Finite Element Solution of Fluid-Structure Interaction Problems

Gordon C. Everstine
Naval Surface Warfare Center, Carderock Div.
Bethesda, Maryland 20817

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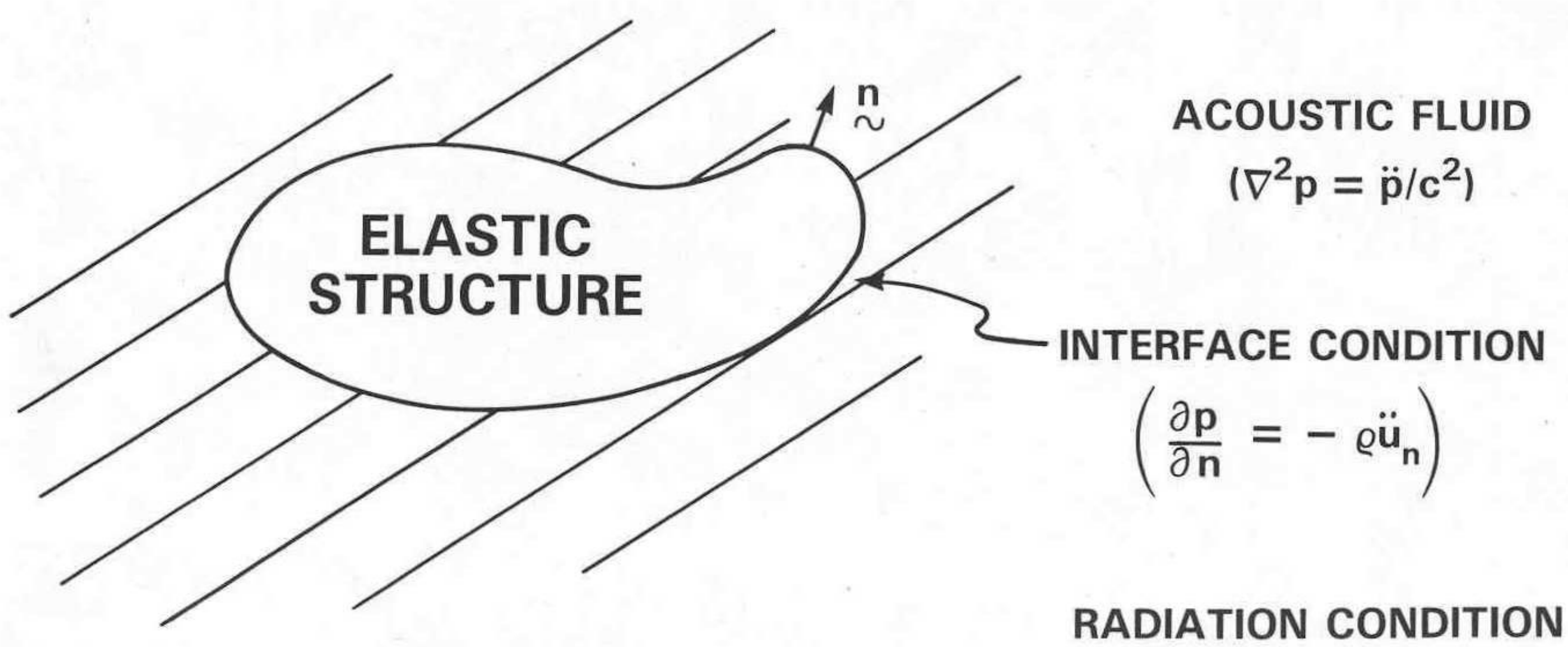
EverstineGC@nswccd.navy.mil

Fluid-Structure Interaction



Exterior Problems: Vibrations, Radiation and Scattering,
Shock Response

Interior Problems: Acoustic Cavities, Piping Systems

Structural Acoustics

Problems: Radiation and Scattering
(Time-Harmonic and Transient)
Fluid-Loaded Vibrations
Acoustic Cavities (e.g., Tanks, Pumps)
Fluid-Filled Piping System Dynamics

