THE ANALYSIS OF PNEUMATIC ENVELOPES

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Includes joint work with W. Collier, K. Brakke (Susquehanna University), W. Schur (WFF/PSL), L. Cadanoti (Princeton University)

Overview

Objective: Outline an approach to modeling pneumatic envelopes, with applications to balloons constructed from flat sheets of thin film

1. Applications of balloon technology

2. Design problem for a statically determinate balloon
(UMN 1950s, J. Smalley 1960-70s)

   (a) 'Designing' a spherical balloon

   (b) Axisymmetric membrane with $\sigma_c = 0$: *natural shape* balloon

      Zero-pressure: $P(z) = bz \Rightarrow P(0) = 0$

      Superpressure: $P(z) = bz + p_0 \Rightarrow P(0) = p_0 > 0$

   (c) Pumpkin balloon (Smalley 1971, Yajima 1998, Schur 1998)

Overview (continued)

3. Model Development (including elasticity)
   (a) Previous work on large balloons*
   (b) Geometric features: folds, wrinkling, lobes, wings.
   (c) Variational formulation of analytical problem
      i. Periodic lobes, no strain (FB, AIAA J 1996)
      ii. Explicit internal fold (FB/Collier, ASME JAM 2000)
      iii. Virtual fold (FB/Brakke, AIAA J 1998)
      iv. Energy relaxation (Collier, 2000 GW doctoral thesis)
      v. Ascent shapes+constraints (FB/Collier, AIAA J 2001)
   (d) Numerical model
      i. *EMsolver* - implemented in Matlab
      ii. *Surface Evolver* (K. Brakke, C) - most features implemented

*W. Schur, applied tension field to balloons, AIAA-91-3668-CP

Overview (continued)

4. Benchmark Comparisons with ABACUS

   (a) Strained zero-pressure natural shape at float,
   EMsolver (with virtual fold) vs. ABACUS with tension field
   (collaboration with W. Schur/WFF/PSL)

   (b) Strained spherical containment vessel for a neutrino detector,
   EMsolver with energy relaxation vs. ABAQUS with tension field
   (collaboration with L. Cadonati, Borexino Project, Princeton U)

5. EMsolver applied to nonstandard problems

   (a) Nonuniqueness of equilibrium shapes: ascent shapes of zero-pressure natural shape designs with and without lobes.

   (b) Pumpkin balloon (with tendon/film mismatch, collaboration with W. Schur)

6. Concluding remarks
   This approach could be applied to other super-light membrane structures (e.g., solar sails, gossamer spacecraft, etc.)

Applications of Balloon Technology

- Terrestrial science
  - Zero-pressure balloons (NASA's standard large scientific balloon)
  - Super-pressure balloons (NASA's Ultra Long Duration Balloon - ULDB)
  - Containment vessel for particle detectors

- Extraterrestrial science
  (Mars, Venus, Uranus, Neptune, Saturn, Jupiter, Io, Titan, ...)
  - Solar Montegolfier balloons
  - Parachutes
  - Solar sails
  - Space inflatables and gossamer structures

Designing a Spherical Balloon

Known Quantities	Units	
Buoyancy (float altitude)	N/m^3	$b_d = g(\rho_a - \rho_g)$
Payload	N	L
Film weight density	N/m^2	w_f
Load tendon weight density	N/m	w_t
Number of tendons (gores)		n_g

Find radius R so that Archimedes' Principle is satisfied:

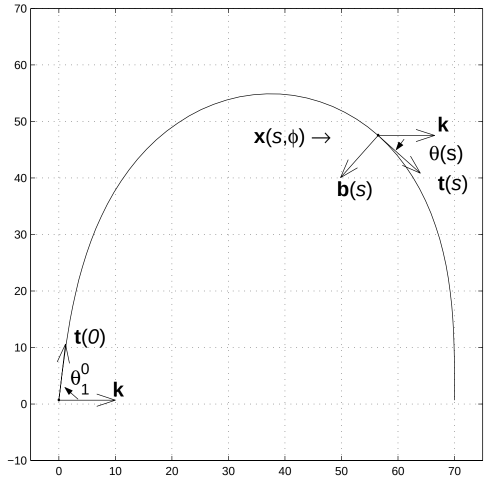
$$\text{Total Lift} = \text{Weight of Balloon System} \tag{1}$$

$$b \cdot \frac{4}{3}\pi R^3 = w_f \cdot 4\pi R^2 + w_t \cdot n_g \pi R + L$$

There is exactly one *positive* solution of (1)

Done!

Equilibrium for an axisymmetric membrane



Balance of Forces on a Patch A with generator $(r(s),0,z(s))$

$$\begin{aligned}\vec{0} \;&=\; \int_0^{\phi} \sigma_m(s)\mathbf{t}(s,\psi)r(s)d\psi - \int_0^{\phi} \sigma_m(s_0)\mathbf{t}(s_0,\psi)r(s_0)d\psi \\ &+\; \int_{s_0}^{s} \sigma_c(\xi)\mathbf{e}_2(\phi)d\xi - \int_{s_0}^{s} \sigma_c(\xi)\mathbf{e}_2(0)d\xi + \int_A \mathbf{f}(\xi,\psi)r(\xi)d\psi d\xi.\end{aligned}$$

$$\mathbf{f}(s,\phi) = -p(s)\mathbf{b}(s,\phi) - w(s)\mathbf{k}$$

$p = bz + p_0$ - hydrostatic pressure, b - buoyancy

$\sigma_m(s)\mathbf{t}(s)$ - meridional contact force (σ_m - meridional stress resultant)

