An Optimization-Based Approach to Membrane Problems Arising in the Analysis of Gossamer Structures

Frank Baginski, Michael Barg & Stacey Vezina, *Department of Mathematics, The George Washington University, Washington, DC 20052, baginski@gwu.edu*

Laura Cadonati, *MIT, LIGO Project, NW17-161, 175 Albany St., Cambridge, MA 02139, cadonati@ligo.mit.edu*

William Collier, *Flight Software Maintenance Group, Computer Sciences Corporation, 7700 Hubble Drive, Lanham-Seabrook, Md. 20706, wcollier@csc.com*

Willi W. Schur, *Physical Sciences Laboratory, NMSU, NASA-GSFC-WFF, Wallops Island, VA 23337, willi.w.schur.1@gsfc.nasa.gov*

Supported with funding from NASA's Balloon Program Office, a mathematical model is being developed for the analysis of large scientific balloons. Initially focusing on fully inflated and partially inflated zero-pressure natural-shape balloons, our approach has proven to be adaptable to a variety of problems that lend themselves to a variational formulation with an optimization-based solution process. Our model is implemented into a program called *EMsolver*.

We present a collection of membrane problems for gossamer structures, including natural-shape balloons, pumpkin balloons, a solar neutrino balloon, and an NGST-like sunshield. In each of these applications, the structure consists of membrane elements and reinforcing tendons. Wrinkling in the membrane is modeled via A. C. Pipkin's energy relaxation approach.

Nomenclature

S - complete balloon shape.
Ω - complete reference configuration.
F_I - discretized configuration in $\mathbb{R}^3$
F_I - flat unstrained reference configuration in $\mathbb{R}^2$.
T - a triangle in F_I
T - a triangle in F_I.
n_g - number of gores in a complete shape.
r_B - bulge radius for the ideal pumpkin gore.
E_T - total energy of the balloon system.
$E_P = -\int_\Omega (\frac{1}{2}bz^2 + p_0 z)\mathbf{k} \cdot d\vec{S}$ - hydrostatic pressure potential energy.
$E_f = \int_S w_f z dA$ - gravitational potential energy of the film.
$E_t = n_g w_t \int_0^{L_t} \tau(s) \cdot \mathbf{k} \ ds$ - gravitational potential energy of the load tendons.
$E_{top} = w_{top} z_{top}$ - gravitational potential of top fitting.
$S_t^* = n_g \int_\Gamma W_t^*(\varepsilon) ds$ - relaxed tendon strain energy.
$S_f^* = \int_\Omega W_f^* dA$ - relaxed film strain energy.
$d\vec{S} = \mathbf{N} dS$, $\mathbf{N}$ is the unit outward normal to S.
dS is surface area measure on S.
dA - area measure in the reference configuration,
ds - arc length measure along Γ,
$\tau(s) \in \mathbb{R}^3$ is the position of the load tendon.

Variational Formulation and Optimization-Based Solution

See Baginski & Collier (2001)

Total potential energy:

$$E_T^*(S) = E_P + E_f + E_t + E_{top} + S_t^* + S_f^*$$

E_P - hydrostatic pressure potential (lifting gas)
E_f - gravitational potential energy of film
E_t - gravitational potential energy of load tendons
E_{top} - gravitational potential energy of top fitting
S_f^* - relaxed strain energy of film
S_t^* - relaxed strain energy of load tendons
S - membrane

Optimization Problem $\star$

$$\textit{For } S \in C, \qquad \begin{array}{ll} \textit{minimize:} & E_T^*(S) \\ \textit{subject to:} & \vec{G}(S) = \vec{0} \end{array}$$

EMsolver uses Matlab's fmincon to solve a discretization of Problem $\star$.

C - class of shapes; G - constraints

Design parameters and related constants for a ULDB Phase IV balloon. Comparing a standard gore design and a supergore design.

Description		ULDB
Top fitting weight (N)	w_{top}	831
Cap weight density (N/m^2)	w_c	0.18387
Cap thickness (μm)	h_c	10.8
Film weight density (N/m^2)	w_f	0.3440
Film weight with caps (N)		16,562
Film thickness (μm)	h_f	38.1
Youngs modulus (MPa)	E	404.2
Poisson ratio	ν	0.825
Tendon weight density (N/m^2)	w_t	0.094
Tendon weight (N)		4,144
Tendon stiffness (MN)	K_t	0.651
Payload (N)	L_{bot}	20393
Specific buoyancy (N/m^3)	b	0.0763
Number of gores	n_g	290
Cap length (m)	l_c	49.45
Constant pressure (Pa)	p_0	135
Target volume (mcm)		0.515
Curved edge length (m)	L_d	152.03
Center gore length (m)	L_c	152.02
Pumpkin generator length (m)	L_p	150.02
Length of tendon (m)	L_t	147.57

Wrinkling and film strain energy relaxation

F. Baginski & W. Collier

See Collier (2000) & Baginski & Collier (2001)

Relaxed membrane strain energy density:

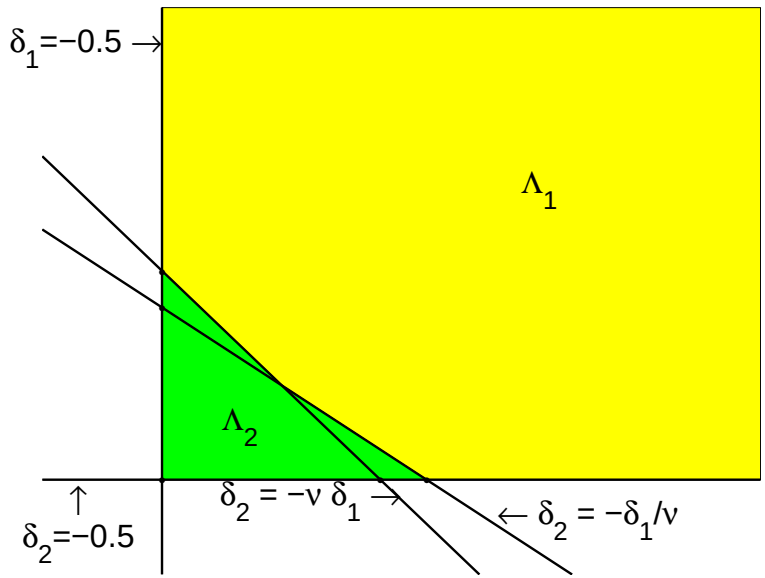
$$W_f^*(T) = \begin{cases} 0, \;\; \delta_1 < 0 \text{ and } \delta_2 < 0 & \textit{(slack)} \\ \frac{1}{2} t E \delta_2^2, \;\; \mu_1 \leq 0 \text{ and } \delta_2 \geq 0 & \textit{(wrinkled)} \\ \frac{1}{2} t E \delta_1^2, \;\; \mu_2 \leq 0 \text{ and } \delta_1 \geq 0 & \textit{(wrinkled)} \\ \frac{tE}{2(1-\nu^2)}(\delta_1^2 + \delta_2^2 + 2\nu\delta_1\delta_2), & \\ \qquad \mu_1 \geq 0 \text{ and } \mu_2 \geq 0 & \textit{(taut)} \end{cases} \tag{1}$$

