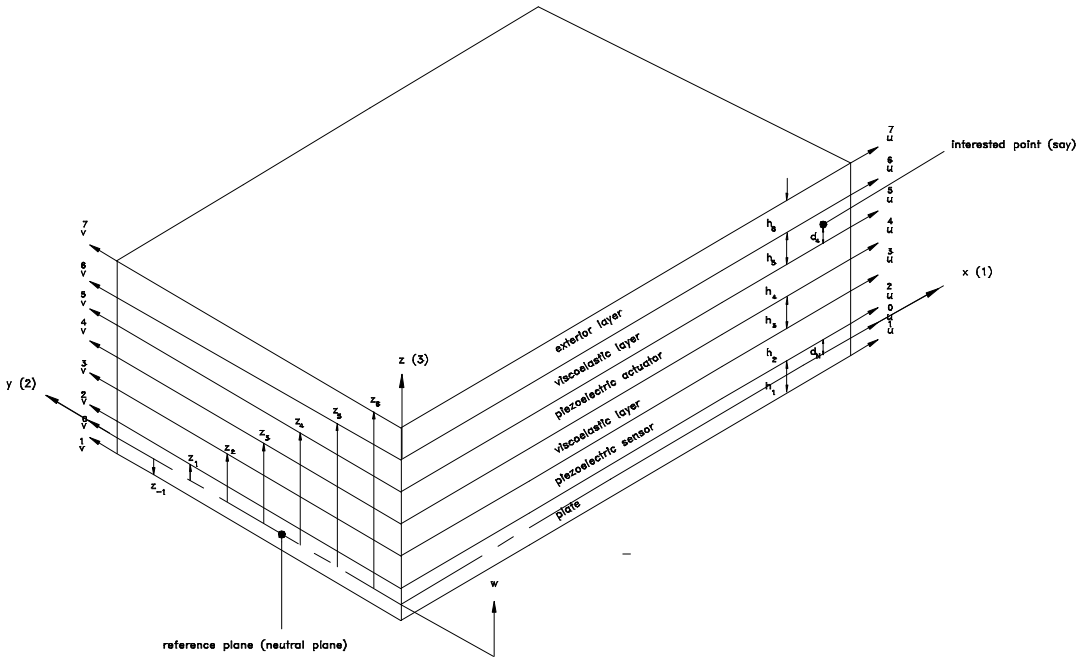
(a)



Theoretical Development
(Formulation)

- Displacement Field
- Displacement-Strain
- Stress-Strain
- Sensor & Actuator Equation
- Equation of Motion

Schematic drawing of six-layer cantilever plate/ACLD system



Composite sandwiched plate coordinate

Displacement field

$$u_1(x,y,z,t) = \overset{0}{u}(x,y,t) + \sum_{j=0}^{j=|k-N|} y_{N\pm j}(z) f_{1_{N\pm j}}(x,y,t)$$

$$u_2(x,y,z,t) = \overset{0}{v}(x,y,t) + \sum_{j=0}^{j=|k-N|} y_{N\pm j}(z) f_{2_{N\pm j}}(x,y,t)$$

$$u_3(x,y,t) = w(x,y,t)$$

where the functions of the thickness $y_i(z)$ and the in-plane coordinate $f_{1_i}(x,y,t)$ and $f_{2_i}(x,y,t)$ are defined as follows:

$$y_i(z) = z - z_{-i}, \qquad z_{-i} \geq z \geq z_{-i-1}$$

$$= z, \qquad z_0 \leq z \leq z_1 \text{ or } z_{-1} \leq z \leq z_0$$

$$= z - z_i, \qquad z_i \leq z \leq z_{i+1}$$

$$f_{1_i}(x,y,t) = -a_{1_i} \frac{\partial w(x,y,t)}{\partial x}$$

$$f_{2_i}(x,y,t) = -a_{2_i} \frac{\partial w(x,y,t)}{\partial y}$$

where the a_{l_i}'s denotes adjustment coefficients which will be determined later. Physically, $a_{1_i} \frac{\partial w}{\partial x}$ and $a_{2_i} \frac{\partial w}{\partial y}$ define the slopes of the i th layer due to bending in the x and y directions, respectively