$\sigma_c(s)\mathbf{e}_2(\phi)$ - circumferential contact force (σ_c - hoop stress resultant)

Natural-Shape Equations ($\sigma_c = 0$, UMN, 1950s)

Further developement and enhancements (J. Smalley, 1960-70s).

$$\frac{\partial}{\partial\phi}\mathbf{e}_2(\phi) = -\mathbf{e}_1(\phi) \quad \Longrightarrow \quad \vec{0} = \frac{\partial}{\partial s}(r\sigma_m\mathbf{t}) - \sigma_c\mathbf{e}_1(\phi) + r\mathbf{f}.$$

Let $T = 2\pi r\sigma_m(s)$

ODEs			Boundary Conditions		
θ'	$=$	$-2\pi r(w\sin\theta + p)/T,$	$\theta(0)$	$=$	$\theta_1^0,$
T'	$=$	$2\pi r w\cos\theta,$	$T(0)$	$=$	$L/\cos\theta_1^0,$
z'	$=$	$\cos\theta,$	$z(0)$	$=$	$0,$
r'	$=$	$\sin\theta.$	$r(0)$	$=$	$0.$

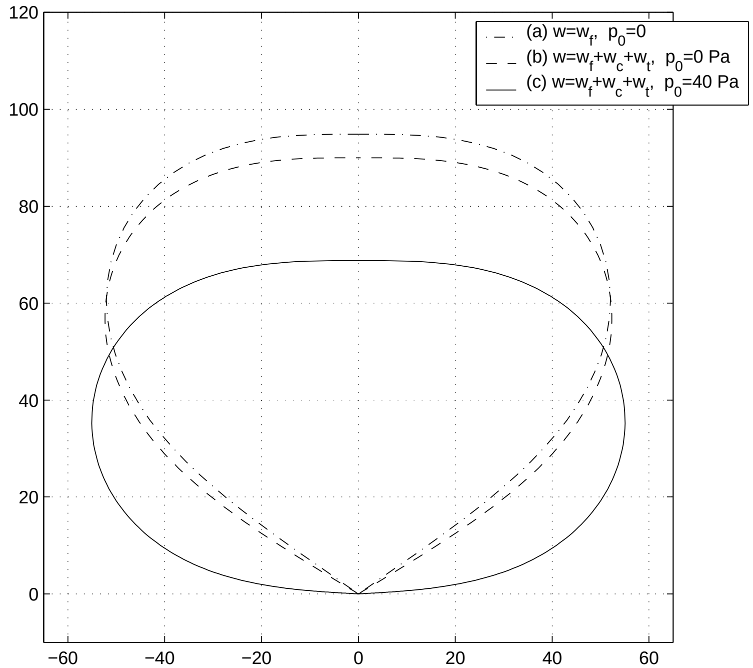
Auxiliary Conditions

$$\begin{aligned} \theta(\theta_1^0,\ell) &= -\tfrac{1}{2}\pi \\ r(\theta_1^0,\ell) &= 0 \end{aligned}$$

$[r(s), z(s), T(s), \theta(s)]$ & (θ_1^0, ℓ) are found via a shooting method

Archimedes satisfied: $bV = W_{film} + L$

Natural-Shape Profiles, $\sigma_c = 0, T \neq$ const.



- Zero-pressure balloons. Typical missions are several days. ZP-balloons are open at base and need significant ballast to maintain altitude (a)-(b)
- Super-pressure balloon. Add sufficient pressure so that day/night volume changes are reduced. (c)
- Available thin films: not strong enough to contain the pressure, too heavy, too expensive
- Use a doubly curved gore with very strong tendons $\Rightarrow$ pumpkin shape.

Observations and Model Assumptions

- Linear stress-strain constitutive law
- Isotropic material (E-Youngs modulus, ν-Poisson's ratio)
- Constant strain model ($T \in S_{Ref} \longleftrightarrow T \in S$)

- Fine wrinkling via energy relaxation - facets are *taut, slack, wrinkled*
- Energy relaxation allows a tension field solution
- Folds can be used to describe distribution of excess material.
- Load tendons behave like sticky linearly elastic strings

- Shapes are characterized by *large deformations* but *small strains*.

- Hydrostatic pressure is shape dependent

Variational Principle for a Strained Balloon

For $S \in C$,
Minimize: $E_{Total}(S) = E_P + E_f + E_t + S_t + S_f$
Subject to: $V = V_0$

(closed system, $P(z) = bz + p_0, p_0$ is obtained from Lagrange multiplier)

E_T	Total energy
E_P	hydrostatic pressure potential
E_f	gravitational potential energy due to film weight
E_t	gravitational potential energy due to tendon weight
S_t	strain energy of tendons
S_f	strain energy of film

Energy Terms

Hydrostatic Pressure: $E_P = -\int_V p\, dV = -\int_S (\frac{1}{2}bz^2 + p_0 z)\vec{k}\cdot d\vec{S},$

Film Weight: $E_f = \int_S w_f z\, dA$

Tendon Weight: $E_t = \sum_{i=1}^{n_g} \int_0^{\ell_d} w_t^i z\, ds$

Tendon Strain: $S_t = \sum_{i=1}^{n_g} \int_0^{\ell_d} W_c^i(s)\, ds, \quad W_c^i(s) = \frac{1}{4}K_t(|\dot{\gamma}_i|^2 - 1)^2.$

