# CONTROL OF VIBRATION AND WAVE PROPAGATION IN SANDWICH STRUCTURES WITH AUXETIC CELLULAR CORES

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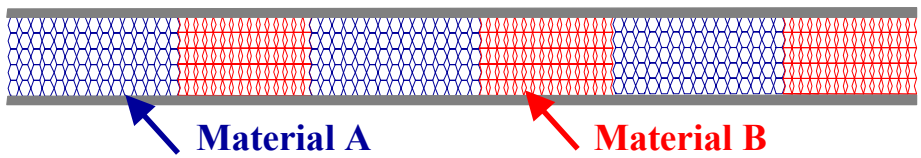
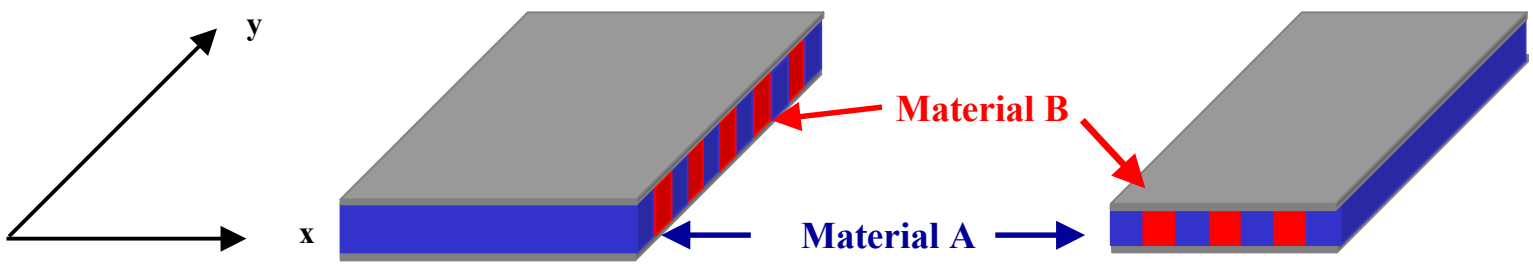
# OVERVIEW and OBJECTIVES

**Analysis of WAVE DYNAMICS in sandwich structures with core of two cellular materials alternating PERIODICALLY along the structure;**

One-dimensional structures - beams



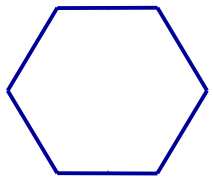

Two-dimensional structures - Plates



# OVERVIEW and OBJECTIVES

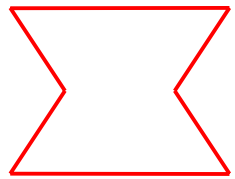
- Elastic and inertial properties of the core vary with elementary geometry of honeycomb structure

**EXAMPLES:**



Regular hexagon

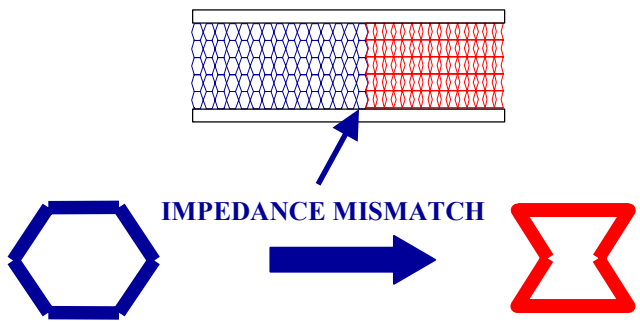
**Material A**



Re-entrant geometry

**Material B**

**Alternating honeycombs of different geometry generates**
**IMPEDANCE MISMATCH ZONES**
**along the structure**



# OVERVIEW and OBJECTIVES

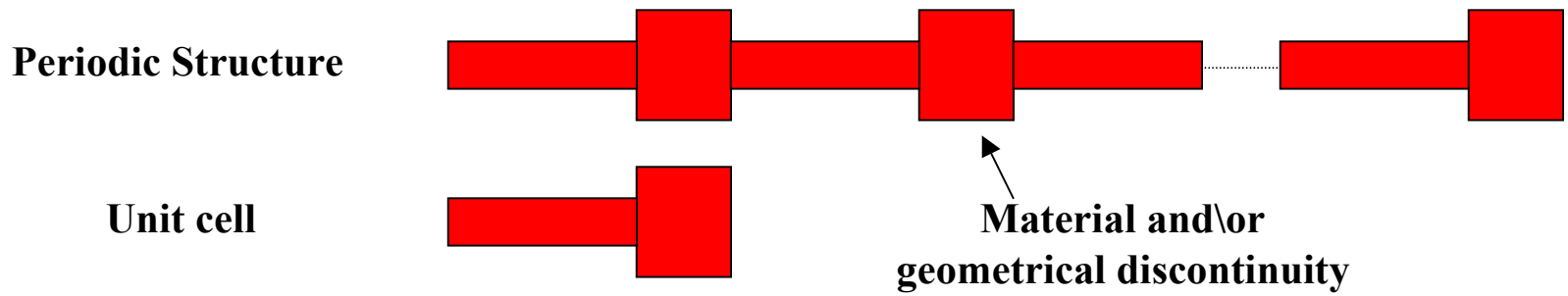
**Periodic impedance mismatch zones generate a**
**PASS AND STOP BAND**
**pattern typical of**
**PERIODIC STRUCTURES**