Relaxed membrane energy:

$$S_{film}^* = \int_\Omega W_f^* dA$$

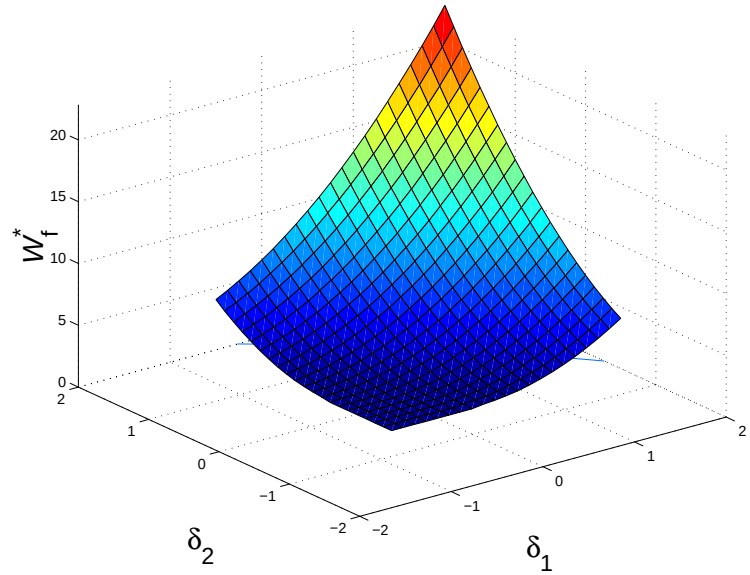
Λ_1-taut, Λ_2-slack, other - wrinkled

$\delta_1 \times \delta_2$-plane

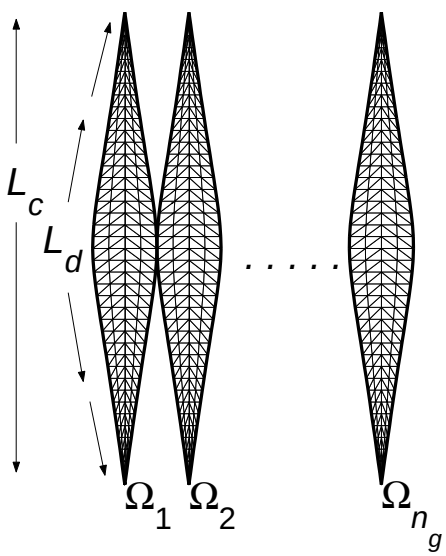


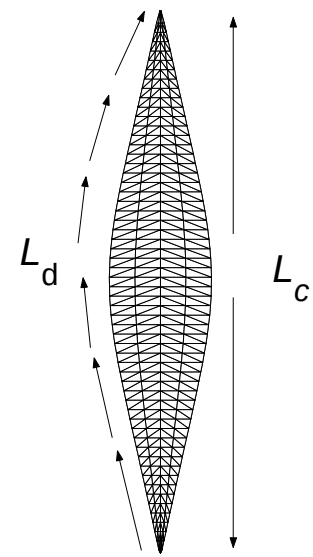
Reference Configuration

W_f^* - quasi-convexification of W_f



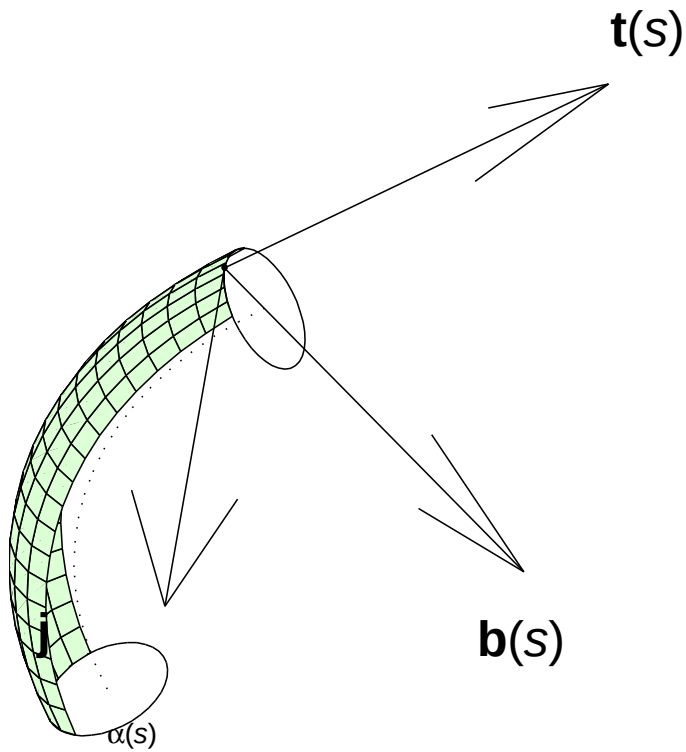