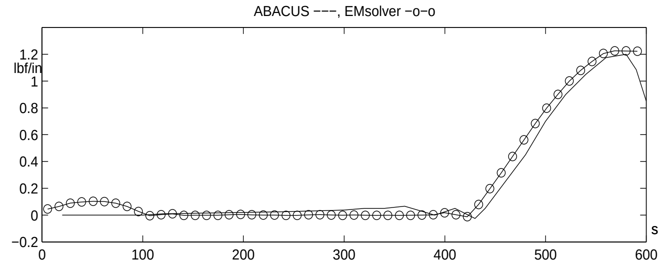
Film Strain: $S_f = \int_\Omega \hat{W}_f(\mathbf{G})dA, \quad \hat{W}_f(\mathbf{G}) = \frac{1}{2}\mathbf{S} : \mathbf{G};$
$\mathbf{G} = \frac{1}{2}(\mathbf{C} - \mathbf{I})$ - Green strain, $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ - Cauchy strain, and $\mathbf{F}$ - def. grad.

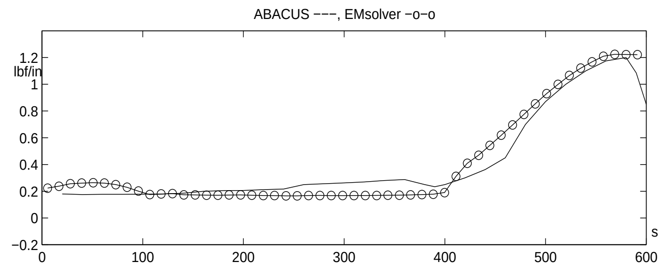
Second Piola-Kirchoff stress tensor

$$\mathbf{S}(\mathbf{G}) = \frac{tE}{1-\nu^2}\left(\mathbf{G} + \nu \text{Cof}(\mathbf{G}^T)\right).$$

Fine wrinkling: replace $\hat{W}_f$ by its relaxation $\hat{W}_f^*$ leading to a *Tension Field*

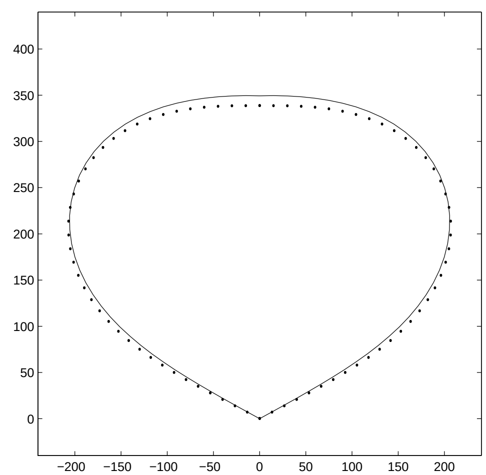
Comparison of EMsolver (virtual fold) with ABACUS (tension field)