Displacement-Strain Relations

$$\overset{p}{e}_{x_i x_j} = \overset{p}{e}_{x_j x_i} = \frac{1}{2}\left(\frac{\partial \overset{p}{u}_i}{\partial x_j} + \frac{\partial \overset{p}{u}_j}{\partial x_i}\right) \qquad (i, j = 1,2,3)\,.$$

$$\begin{bmatrix} e_1 \\ e_2 \\ g_6 \end{bmatrix} = \begin{bmatrix} \overset{0}{e}_1 \\ \overset{0}{e}_2 \\ \overset{0}{g}_6 \end{bmatrix} + \sum_{i=1}^{i=k} y_i \begin{bmatrix} \mathrm{K}_1 \\ \mathrm{K}_2 \\ \mathrm{K}_6 \end{bmatrix}_i + \begin{bmatrix} d_{31}\mathrm{E}_3\Lambda(x,y) \\ d_{32}\mathrm{E}_3\Lambda(x,y) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} g_5 \\ g_4 \end{bmatrix}_k = \begin{bmatrix} (1-a_{1_k})\dfrac{\partial w}{\partial x} \\ (1-a_{2_k})\dfrac{\partial w}{\partial y} \end{bmatrix} + \begin{bmatrix} \dfrac{d_{31}\mathrm{E}_3\int_l \Lambda(x,y)dx}{h_{vem}} \\ \dfrac{d_{31}\mathrm{E}_3\int_w \Lambda(x,y)dy}{h_{vem}} \end{bmatrix}$$

Stress-Strain Relation

$$\begin{bmatrix} s_1 \\ s_2 \\ s_6 \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} e_1 \\ e_2 \\ g_6 \end{bmatrix}_k$$

$$\begin{bmatrix} s_5 \\ s_4 \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{55} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{44} \end{bmatrix}_k \begin{bmatrix} g_5 \\ g_4 \end{bmatrix}_k .$$

Sensor & Actuator Equation

$$q(t) = \int_A \begin{Bmatrix} d_{31}^0 \\ d_{32}^0 \\ 0 \end{Bmatrix}^T \begin{Bmatrix} s_1 \\ s_2 \\ s_6 \end{Bmatrix} dA$$

$$V_s = \frac{1}{C} q(t) = \frac{1}{C} \int_A \begin{Bmatrix} d_{31}^0 \\ d_{32}^0 \\ 0 \end{Bmatrix}^T \begin{Bmatrix} s_1 \\ s_2 \\ s_6 \end{Bmatrix} dA$$

$$V_C = -(K_p + iwK_d)V_S$$

Equation of Motion & Eigenvalue Analysis

$$[M_G]\left\{\frac{\P^2 \Delta_G}{\P t^2}\right\} + [K_G]\{\Delta_G\} = \{F_G\} + \{\tilde{F}_G\}$$

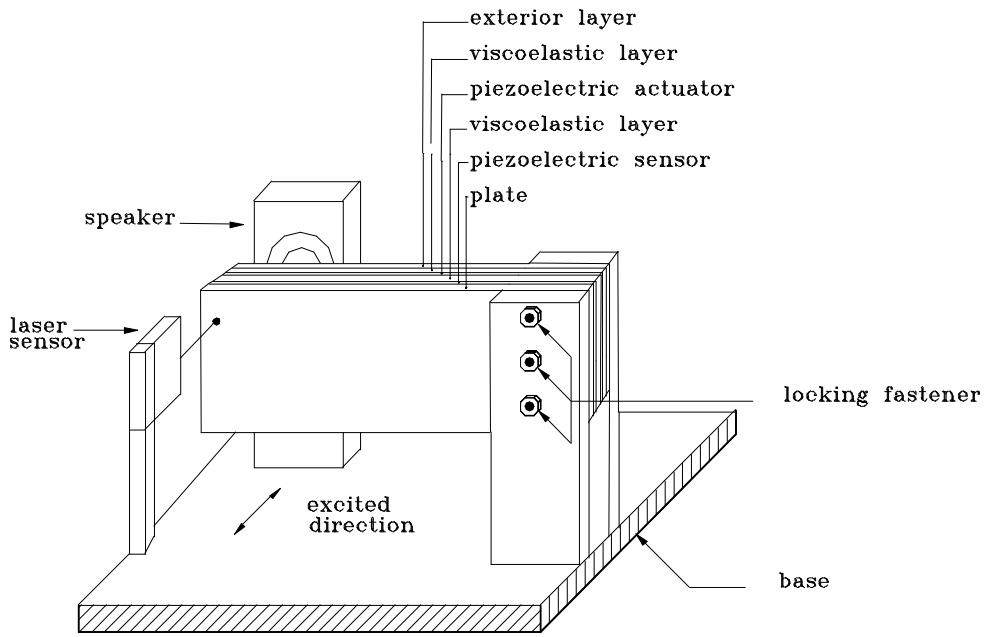
$$\{[K_G] - w^{*2}[M_G]\}\{\Phi\} = 0$$

where w^{*2} are complex eigenvalues.