Natural state of single gore



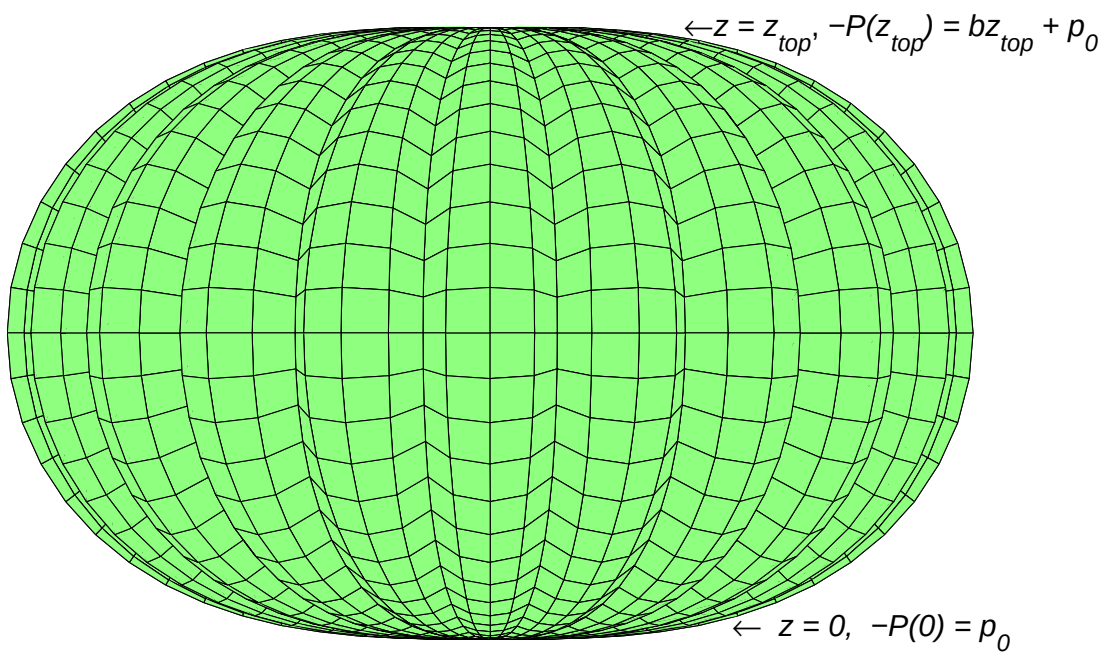


The Pumpkin Balloon
F. Baginski & W. Schur

Section of pumpkin gore

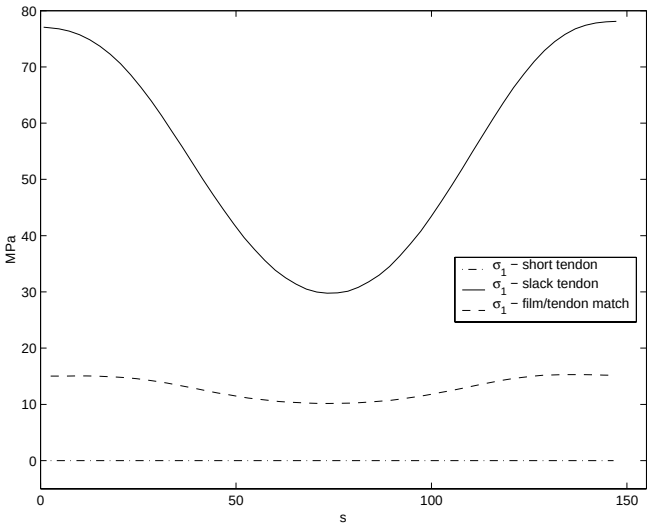


Pumpkin Balloon

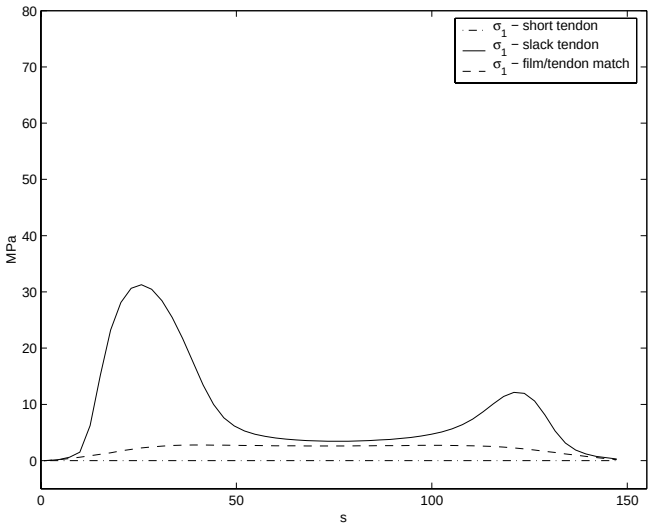


Averaged principal stresses of natural and pumpkin shapes

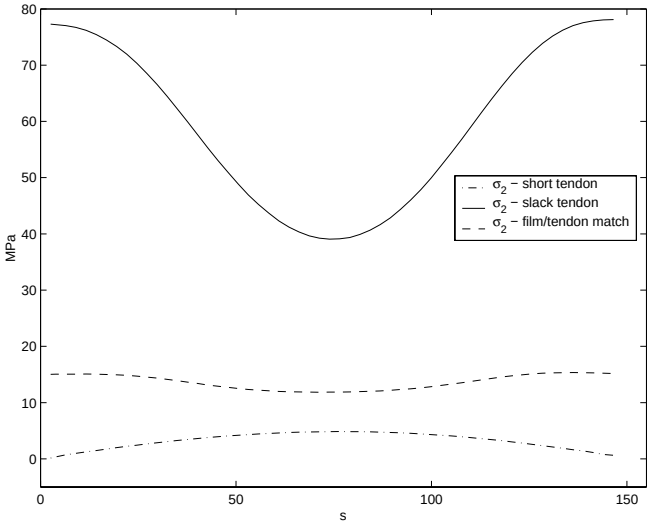
(a) $\bar{\sigma}_1(s) \approx$ meridional stress