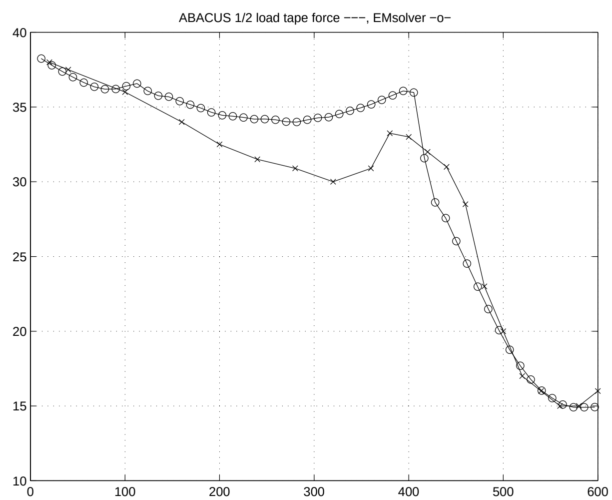




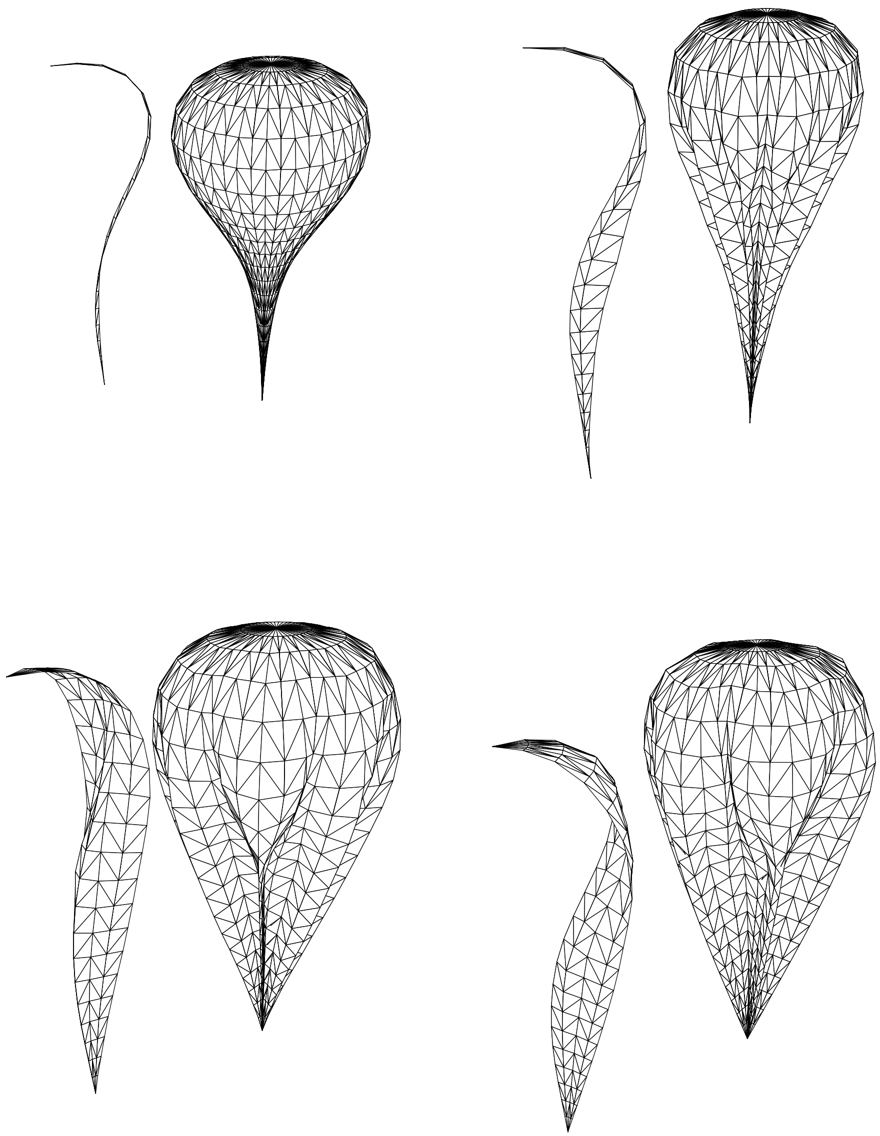
Parameters

159 gores	Gore length 182 m
$b = 0.05429$N/m^3	$\nu = 0.82$
$E = 124$ MPa	$E_t = 26.24$ kN
$m_f = 18.7$ g/m^2	$m_t = 0.0313$ g/m
$V = 832515$m^3	(zero-slackness)

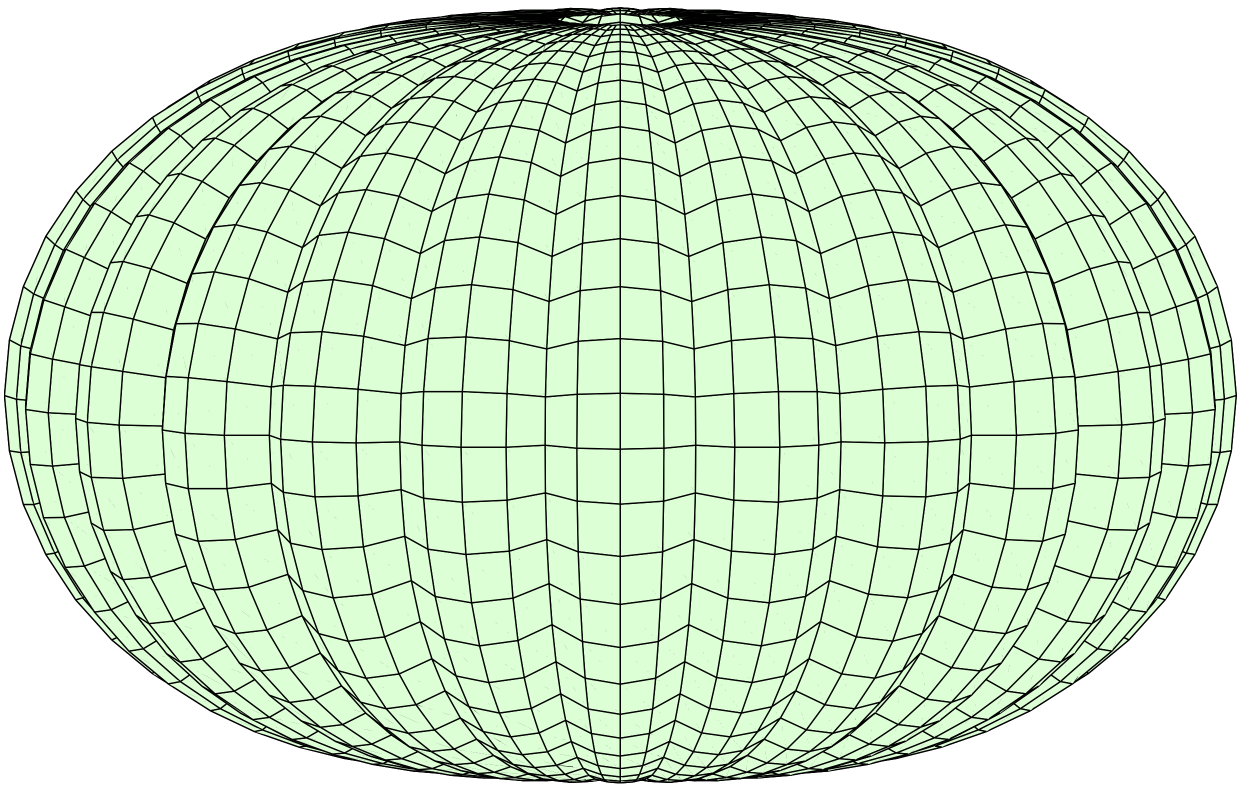




Strained lobed ascent shapes (multiple solutions for same loading)