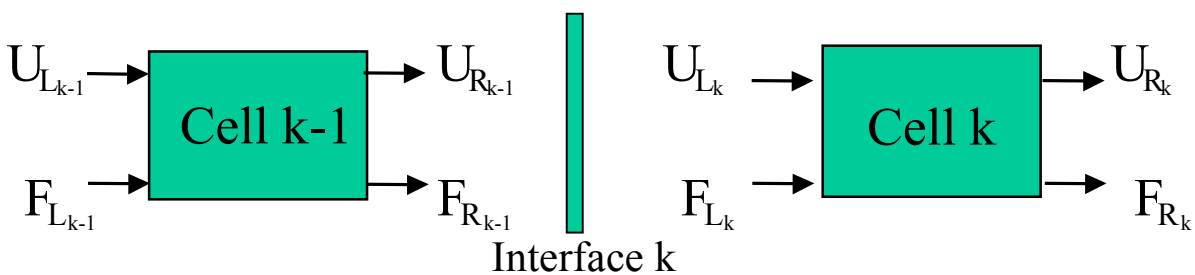
**This concept can be used to design**
**COMPOSITE PANELS**
**that behave as**
**DIRECTIONAL MECHANICAL FILTERS**
**without requiring additional devices for active and passive vibration and wave propagation control.**

**SIZE AND WEIGHT**
**OF PANELS ARE NOT COMPROMISED**

# CONTROL OF WAVE PROPAGATION IN SANDWICH BEAMS WITH AUXETIC CORE

# Wave propagation in 1-Dimensional periodic structures




$$\begin{Bmatrix} U_L \\ F_L \end{Bmatrix}_k = [T_k] \cdot \begin{Bmatrix} U_L \\ F_L \end{Bmatrix}_{k-1} \quad \textbf{or} \quad Y_k = [T_k] \cdot Y_{k-1}$$

where $Y_k$ = state vector and $T_k$ = transfer matrix

# Wave propagation in 1-Dimensional periodic structures

Eigenvalue Problem:

$$[T_k] \cdot Y_k = \lambda \cdot Y_k$$

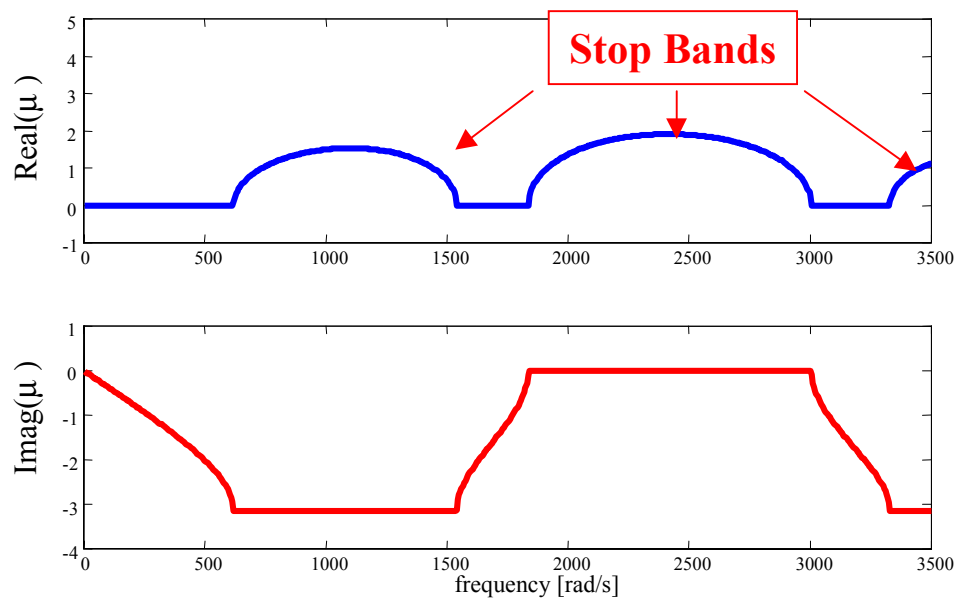
$$Y_k = \lambda \cdot Y_{k-1} = e^{\mu} \cdot Y_{k-1}$$