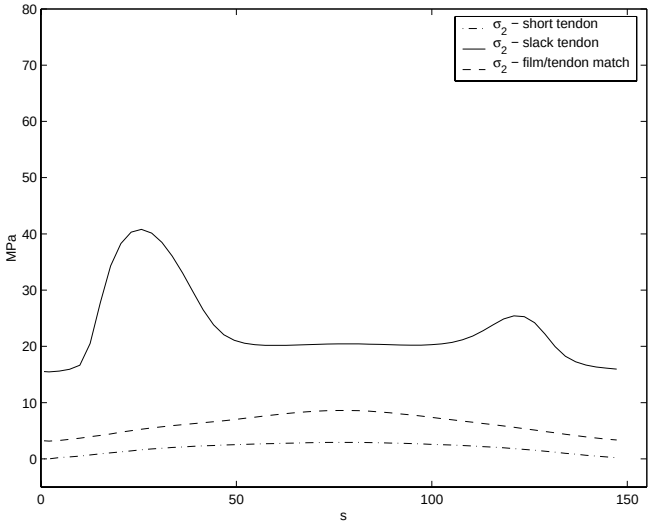
(b) $\bar{\sigma}_1(s)$



(c) $\bar{\sigma}_2(s) \approx$ hoop stress



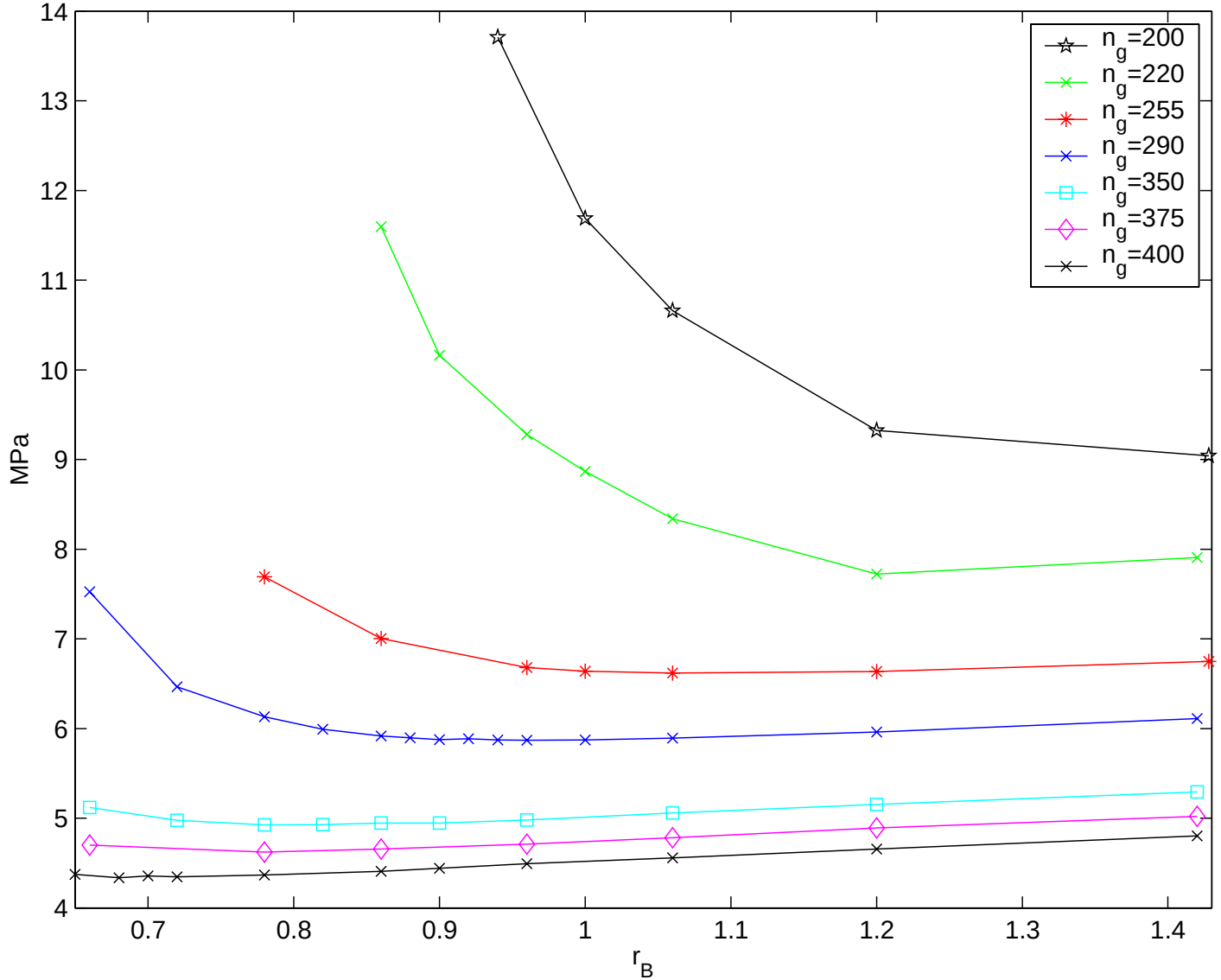
(d) $\bar{\sigma}_2(s)$



(a)-(b) slack tendons; (c)-(d) short tendons (structural lack-of-fit)

Max. principal stress μ_2 vs. bulge radius r_B for n_g-gores.