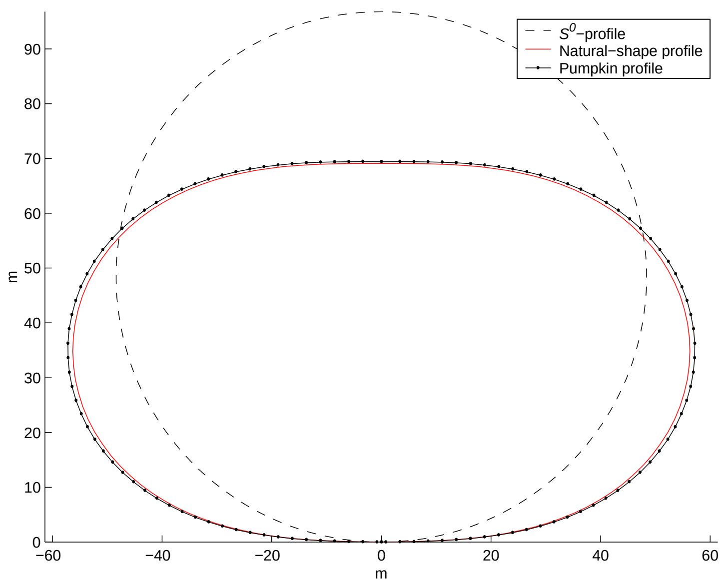
The Pumpkin Balloon

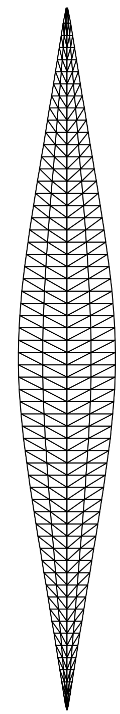


Background on the Pumpkin Balloon

- J. Smalley coined the term *pumpkin balloon*. Extensibility of the film is used to achieve the doubly curved pumpkin gore (early 1970s).
- CNES built several small pumpkin balloons, cutting half-gore panels with extra material (M. Rougeron, CNES/France, mid-late 1970s)
- Sewing techniques to gather material at gore seams (N. Yajima, Japan, 1998, see Adv. in Space Res., 2000).
- NASA/ULDB - structural lack-of-fit (shorten tendons) + material properties (W. Schur, PSL/WFF, 1998, see AIAA-99-1526).

Strained Pumpkin Balloon



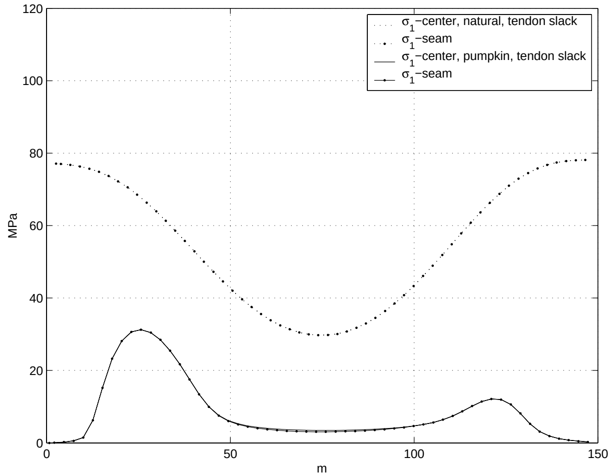


Principal stresses for superpresure natural-shaped and pumpkin balloons.

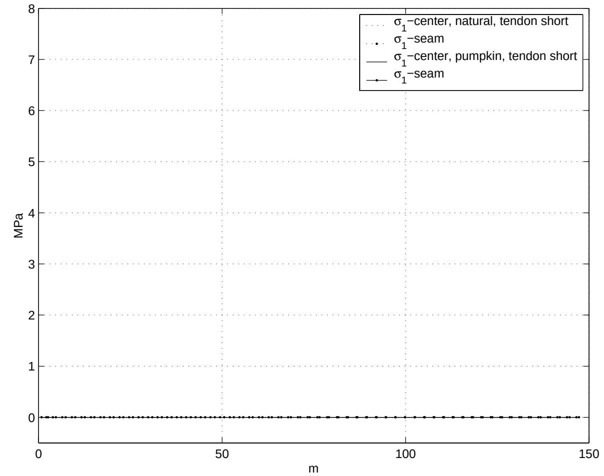
(a)-(b) 2.9% slackness; (c)-(d) 2.2% tendon shortening.

(joint work with W. Schur)

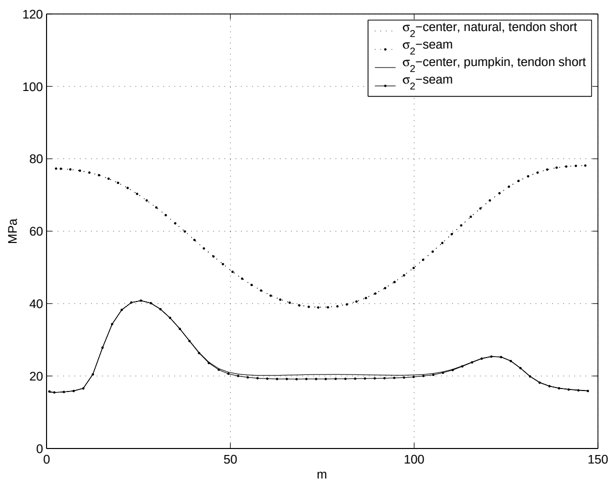
(a) “Meridional” stresses-slack tendons



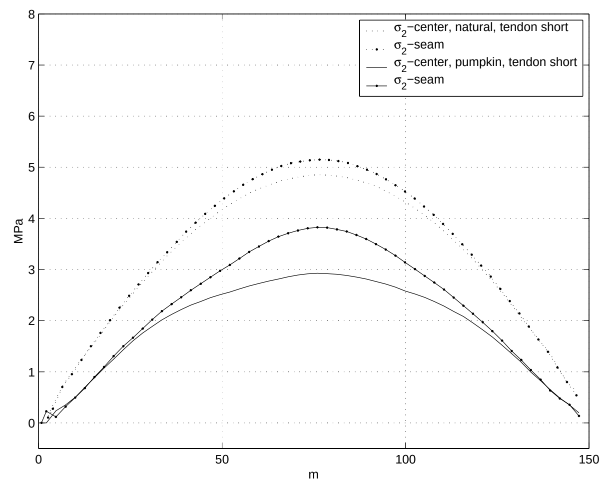
(c) “Meridional” stresses - tendon shortening