Approaches: Finite Element
Finite Element/Boundary Element
Finite Element/Infinite Element
Doubly Asymptotic Approximations (DAA)
Retarded Potential Integral Equation
(Transient)

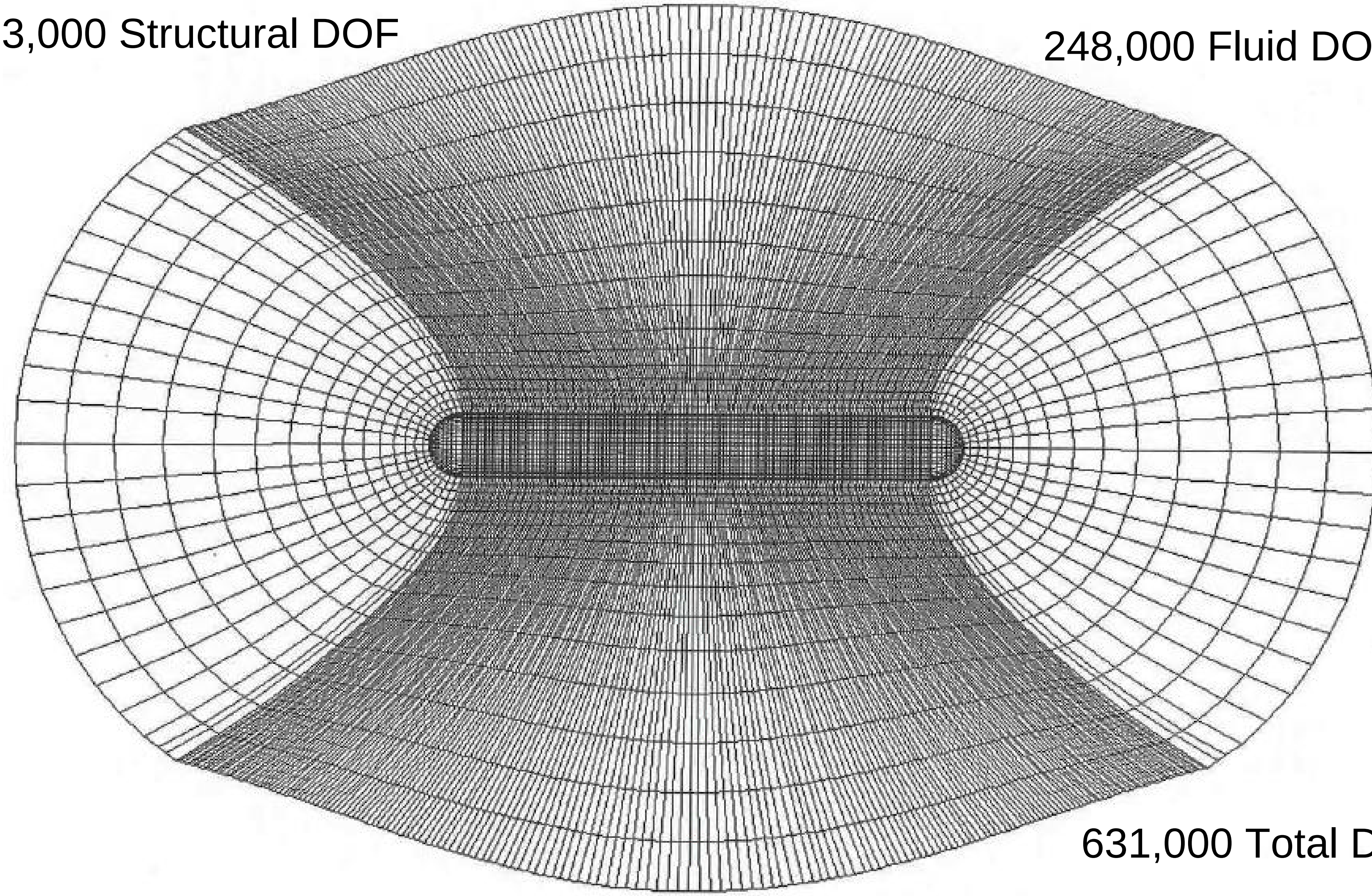
Large-Scale Fluid-Structure Modeling Approaches