Where:

$\mu$ : PROPAGATION CONSTANT

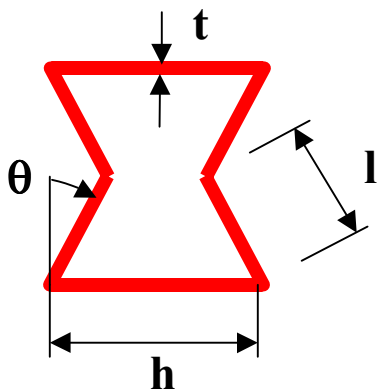
Real($\mu$ ): Amplitude decay

Imag($\mu$ ): phase difference between adjacent cells



# Auxetic solids

**Geometric Layout of an elementary cell of honeycomb material:**

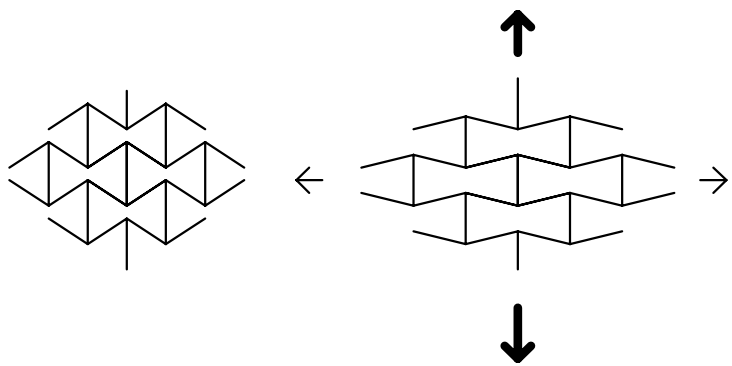



$\alpha = h/l$

$\beta = t/l$

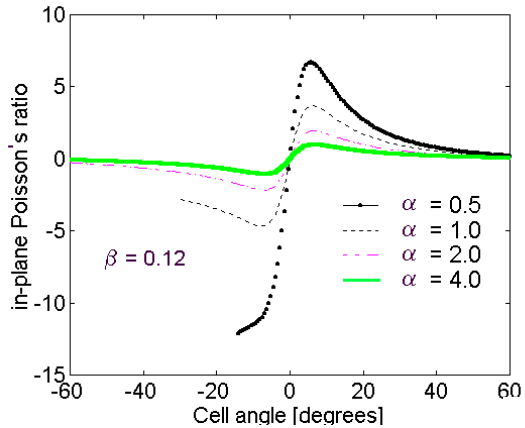
**$\theta < 0$: Re-entrant geometry (AUXETIC SOLID)**