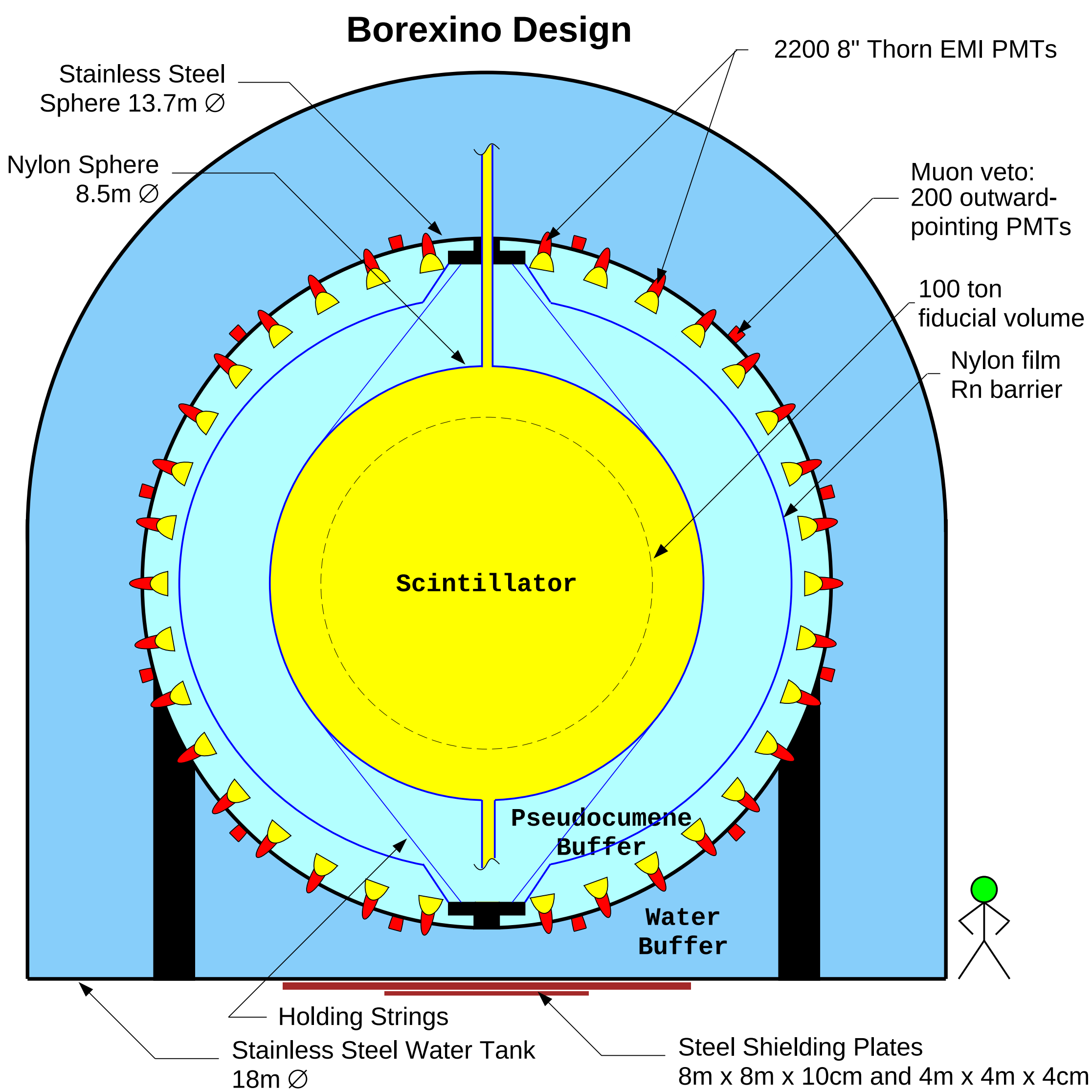
(b) “Hoop” stresses - slack tendons



(d) “Hoop” stresses - tendon shortening



Borexino Containment Vessel (joint work with L. Cadonati)