- Structure
  - Finite elements
- Fluid
  - Boundary elements
  - Finite elements with absorbing boundary
    - Infinite elements
    - ?c impedance
  - Doubly asymptotic approximations (shock)
  - Retarded potential integral equation (transient)

Exterior Fluid Mesh

383,000 Structural DOF

248,000 Fluid DOF



631,000 Total DOF

Structural-Acoustic Analogy

General Equation: $\nabla^2\phi + g = a\,\ddot{\phi} + b\,\dot{\phi}$

Navier's Equation of Elasticity:

$$\frac{\lambda + 2G}{G}u_{,xx} + u_{,yy} + u_{,zz} + \frac{\lambda + G}{G}(v_{,xy} + w_{,xz}) + \frac{1}{G}f_x = \frac{\rho}{G}\ddot{u}$$

$$\text{where } \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad \text{(Lamé constant)}$$

Analogy: $u = \phi, \quad v \equiv w \equiv 0, \quad \lambda = -G \;\Rightarrow$

$$G_e \text{ arbitrary}, \; E_e = \alpha G_e, \; \rho_e = aG_e, \; \alpha = \begin{cases} 10^{-5} & \text{(2-D)} \\ 10^{20} & \text{(3-D)} \end{cases}$$

$$F_x = f_x V = G_e g V - (G_e b V)\dot{\phi} \quad \text{(load + dashpot)}$$

Fluid-Structure Interaction Equations

STRUCTURE: $M\,\ddot{u} + B\,\dot{u} + K\,u = -Ap + f_1$

COMPRESSIBLE FLUID: $\nabla^2 p = \ddot{p}/c^2,\ \partial p/\partial n = -\rho \ddot{u}_n$

$$Q\ddot{p} + Hp = \rho A^T \ddot{u}$$

COUPLED EQ:

$$\begin{bmatrix} M & O \\ -\rho A^T & Q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} B & O \\ O & C \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} K & A \\ O & H \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

SYMMETRIC POTENTIAL FORMULATION: $q = \int p\, dt$

$$\begin{bmatrix} M & O \\ O & Q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} B & A \\ A^T & C \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} K & O \\ O & H \end{bmatrix} \begin{Bmatrix} u \\ q \end{Bmatrix} = \begin{Bmatrix} f_1 \\ g_2 \end{Bmatrix}$$

Fluid Finite Elements

- Pressure Formulation
  - $E_e = 10^{20}G_e$, $\rho_e = G_e/c^2$, G_e arbitrary
  - Direct input of areas in K and M matrices
- Symmetric Potential Formulation
  - u_z represents velocity potential
  - New unknown: $q = \int p \, dt$ (velocity potential)
  - $G_e = -1/\rho$, $E_e = -10^{20}/\rho$, $\rho_e = -1/(\rho c^2)$
  - Direct input of areas in B (damping) matrix

Finite Element Formulations of FSI

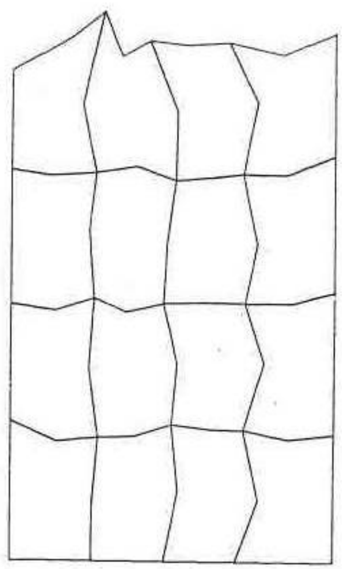
	Displacement	Pressure	Velo. Pot.
Fluid unknown	Total disp.	p_s	$q_s = \int p_s \, dt$
Fluid DOF/pt.	3	1	1
Coef. matrices	Symmetric	Nonsymmetric	Symmetric
Spurious modes	Yes	No	No
F-S interface cond.	MPC	Matrix entry	Matrix entry

