Finite Element Solution of Fluid-Structure Interaction Problems

Gordon C. Everstine
Naval Surface Warfare Center, Carderock Div.
Bethesda, Maryland 20817
18 May 2000
EverstineGC@nswccd.navy.mil
Fluid-Structure Interaction

Exterior Problems: Vibrations, Radiation and Scattering, Shock Response

Interior Problems: Acoustic Cavities, Piping Systems
Structural Acoustics

Problems: Radiation and Scattering
(Time-Harmonic and Transient)
Fluid-Loaded Vibrations
Acoustic Cavities (e.g., Tanks, Pumps)
Fluid-Filled Piping System Dynamics

Approaches: Finite Element
Finite Element/Boundary Element
Finite Element/Infinite Element
Doubly Asymptotic Approximations (DAA)
Retarded Potential Integral Equation (Transient)
Large-Scale Fluid-Structure Modeling Approaches

- **Structure**
  - Finite elements

- **Fluid**
  - Boundary elements
  - Finite elements with absorbing boundary
    - Infinite elements
    - ?c impedance
  - Doubly asymptotic approximations (shock)
  - Retarded potential integral equation (transient)
Exterior Fluid Mesh

383,000 Structural DOF

248,000 Fluid DOF

631,000 Total DOF
Structural-Acoustic Analogy

General Equation: \( \nabla^2 \phi + g = a \ddot{\phi} + b \dot{\phi} \)

Navier’s Equation of Elasticity:
\[
\frac{\lambda + 2G}{G} u_{,xx} + u_{,yy} + u_{,zz} + \frac{\lambda + G}{G} (v_{,xy} + w_{,xz}) + \frac{1}{G} f_x = \frac{\rho}{G} \dddot{u}
\]

where \( \lambda = \frac{E \nu}{(1 + \nu)(1 - 2\nu)} \) (Lamé constant)

Analogy: \( u = \phi, \ v = w = 0, \ \lambda = -G \Rightarrow \)

\( G_e \) arbitrary, \( E_e = \alpha G_e, \ \rho_e = \alpha G_e, \ \alpha = \begin{cases} 10^{-5} & (2-D) \\ 10^{20} & (3-D) \end{cases} \)

\( F_x = f_x V = G_e g V - (G_e b V) \dot{\phi} \) (load + dashpot)
Fluid-Structure Interaction Equations

**STRUCTURE:**  \[ M \ddot{u} + B \dot{u} + K u = -Ap + f_1 \]

**COMPRESSIBLE FLUID:**  \[ \nabla^2 p = \ddot{p}/c^2, \ \partial p/\partial n = -\rho \ddot{u}_n \]

\[ Q \ddot{p} + H p = \rho A^T \dddot{u} \]

**COUPLED EQ:**
\[
\begin{bmatrix}
M & O \\
-\rho A^T & Q
\end{bmatrix}
\begin{Bmatrix}
\dddot{u} \\
\ddot{p}
\end{Bmatrix}
+
\begin{bmatrix}
B & O \\
O & C
\end{bmatrix}
\begin{Bmatrix}
\dddot{u} \\
\ddot{p}
\end{Bmatrix}
+
\begin{bmatrix}
K & A \\
O & H
\end{bmatrix}
\begin{Bmatrix}
u \\
p
\end{Bmatrix}
=
\begin{Bmatrix}
f_1 \\
f_2
\end{Bmatrix}
\]

**SYMMETRIC POTENTIAL FORMULATION:**  \[ q = \int p \, dt \]

\[
\begin{bmatrix}
M & O \\
O & Q
\end{bmatrix}
\begin{Bmatrix}
\dddot{u} \\
\dddot{q}
\end{Bmatrix}
+
\begin{bmatrix}
B & A \\
A^T & C
\end{bmatrix}
\begin{Bmatrix}
\dddot{u} \\
\dddot{q}
\end{Bmatrix}
+
\begin{bmatrix}
K & O \\
O & H
\end{bmatrix}
\begin{Bmatrix}
u \\
q
\end{Bmatrix}
=
\begin{Bmatrix}
f_1 \\
g_2
\end{Bmatrix}
\]
Fluid Finite Elements

• Pressure Formulation
  – $E_e = 10^{20}G_e, \rho_e = G_e/c^2, G_e$ arbitrary
  – Direct input of areas in $K$ and $M$ matrices

• Symmetric Potential Formulation
  – $u_z$ represents velocity potential
  – New unknown: $q = \int p \, dt$ (velocity potential)
  – $G_e = -1/\rho, \quad E_e = -10^{20}/\rho, \quad \rho_e = -1/(\rho c^2)$
  – Direct input of areas in $B$ (damping) matrix
Finite Element Formulations of FSI

<table>
<thead>
<tr>
<th>Fluid unknown</th>
<th>Displacement</th>
<th>Pressure</th>
<th>Velo. Pot.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid DOF/pt.</td>
<td>Total disp.</td>
<td>$p_s$</td>
<td>$q_s = \int p_s , dt$</td>
</tr>
<tr>
<td>Coef. matrices</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Spurious modes</td>
<td>Symmetric</td>
<td>Nonsymmetric</td>
<td>Symmetric</td>
</tr>
<tr>
<td>F-S interface cond.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>MPC</td>
<td>Matrix entry</td>
<td>Matrix entry</td>
</tr>
</tbody>
</table>