**Auxetic cellular solids feature NEGATIVE POISSON'S RATIO behavior**

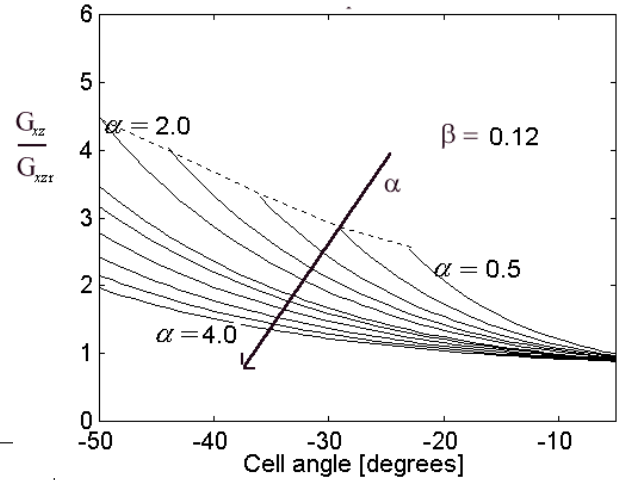


# Auxetic solids

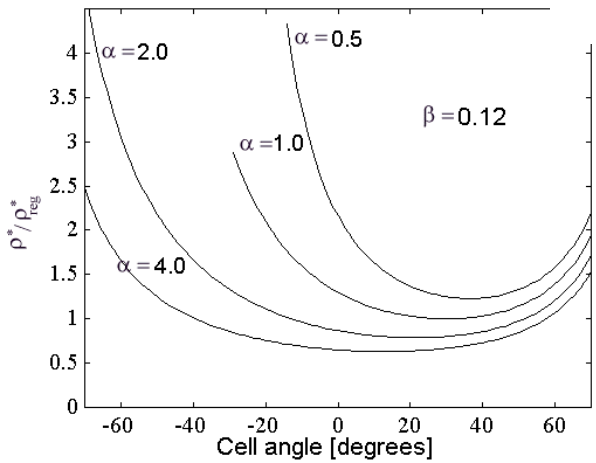
## Poisson's ratio vs. angle θ




## Shear Modulus vs. angle θ

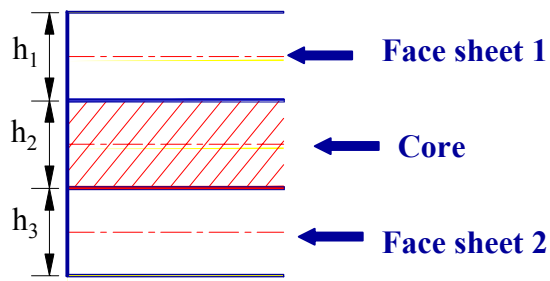


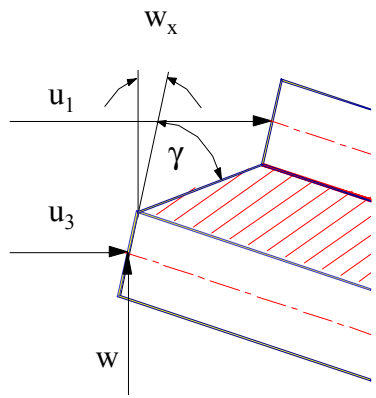

## Density vs. angle θ



# Spectral Finite Element (SFE) modeling of sandwich beam

**Geometric and kinematic relationships:**




**Shear deformation and displacement of the core:**

$$\gamma = \frac{1}{h_2}[u_1 - u_3 + h \cdot w_x]$$

$$u_2 = \frac{1}{2}[u_1 + u_3 + (\frac{h_1 - h_3}{2}) \cdot w_x]$$

# Spectral Finite Element (SFE) modeling of sandwich beam

**Hamilton's Principle:**

$$\int_{t_1}^{t_2} \delta(T - U)dt = 0$$

**Strain energy:**

$$U = \frac{1}{2} \cdot D_t \int_0^L w_{xx}^2 dx + \frac{1}{2}\int_0^L E_1 A_1 u_{1x}^2 dx + \frac{1}{2}\int_0^L E_3 A_3 u_{3x}^2 dx + \frac{1}{2}\int_0^L G_2 \cdot A_2 \cdot \gamma^2 dx$$

**BENDING** **EXTENSION** **SHEAR**

**Kinetic energy:**

$$T = \frac{1}{2}\int_0^L m\dot{w}^2 \cdot dx + \frac{1}{2}\int_0^L m_1 \dot{u}_1^2 \cdot dx + \frac{1}{2}\int_0^L m_3 \dot{u}_3^2 \cdot dx$$

# Spectral Finite Element (SFE) modeling of sandwich beam

**Equations of motion:**

$$u_{1_{xx}} = \frac{1}{K_1}\left[\frac{G_2A_2}{h_2^2} - \omega^2\left(m_1 + \frac{1}{4}m_2\right)\right]u_1 - \frac{1}{K_1}\left(\frac{G_2A_2}{h_2^2} + \frac{1}{4}\omega^2 m_2\right)\cdot u_3 + \frac{1}{K_1}\left(\frac{G_2A_2}{h_2^2}h + \frac{h_1 - h_3}{8}\omega^2 m_2\right)w_x$$

$$u_{3_{xx}} = -\frac{1}{K_3}\left(\frac{G_2A_2}{h_2^2} + \frac{1}{4}\omega^2 m_2\right)\cdot u_1 + \frac{1}{K_3}\left[\frac{G_2A_2}{h_2^2} - \omega^2\left(m_3 + \frac{1}{4}m_2\right)\right]u_3 + \frac{1}{K_3}\left(\frac{G_2A_2}{h_2^2}h - \frac{h_1 - h_3}{8}\omega^2 m_2\right)w_x$$

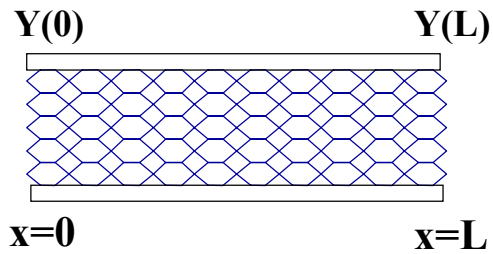
$$w_{xxxx} = \frac{1}{D_t}\left(\frac{G_2A_2h}{h_2^2} - \frac{1}{8}\omega^2 m_2\right)\cdot u_{1x} - \frac{1}{D_t}\left(\frac{G_2A_2h}{h_2^2} - \frac{1}{8}\omega^2 m_2\right)\cdot u_{3x} + \omega^2\frac{m}{D_t}w + \frac{1}{D_t}\left(\frac{G_2A_2h^2}{h_2^2} - \frac{(h_1 - h_3)^2}{16}\omega^2 m_2\right)w_{xx}$$