+ 3-variable formulations

Displacement Formulation

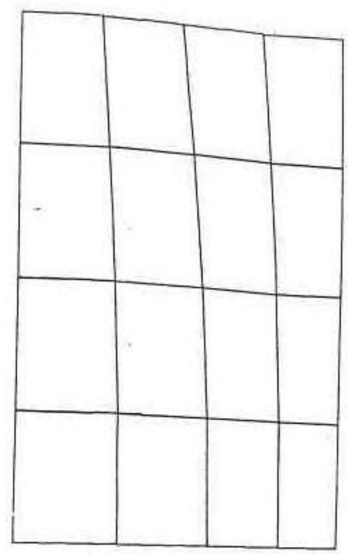
- Fundamental unknown: fluid displacement (3 DOF/point)
- Model fluid domain with elastic F.E. (e.g., elastic solids in 3-D, membranes in 2-D)
- Any coordinate systems; constrain rotations (DOF 456)
- Material properties (3-D): $G_e \approx 0 \;\Rightarrow\; E_e=(6\varepsilon)\rho c^2$, $\nu_e=½-\varepsilon$, $\rho_e= \rho$, where $\varepsilon=10^{-4}$
- Boundary conditions:
  - Free surface: natural B.C.
  - Rigid wall: $u_n=0$ (SPC or MPC)
  - Accelerating boundary: u_n continuous (MPC), slip
- Real and complex modes, frequency and transient response
- 3 DOF/point, spurious modes