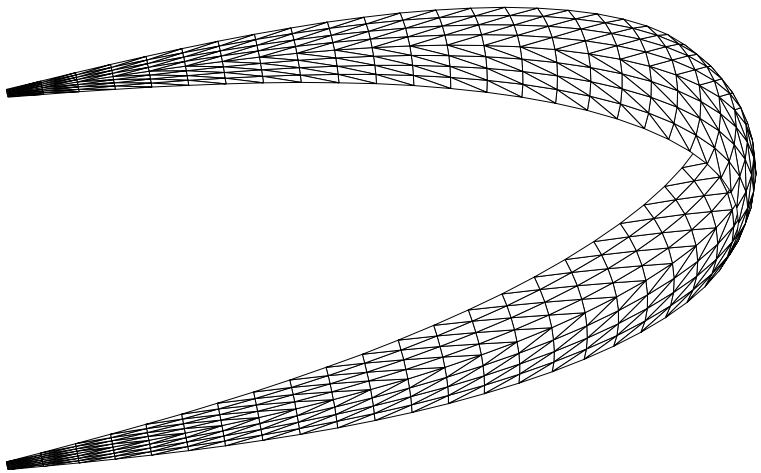
F. Baginski & M. Barg



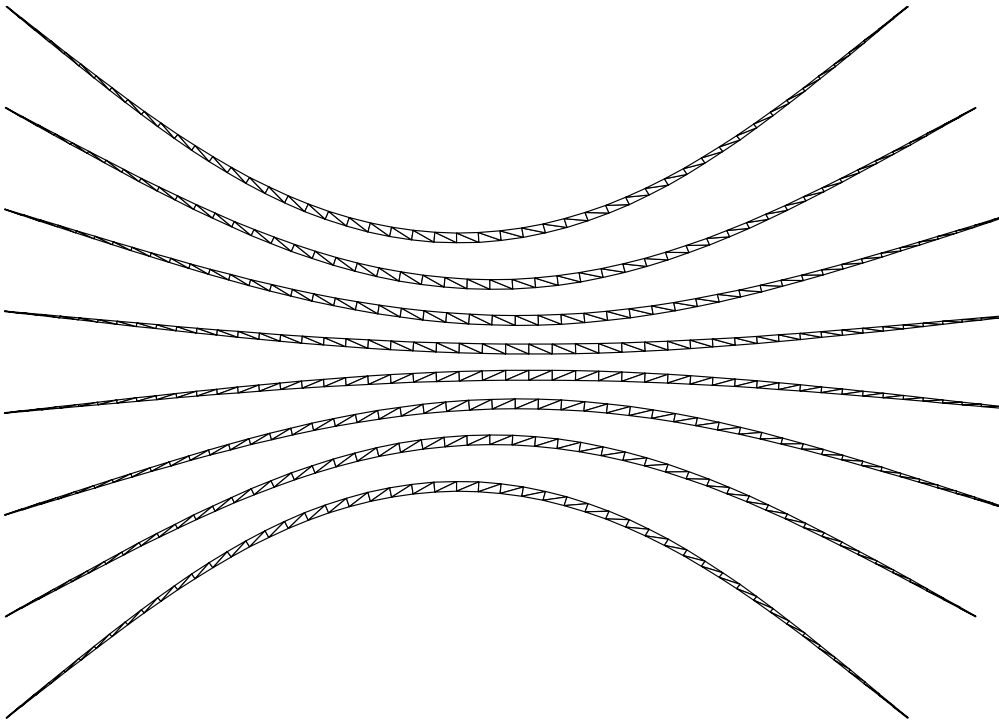
“Molded” Pumpkin Supergore

F. Baginski & W. Schur

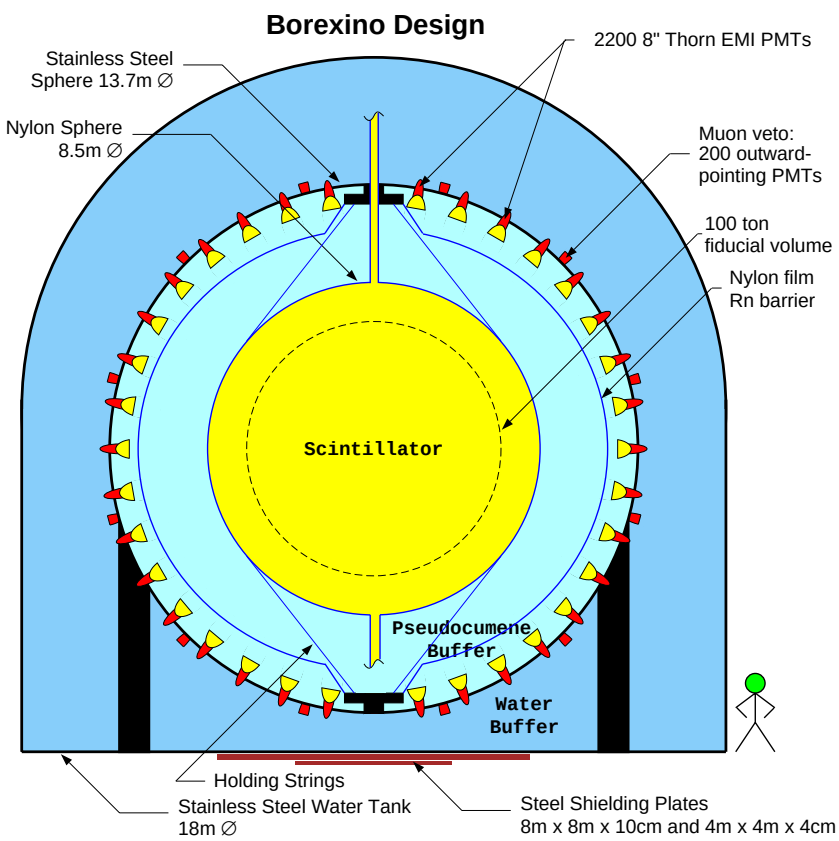
Molded Pumpkin Supergore