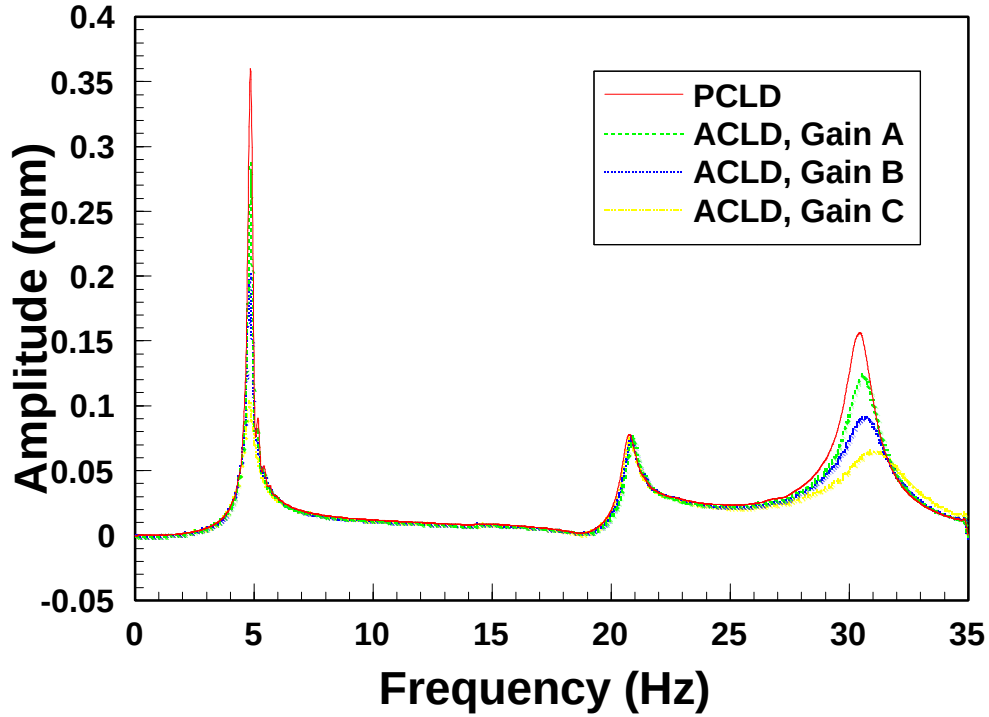
The n th eigenvalue is written as follows:

$$w_n^{*2} = w_n^2(1 + ih_n)$$

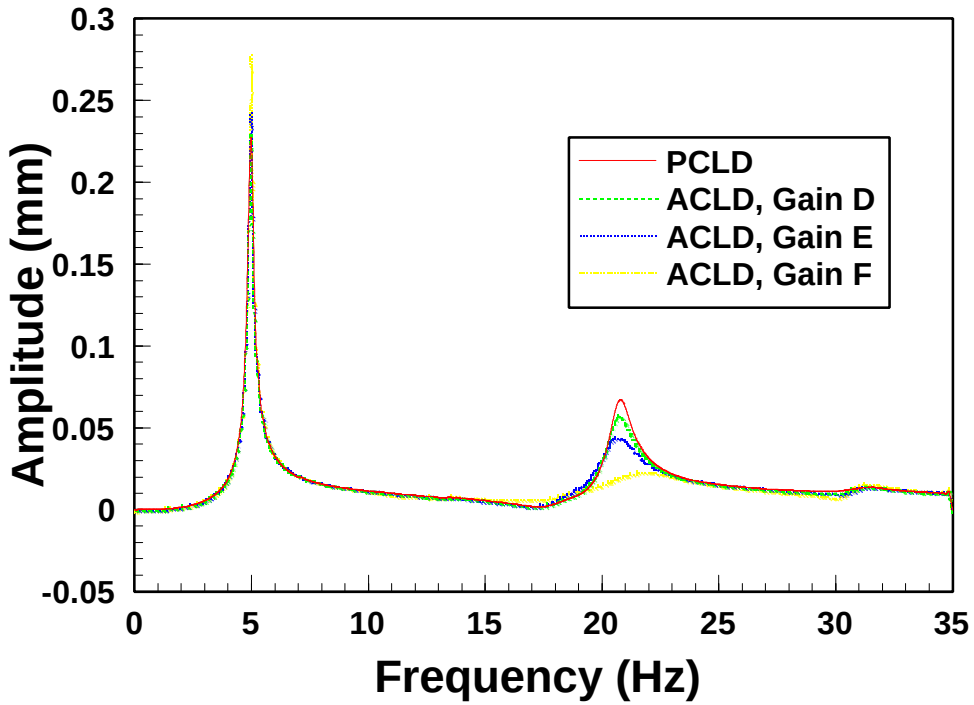
Experimental setup for composite sandwiched plate/ACLD