Displacement Method Mode Shapes



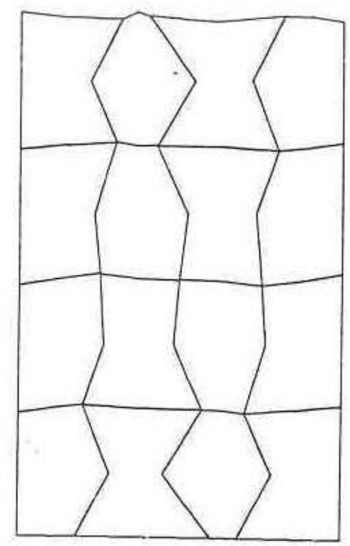
0 Hz

Spurious



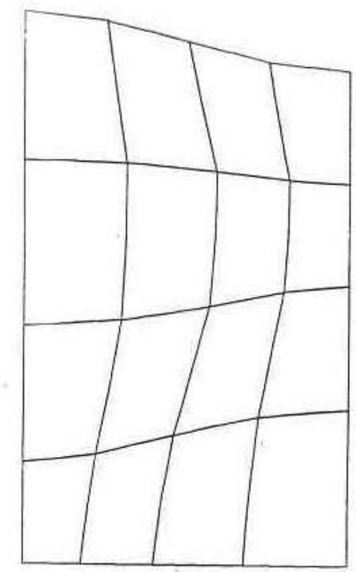
1506 Hz

Good



1931 Hz

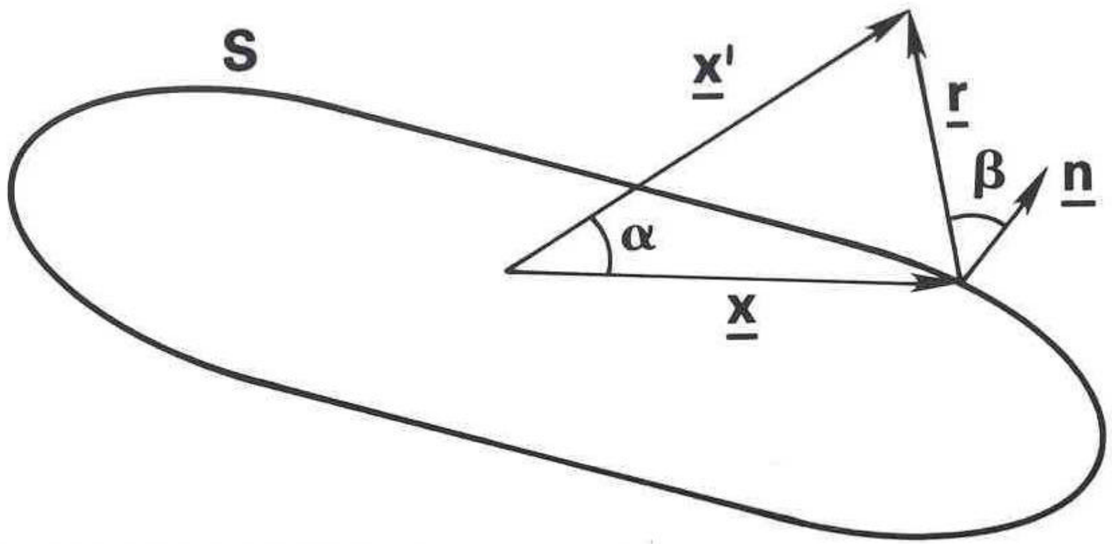
Spurious



1971 Hz

Good

Helmholtz Integral Equations



$$k = \frac{\omega}{c}$$

$$\int_S \left(p \frac{\partial D}{\partial n} - \frac{\partial p}{\partial n} D \right) dS = \begin{cases} 1/2\, p - p_I & (S) \\ p - p_I & (E) \\ - p_I & (I) \end{cases}$$