**State Space Formulation & Solution:**

$$\mathbf{Z_x}(x) = \mathbf{A}\cdot\mathbf{Z}(x) \quad \Longrightarrow \quad \mathbf{Z}(x) = e^{\mathbf{A}\cdot x}\cdot\mathbf{Z}(0)$$

where: $\mathbf{Z} = [u_1 \quad u_3 \quad w \quad w_x \quad u_{1x} \quad u_{3x} \quad w_{xxx} \quad w_{xx}]^T$

# Transfer Matrix and propagation constants




**State Vector:**

$$\mathbf{Y} = [\underbrace{u_1 \quad u_3 \quad w \quad w_x}_{\text{Generalized Displacements}} \quad \overbrace{N_1 \quad N_3 \quad S \quad M}^{\text{Generalized Forces}}]^T$$

$$\mathbf{Y}(L) = \mathbf{G} \cdot e^{\mathbf{A} \cdot L} \cdot \mathbf{G}^{-1} \cdot \mathbf{Y}(0) \quad \Rightarrow \quad \mathbf{T} = \mathbf{G} \cdot e^{\mathbf{A} \cdot L} \cdot \mathbf{G}^{-1} \text{(TRANSFER MATRIX)}$$

where:

$$\mathbf{G} = \mathrm{diag}([1 \quad 1 \quad 1 \quad 1 \quad -K_1 \quad -K_3 \quad D_t \quad -D_t])$$

# MODEL VALIDATION: Beam with Uniform Core

**Beam Geometry:**

| | Material | Length [m] | Thickness [m] | Width [m] |
|---|---|---|---|---|
| **Face sheet 1** | Aluminum | 1 | 1e-3 | 0.04 |
| **Core** | Regular Nomex | 1 | 2e-3 | 0.04 |
| **Face sheet 2** | Aluminum | 1 | 1e-3 | 0.04 |