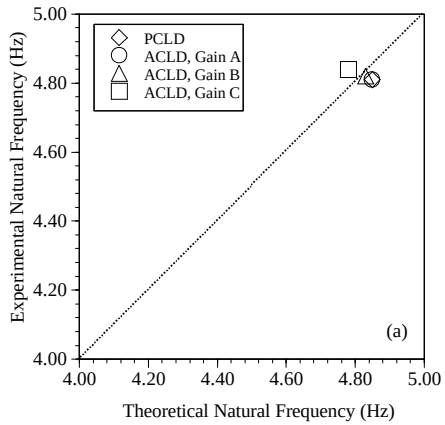
Response to random excitations for bending control

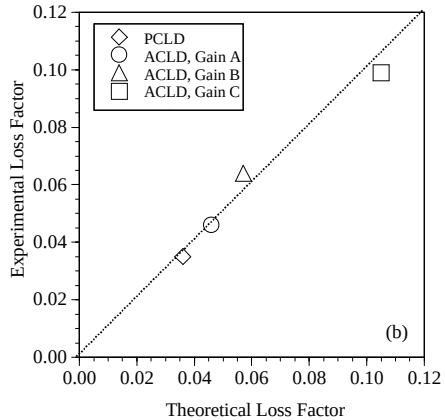


Response to random excitations for torsion control

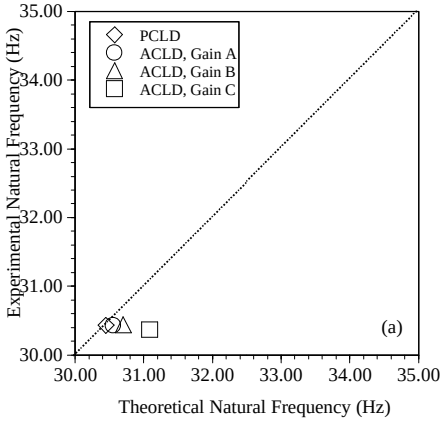


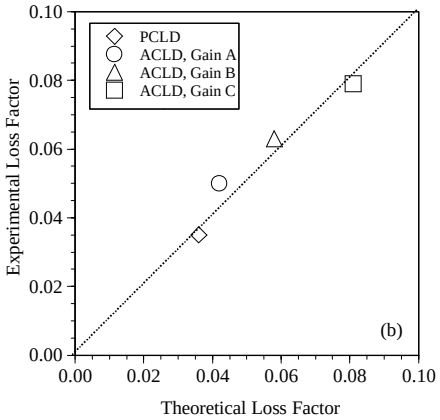
Comparison between theory & experiments for first bending mode with different control gains



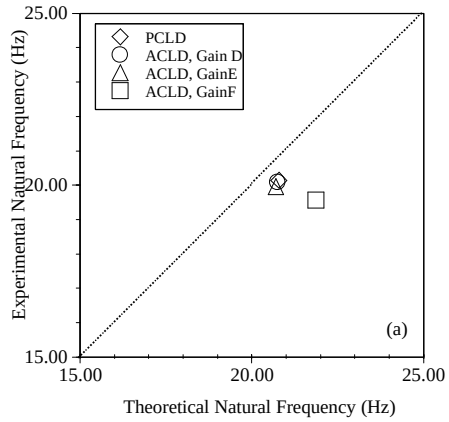


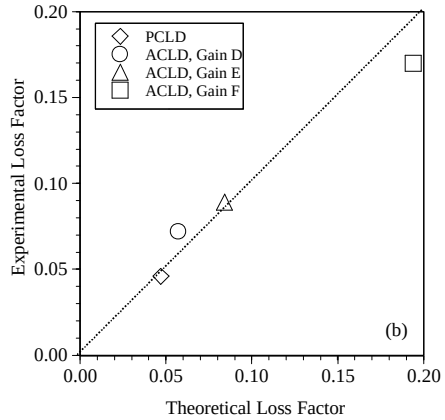
Comparison between theory & experiments for second bending mode with different control gains





Comparison between theory & experiments for torsional mode with different control gains





Summary

- Introduced a a new class of composite sandwiched plates
- Developed the theoretical analysis of a composite plate/ACLD
- Theoretical Analysis was validated experimentally
- The ACLD is found to be effective in controlling the vibration of the plates