$$D = e^{-ikr}/4\pi r \quad \text{(GREEN'S FUNCTION)}$$

$$\text{SURFACE:} \quad Ep = Cv_n + p_I$$

Matrix Formulation of Fluid-Structure Problem

Structure: $Zv = F - GAp$

$Z = (-\omega^2 M + i\omega B + K)/(i\omega)$

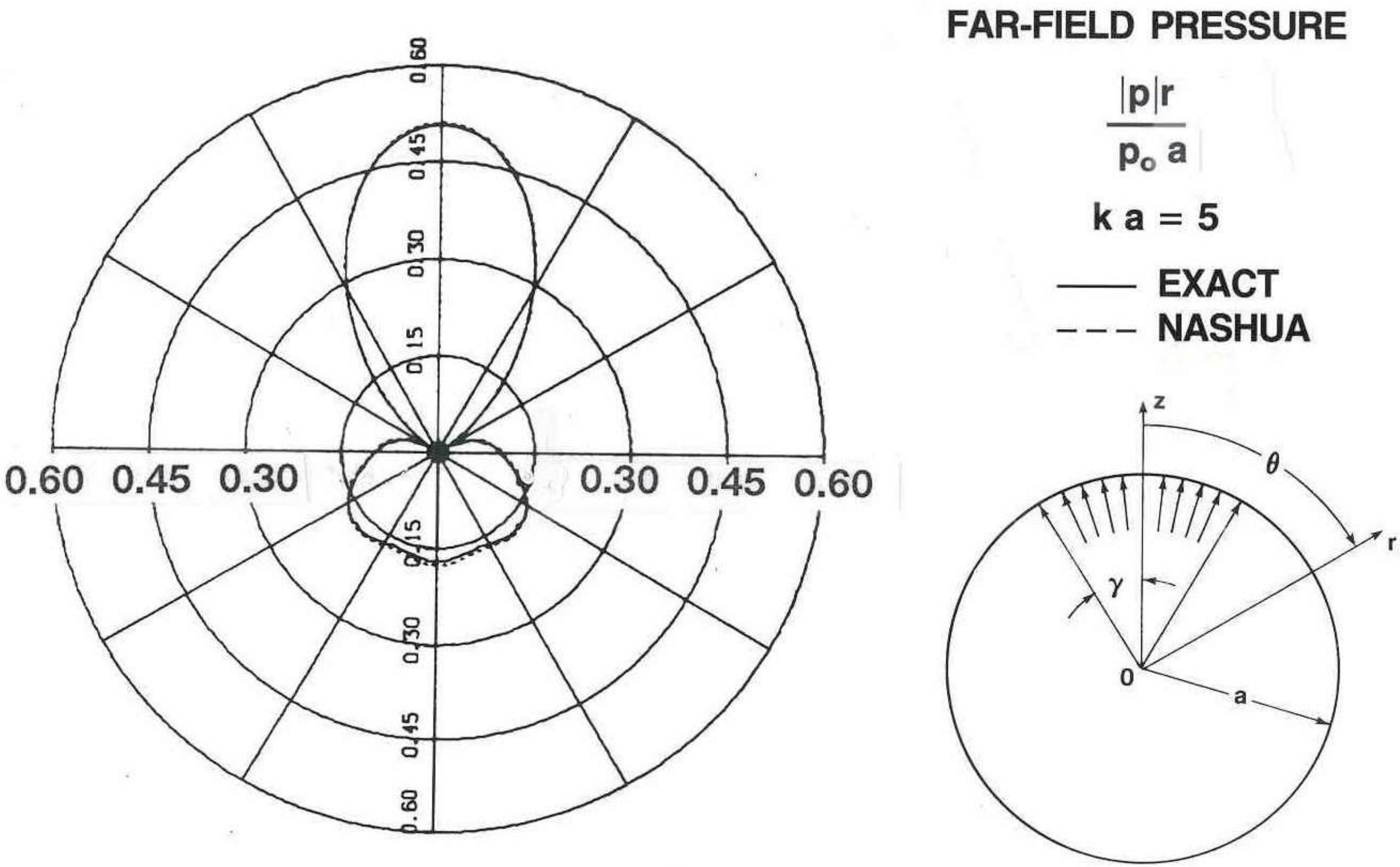
Fluid: $Ep = Cv_n + p_i$

F-S Transformation: $F = GF_n, \quad v_n = G^T v$

Coupled Equations: $(E + CG^T Z^{-1} GA)p = CG^T Z^{-1} F + p_i$

Velocity: $v_n = G^T Z^{-1} F - G^T Z^{-1} GAp$

Spherical Shell With Sector Drive



Added Mass by Boundary Elements

Helmholtz equation: $$\oint_S \left(p\frac{\partial D}{\partial n} - D\frac{\partial p}{\partial n} \right) dS = \frac{p}{2}$$