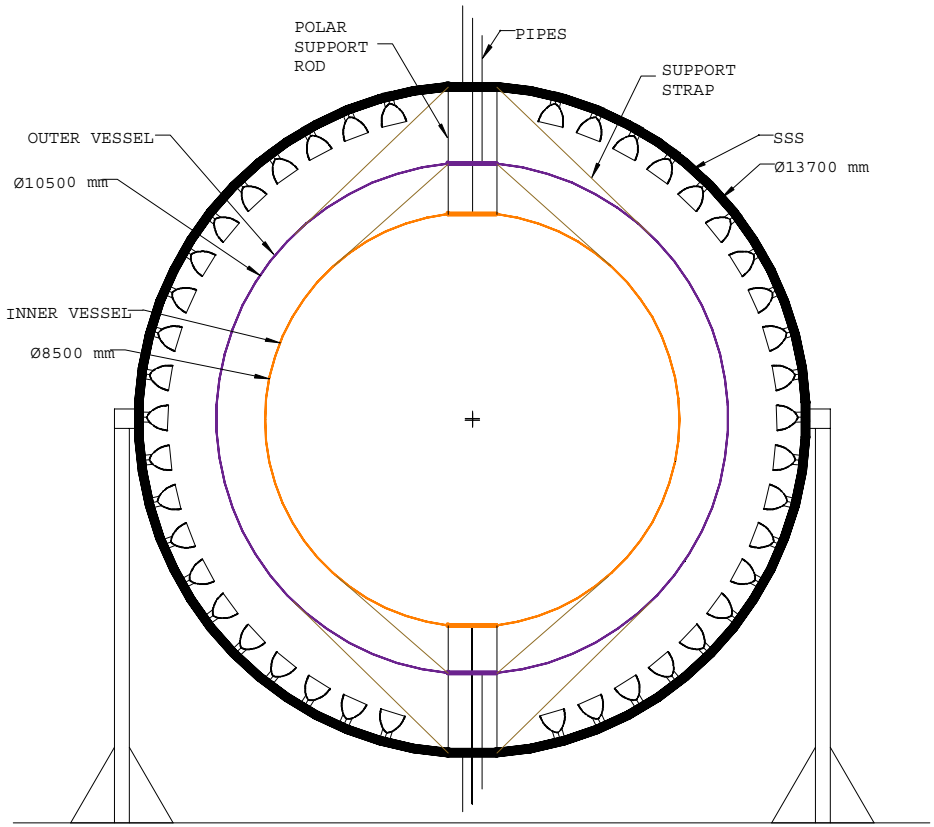


Flat unassembled configuration

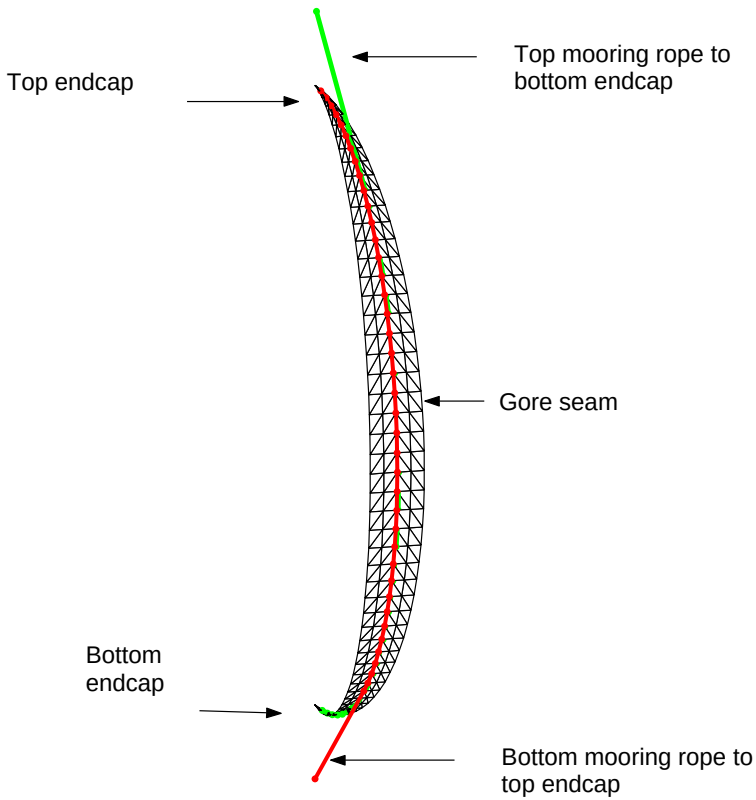


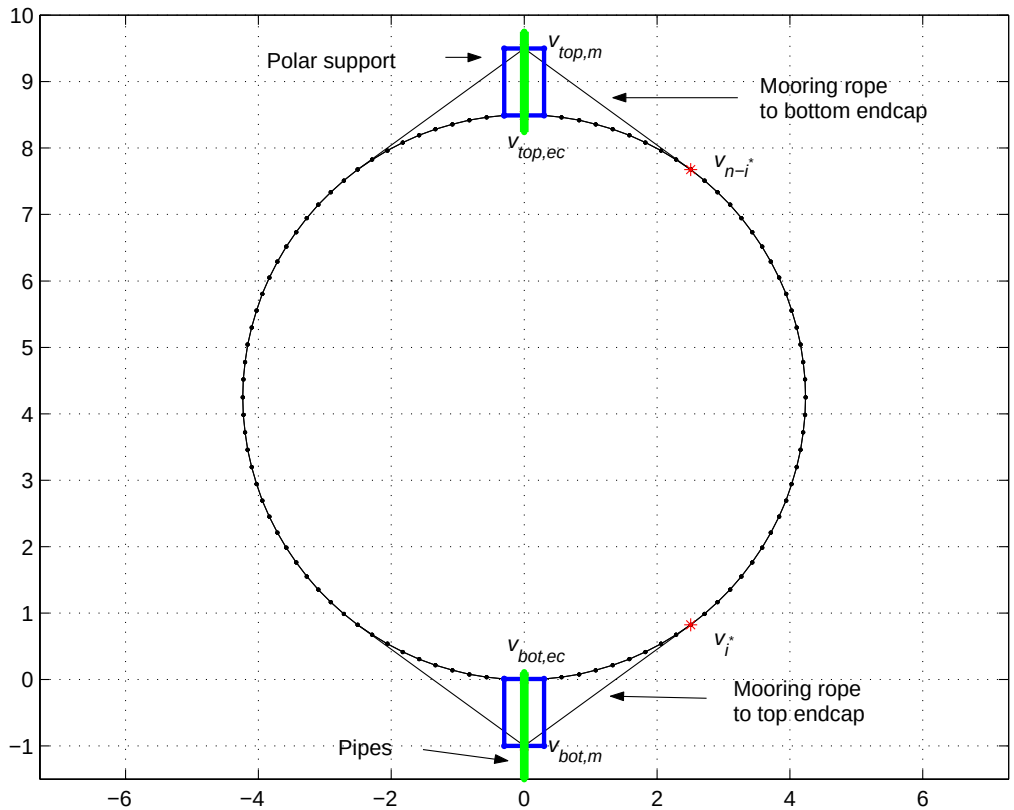
Containment vessel for a solar neutrino detector
L. Cadonati & F. Baginski