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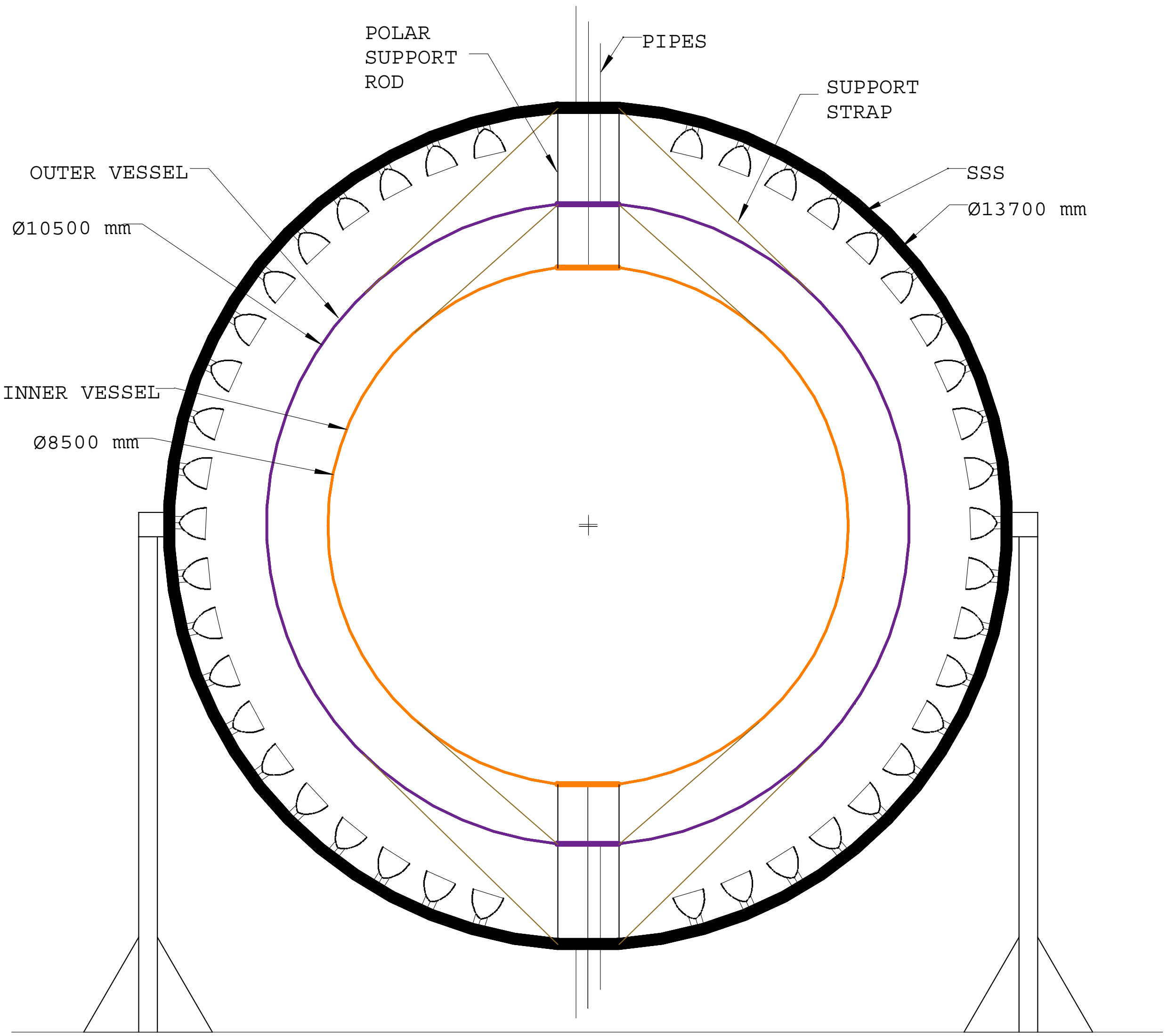
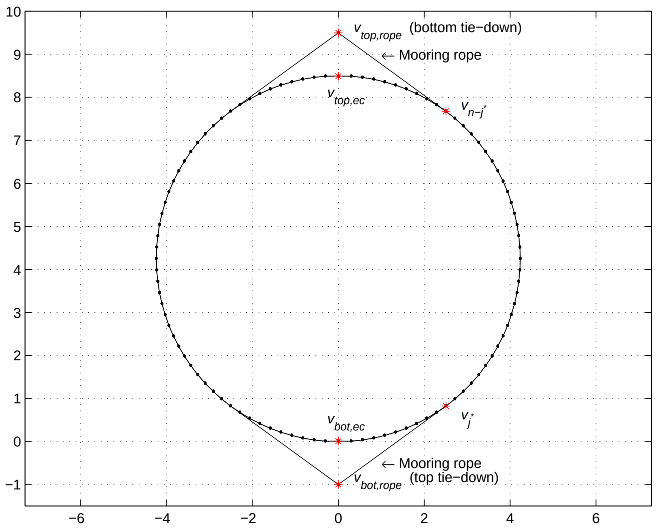
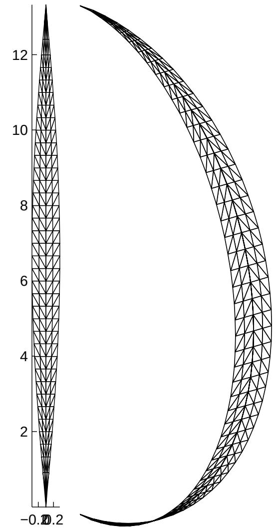


Borexino (continued)