$$D=\frac{e^{-ikr}}{4\pi r} \quad \text{(Green's function)}$$

Boundary element equation: $$Ep = Cv_n$$

Added mass matrix: $$M_a = GAE^{-1}\left(\frac{C}{i\omega}\right)G^T$$

Eigenproblem: $$(M + M_a)\ddot{u} + Ku = 0$$

Frequencies of Submerged Cylindrical Shell

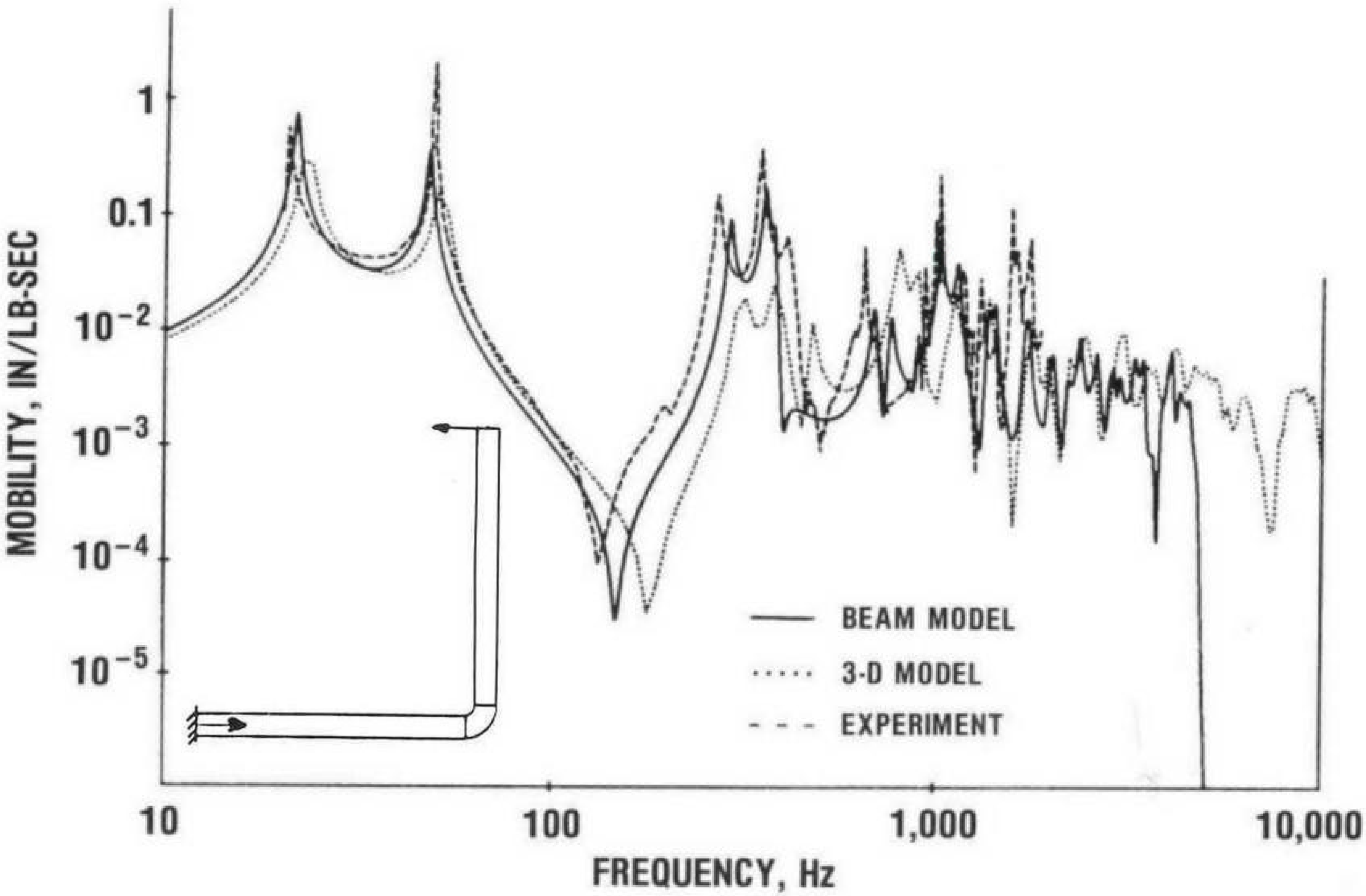
Mode	N	M	L	F.E.	B.E.	Approx.
1	1	0		0.00	0.00	0.00
2	2	1		1.13	1.13	1.11
3	0		1	1.63	1.44	1.38
4	3	1		1.79	1.81	1.77
5	4	1		3.61	3.67	3.57
6	1		1	4.44	4.26	4.22
7	4	3		4.81	4.82	4.70
8	3	3		4.94	4.93	4.82
9	5	1		6.31	6.38	6.18
10	5	3		6.83	6.86	6.67

N=circumferential, M=longitudinal, L=radial (end)

Low Frequency F.E. Piping Model

- Beam model for pipe
- 1-D acoustic fluid model for fluid (rods)
- Two sets of coincident grid points
- Pipe and fluid have same transverse motion
- Elbow flexibility factors are used
- Adjusted fluid bulk modulus for fluid in elastic pipes $E=B/[1+BD/E_s t)]$
- Arbitrary geometry, inputs, outputs
- Applicable below first lobar mode

Planar Piping System: Free End Response



Needs

- Link between CAD model and FE model
- Infinite elements
- Meshing (e.g., between hull and outer fluid FE surface
- Modeling difficulties (e.g., joints, damping, materials, mounts)
- Error estimation and adaptive meshing

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