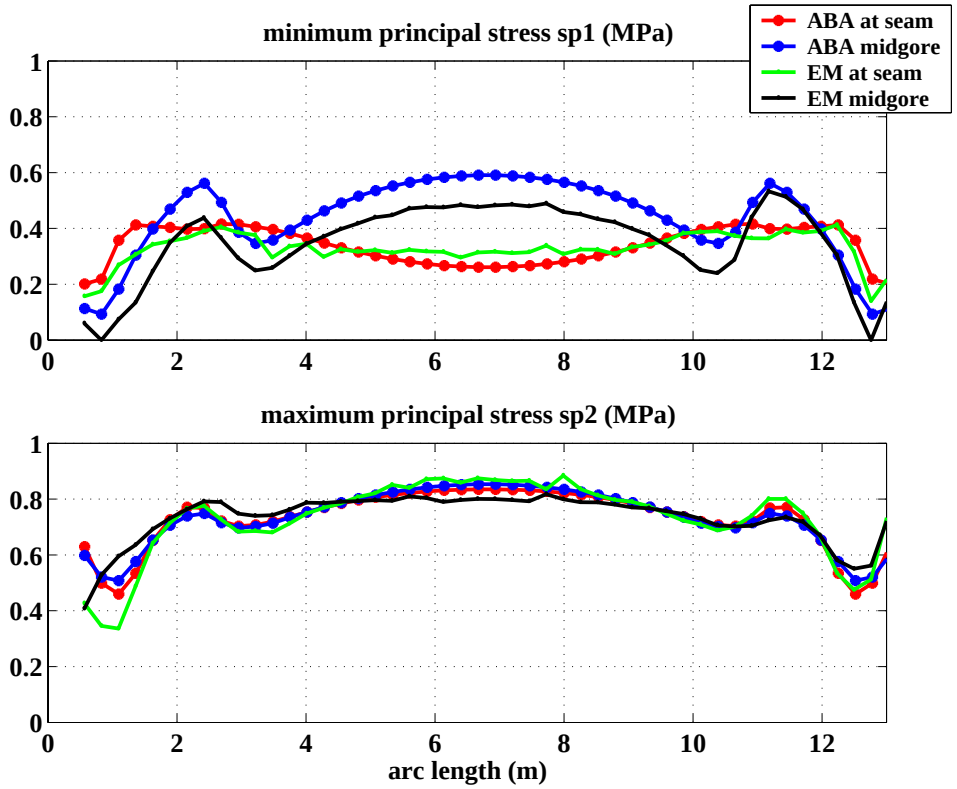
Tendon/gore configuration in Borexino



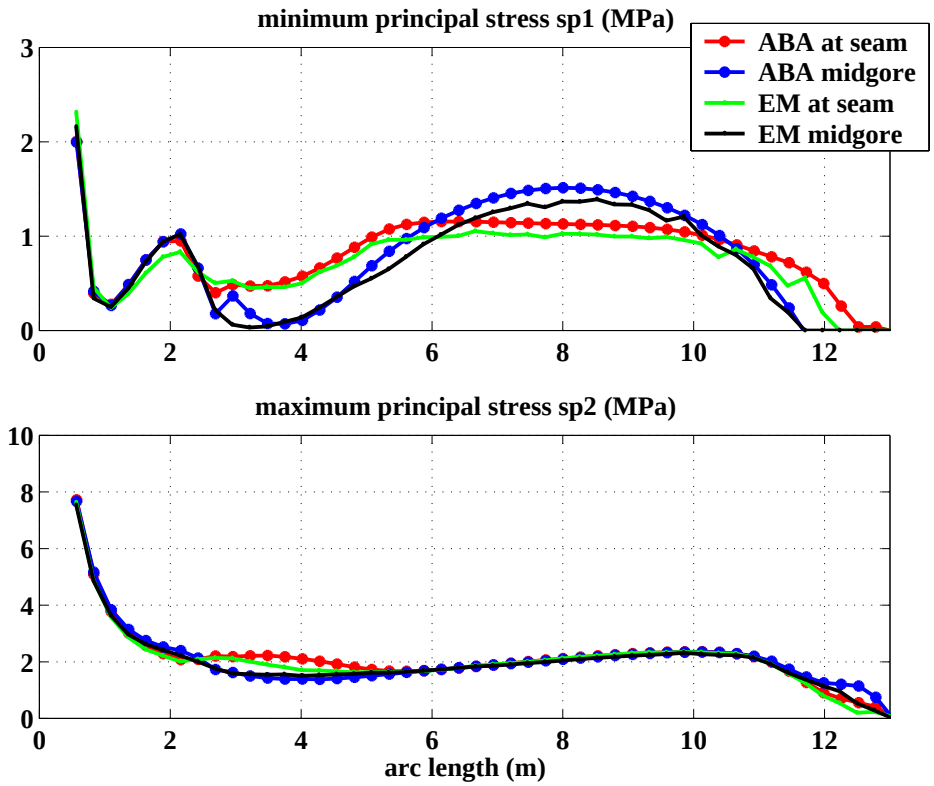


EMsolver and ABACUS Results Comparison

Constant Pressure

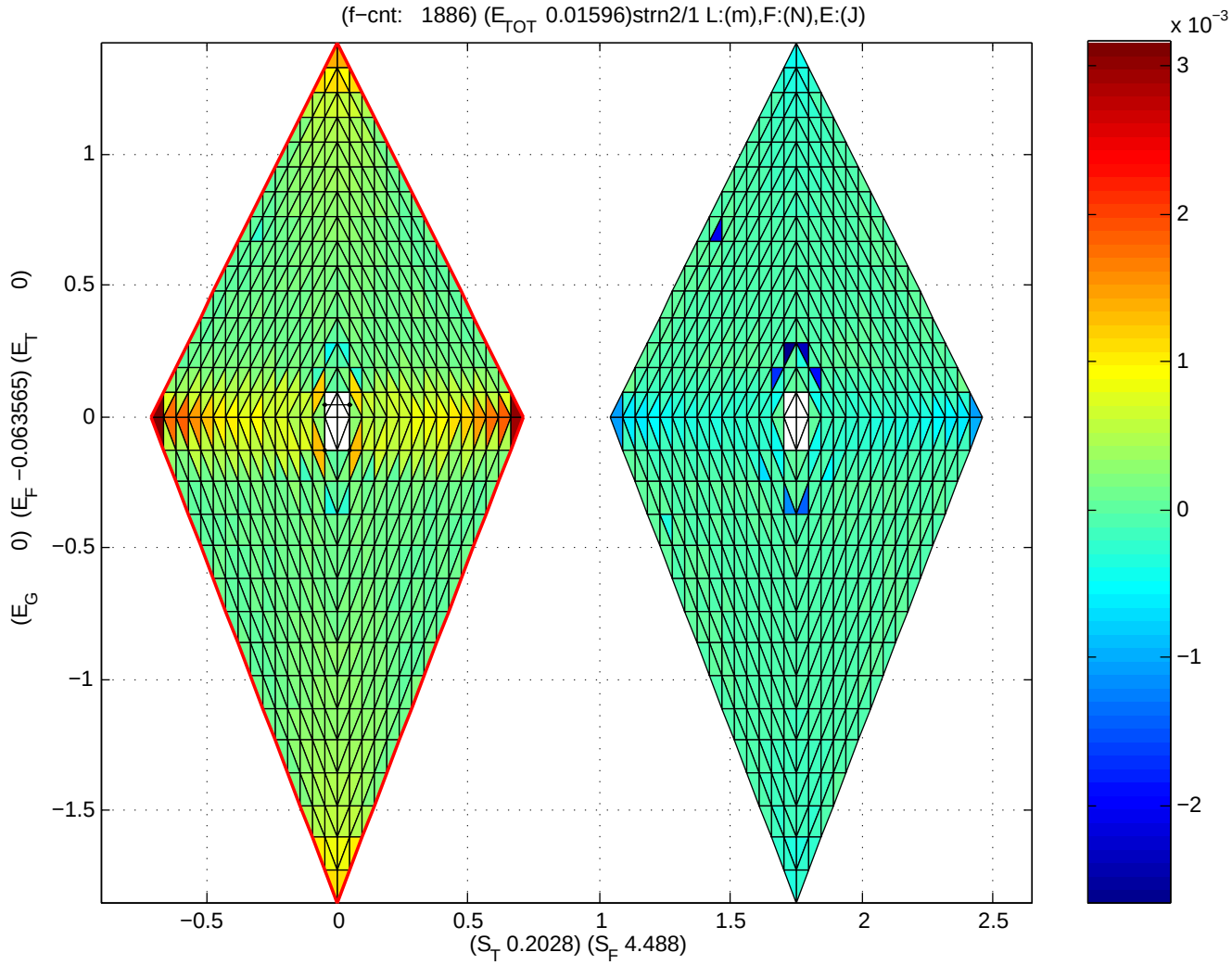


Buoyant Force + Constant Pressure



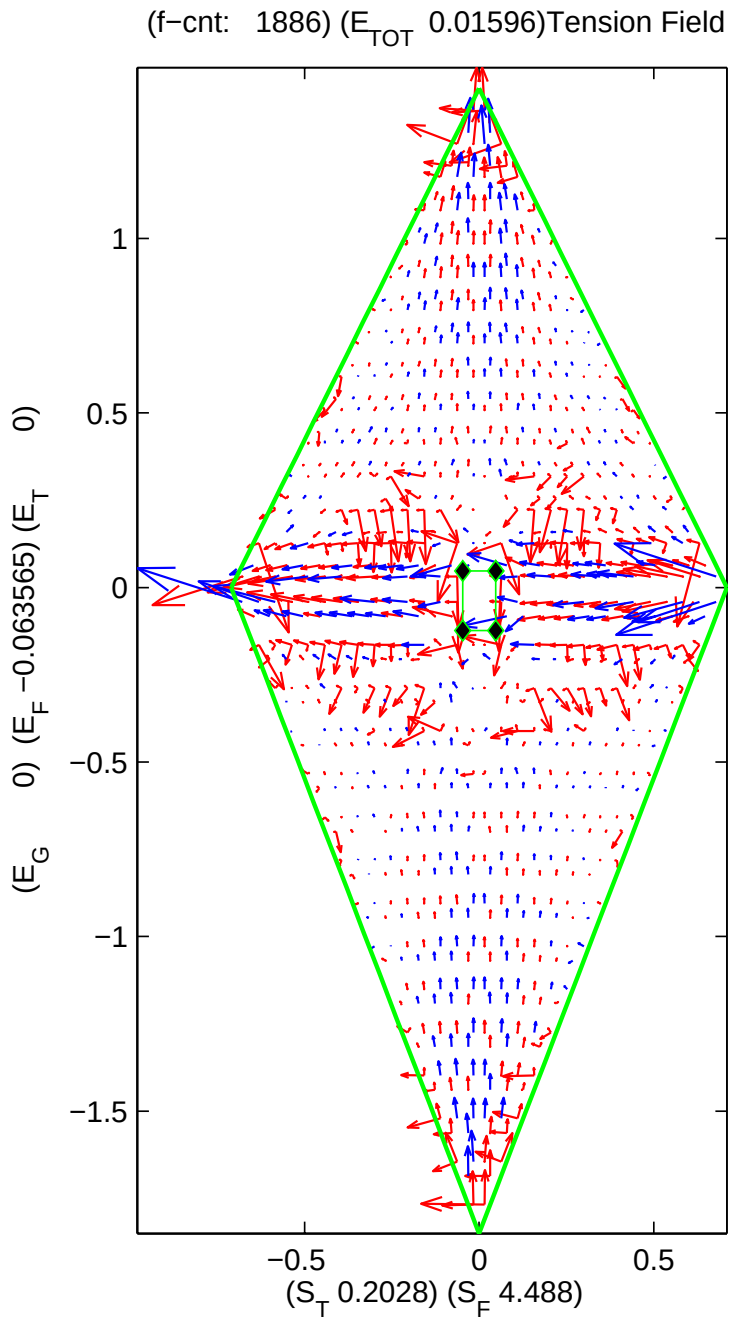
NGST-like sunshield
F. Baginski & S. Vezina

Principal strains



Initially flat membrane. Corners displaced 2.5 cm out-of-plane; Fixed center box.

Wrinkle Pattern

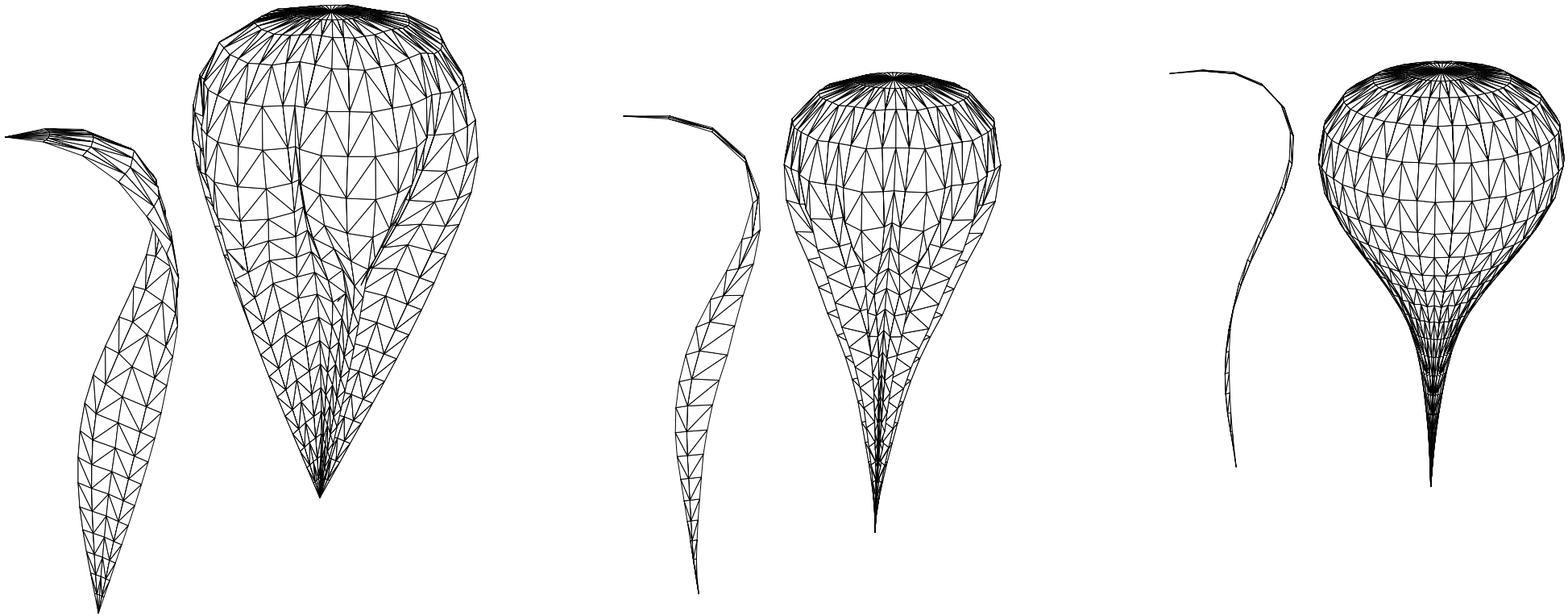


Tension Field − blue
Taut −red

Corners: 2.5 cm out−of−plane
0.2 mm in−plane
Center square fixed

Nonuniqueness of Ascent Shapes

Baginski (2002)



References

- F. Baginski and W. Collier, “Modeling the shapes of constrained partially inflated high altitude balloons,” *AIAA J.*, Vol. 39, No. 9, September 2001, pp. 1662-1672. Errata: *AIAA J.*, Vol. 40, No. 9, September 2002, pp. 1253.
- F. Baginski and W. Schur, “Structural Analysis of Pneumatic Envelopes: A Variational Formulation and Optimization-Based Solution Process,” inflated high altitude balloons,” *AIAA J.*, Vol. 41, No. 2, February 2003, pp. 304-311.

F. Baginski, “Nonuniqueness of strained ascent shapes of high altitude balloons,” COSPAR02-A-01591, The Second World Space Congress/34th COSPAR Scientific Assembly, Houston, Texas 10-19 October 2002.

- F. Baginski and J. Winker, “The natural shape balloon and related models,” 34th COSPAR Scientific Assembly, PSB1-0009-02, Houston, Texas, 10-19 October 2003.
- W. Collier, *Applications of Variational Principles to Modeling a Partially Inflated Scientific Research Balloon*, Ph.D. Thesis, Department of Mathematics, The George Washington University, January 2000.
- W. W. Schur, “Analysis of load tape constrained pneumatic envelopes”, 40th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, AIAA-99-1526, St. Louis, MO, April 1999.
- A. C. Pipkin, “Relaxed energy densities for large deformations of membranes,” *IMA Journal of Applied Mathematics*, Vol. 52, 1994, pp. 297-308.

Acknowledgement

F. Baginski, M. Barg, W. Collier and S. Vezina were supported in part by

NASA Awards NAG5-5292 and NAG5-5353.