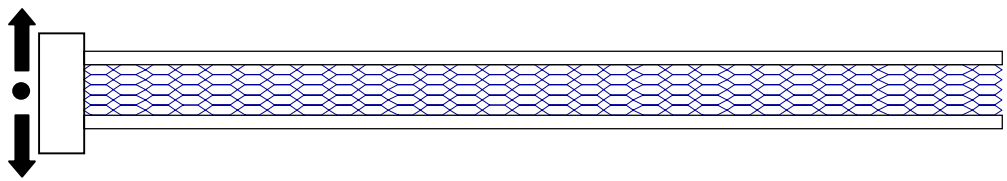
**ANSYS Mesh:**

- **Core: 480 PLANE42 elements;**
- **Face Sheets: 240 BEAM54 elements.**

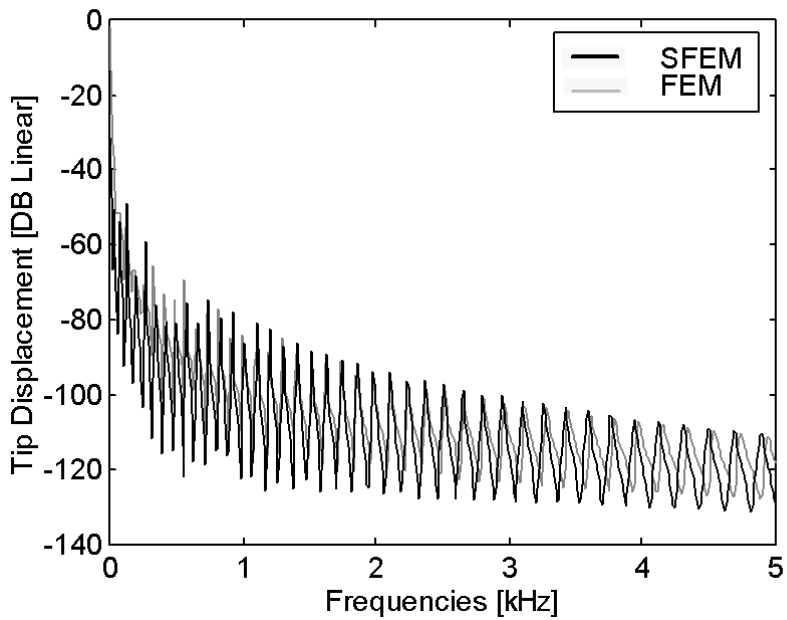
**MATLAB Mesh:**

- **20 Sandwich-beam elements.**

# MODEL VALIDATION: Beam with Uniform Core

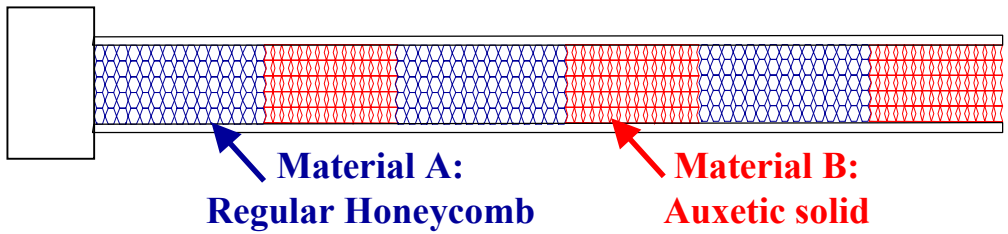


**Tip response for harmonic base motion**

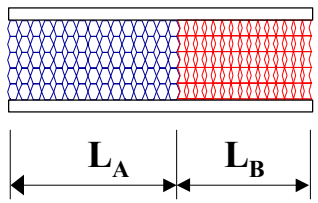


# Performance of sandwich beams with periodic core

**Periodic Sandwich Beam:**




**Elementary cell:**




**Cell length:**
**$L = L_A + L_B$**

**Performance is evaluated for:**

- **MATERIAL A: given regular honeycomb;**
- **MATERIAL B: Auxetic solids of different geometries (angle $\theta$);**
- **Varying length ratios ($L_B/L$).**