+ 3-variable formulations
Displacement Formulation

- Fundamental unknown: fluid displacement (3 DOF/point)
- Model fluid domain with elastic F.E. (e.g., elastic solids in 3-D, membranes in 2-D)
- Any coordinate systems; constrain rotations (DOF 456)
- Material properties (3-D): $G_e \approx 0 \Rightarrow E_e = (6\varepsilon)\rho c^2$, $\nu_e = \frac{1}{2} - \varepsilon$, $\rho_e = \rho$, where $\varepsilon = 10^{-4}$
- Boundary conditions:
  - Free surface: natural B.C.
  - Rigid wall: $u_n = 0$ (SPC or MPC)
  - Accelerating boundary: $u_n$ continuous (MPC), slip
- Real and complex modes, frequency and transient response
- 3 DOF/point, spurious modes
Displacement Method Mode Shapes

0 Hz  
Spurious

1506 Hz  
Good

1931 Hz  
Spurious

1971 Hz  
Good
Helmholtz Integral Equations

\[ \int_S \left( p \frac{\partial D}{\partial n} - \frac{\partial p}{\partial n} D \right) dS = \begin{cases} \frac{1}{2} p - p_l & (S) \\ p - p_l & (E) \\ -p_l & (I) \end{cases} \]

\[ D = e^{-ikr/2\pi} \quad \text{(GREEN'S FUNCTION)} \]

SURFACE: \[ Ep = C v_n + p_l \]
Matrix Formulation of Fluid-Structure Problem

Structure: \[ Zv = F - GAp \]
\[ Z = \left(-\omega^2M + i\omega B + K\right)/(i\omega) \]

Fluid: \[ Ep = Cv_n + p_i \]

F-S Transformation: \[ F = GF_n, \quad v_n = G^Tv \]

Coupled Equations: \[ (E+CG^TZ^{-1}GA)p = CG^TZ^{-1}F + p_i \]

Velocity: \[ v_n = G^TZ^{-1}F - G^TZ^{-1}GAp \]
Spherical Shell With Sector Drive

FAR-FIELD PRESSURE

\[ \frac{|p|}{r} \]

\[ \frac{p}{p_o a} \]

\[ k a = 5 \]

--- EXACT

--- --- NASHUA

Everstine 5/18/00
Added Mass by Boundary Elements

Helmholtz equation: \[ \int_S \left( p \frac{\partial D}{\partial n} - D \frac{\partial p}{\partial n} \right) dS = \frac{p}{2} \]

\[ D = \frac{e^{-ikr}}{4\pi r} \quad \text{(Green’s function)} \]

Boundary element equation: \[ E_p = C' v_n \]

Added mass matrix: \[ M_a = GAE^{-1} \left( \frac{C}{i\omega} \right) G^T \]

Eigenproblem: \[ (M + M_a)\ddot{u} + Ku = 0 \]
### Frequencies of Submerged Cylindrical Shell

<table>
<thead>
<tr>
<th>Mode</th>
<th>N</th>
<th>M</th>
<th>L</th>
<th>F.E.</th>
<th>B.E.</th>
<th>Approx.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td>1.13</td>
<td>1.13</td>
<td>1.11</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td></td>
<td>1.63</td>
<td>1.44</td>
<td>1.38</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td></td>
<td>1.79</td>
<td>1.81</td>
<td>1.77</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>1</td>
<td></td>
<td>3.61</td>
<td>3.67</td>
<td>3.57</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td></td>
<td>4.44</td>
<td>4.26</td>
<td>4.22</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>3</td>
<td></td>
<td>4.81</td>
<td>4.82</td>
<td>4.70</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>3</td>
<td></td>
<td>4.94</td>
<td>4.93</td>
<td>4.82</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>1</td>
<td></td>
<td>6.31</td>
<td>6.38</td>
<td>6.18</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>3</td>
<td></td>
<td>6.83</td>
<td>6.86</td>
<td>6.67</td>
</tr>
</tbody>
</table>

*N=circumferential, M=longitudinal, L=radial (end)*
Low Frequency F.E. Piping Model

- Beam model for pipe
- 1-D acoustic fluid model for fluid (rods)
- Two sets of coincident grid points
- Pipe and fluid have same transverse motion
- Elbow flexibility factors are used
- Adjusted fluid bulk modulus for fluid in elastic pipes
  \[ E = \frac{B}{1 + BD/E_s t} \]
- Arbitrary geometry, inputs, outputs
- Applicable below first lobar mode
Planar Piping System: Free End Response
Needs

• Link between CAD model and FE model
• Infinite elements
• Meshing (e.g., between hull and outer fluid FE surface)
• Modeling difficulties (e.g., joints, damping, materials, mounts)
• Error estimation and adaptive meshing
References


http://ocean.dt.navy.mil/pubs/__pubs.htm