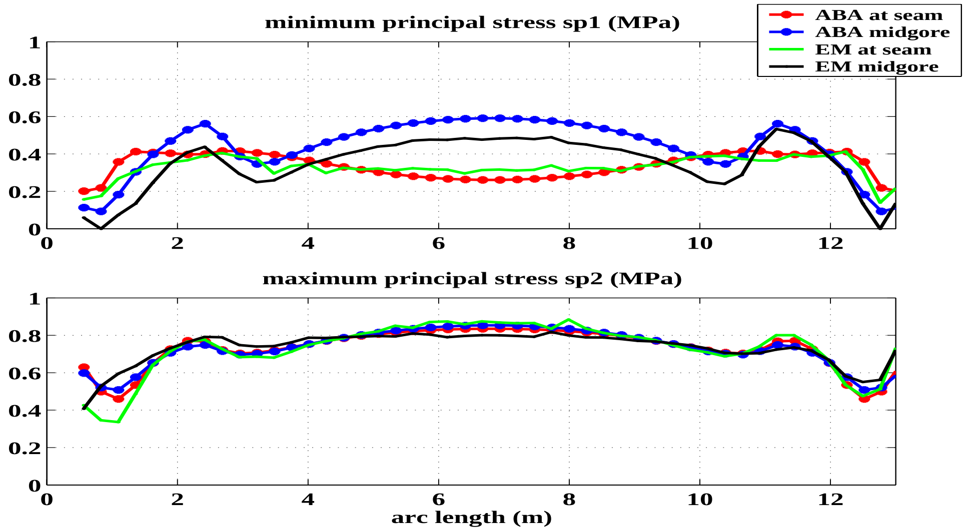
Schematic



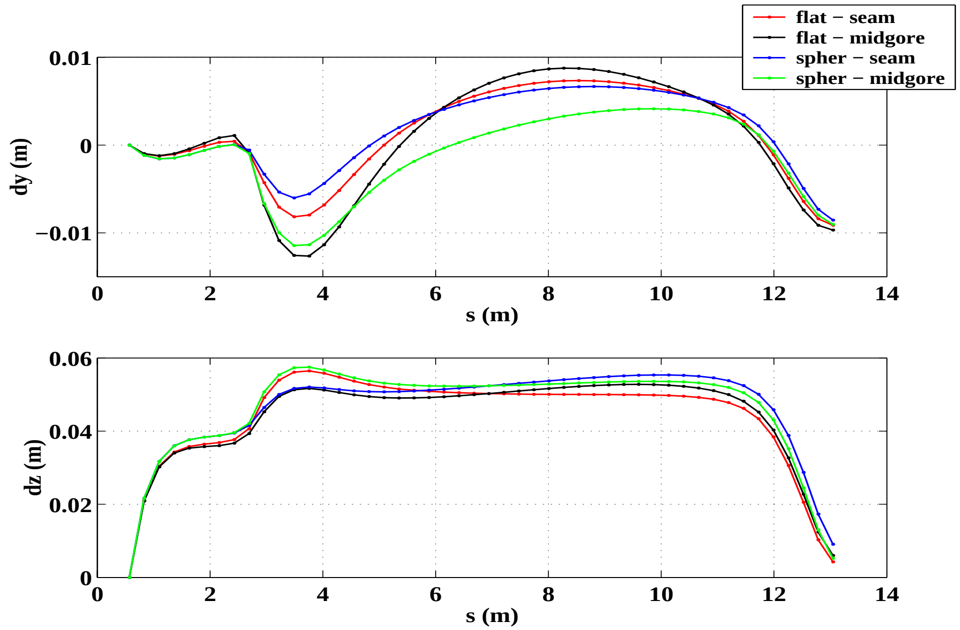
Reference/Initial Configs.



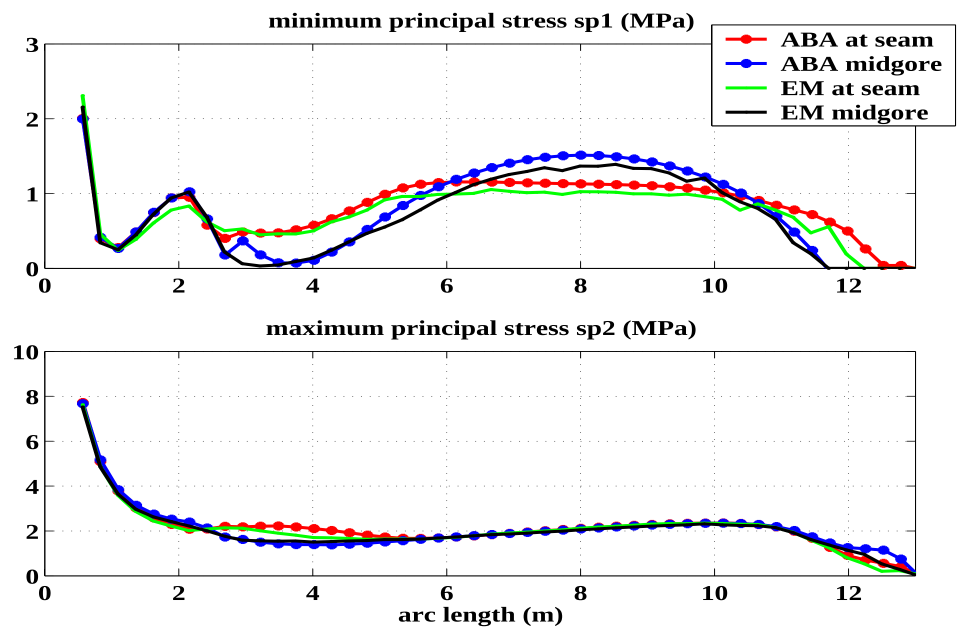
Principal Stress Resultants, $P(z) = 50$ Pa



ABACUS displacements ($b \neq 0$, 0.1% density difference)



Prin. Stress Resultants, open system, $P(0) = 96$ Pa, $P(2R) = 170$ Pa



Future Balloon Research

- Pumpkin - Deployment problems, new features observed during ascent and launch (stress raisers?), optimal gore design.
- Validation - compare actual strain measurements to EMsolver predictions.
- Aerodynamic loading of a strained balloon (link computational fluid dynamics and structural analysis).
- Apply our approach to space inflatables, solar sails & gossamer structures.