# Properties of sandwich core

**(A) - Regular Honeycomb (Hexagonal NOMEX):**

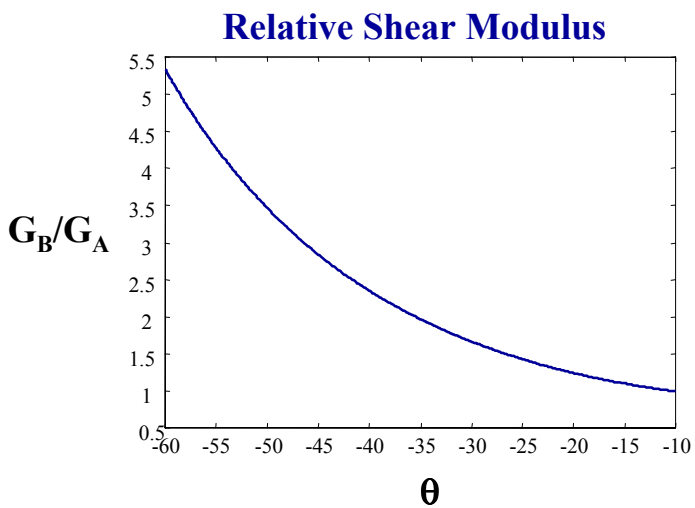
$\theta = 30^o$

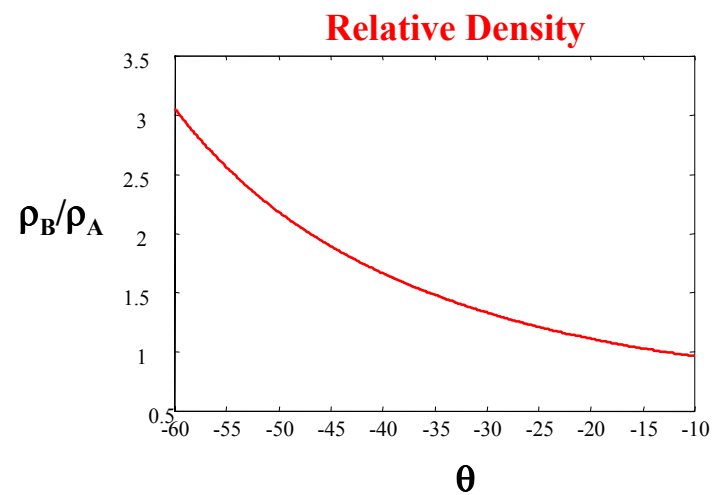
$\alpha = 1;\ \beta = 0.05$

**(B) - Auxetic Honeycomb:**

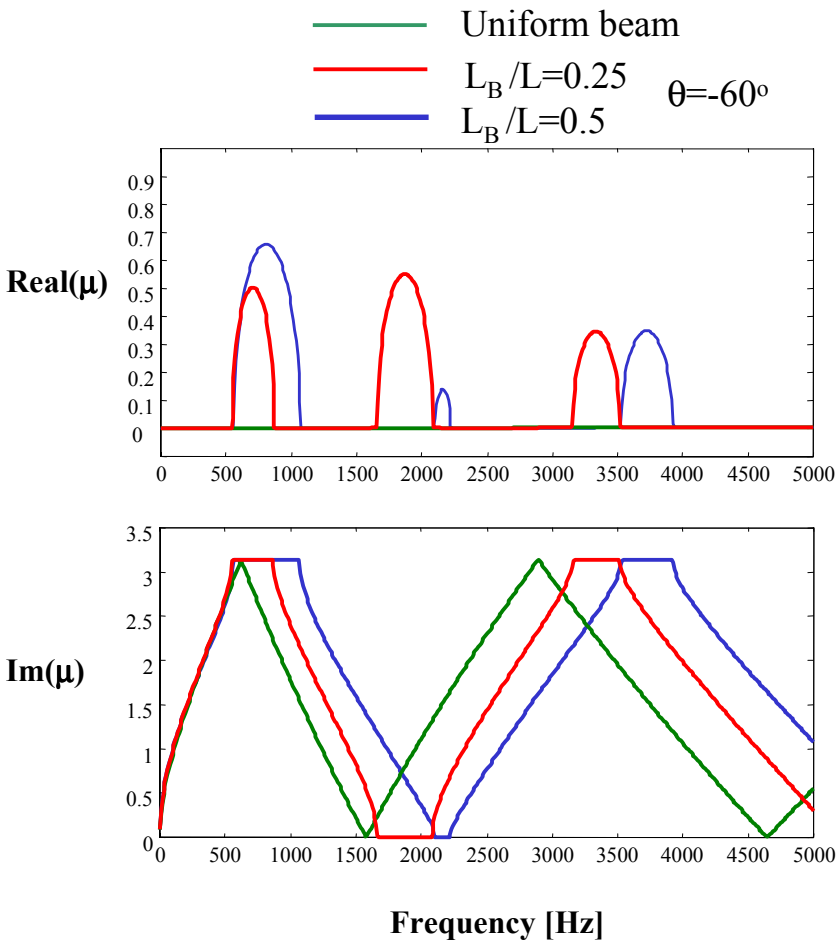
$\theta = -60^o : -10^o$

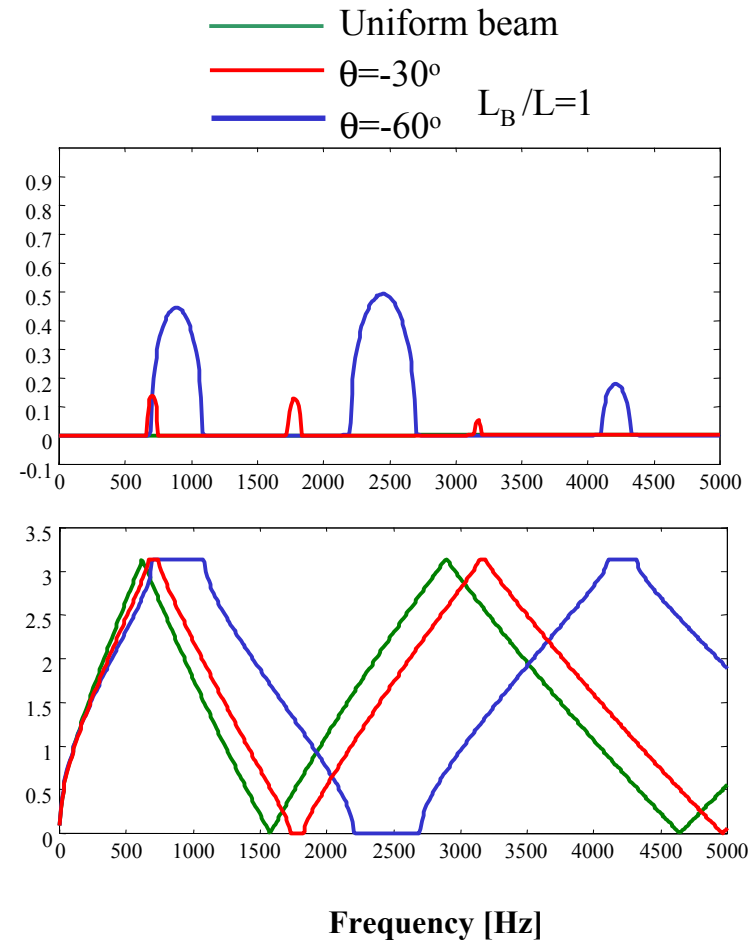
$\alpha = 2;\ \beta = 0.05$



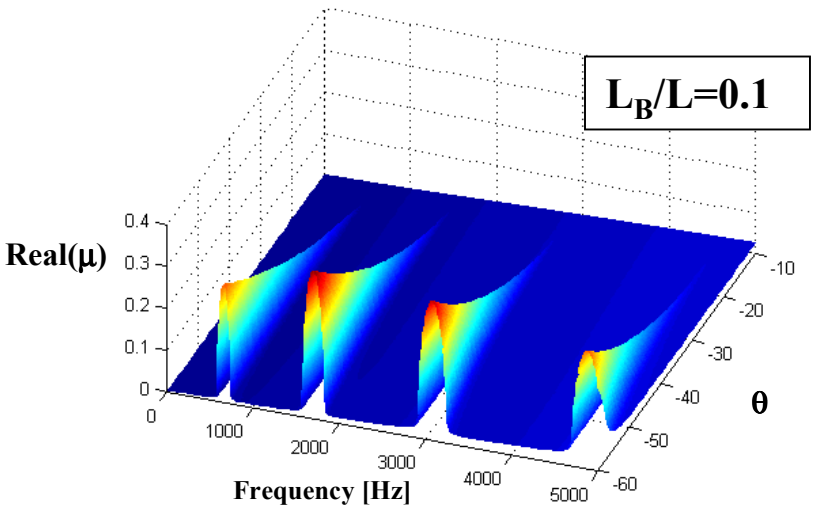



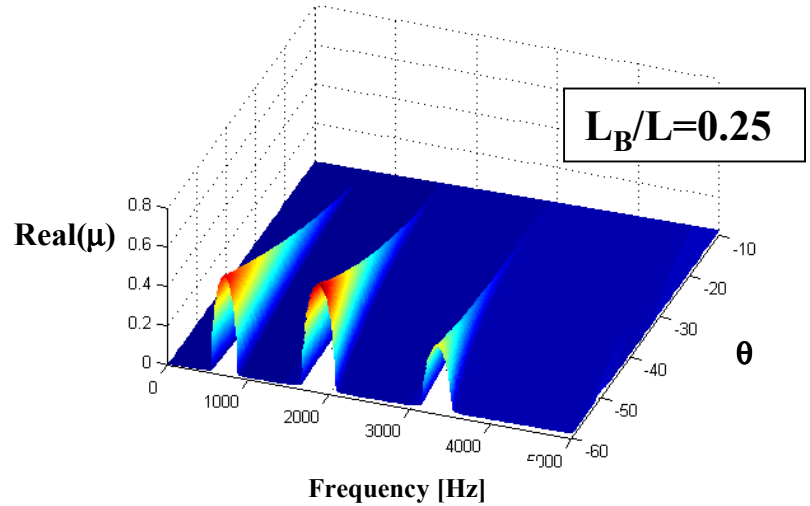
# Propagation Constants

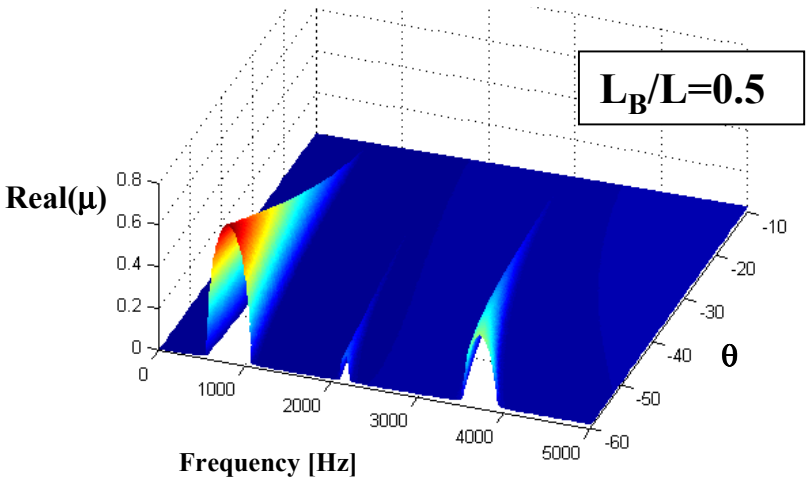


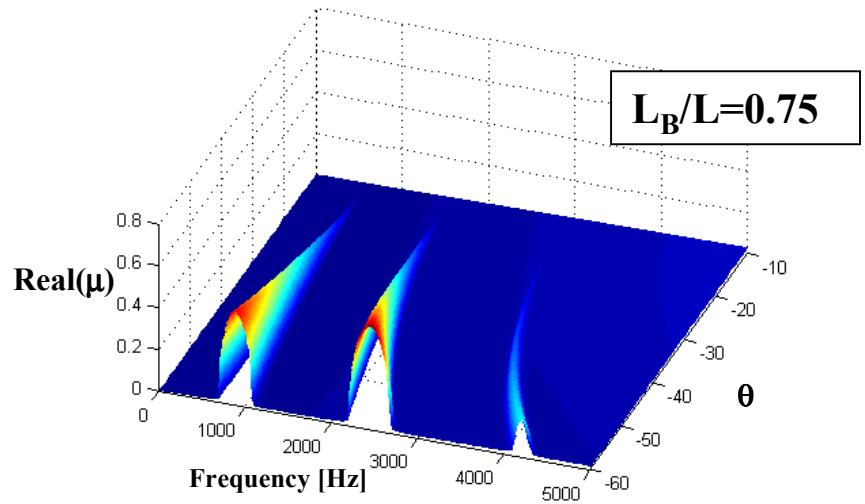



# Propagation Constants

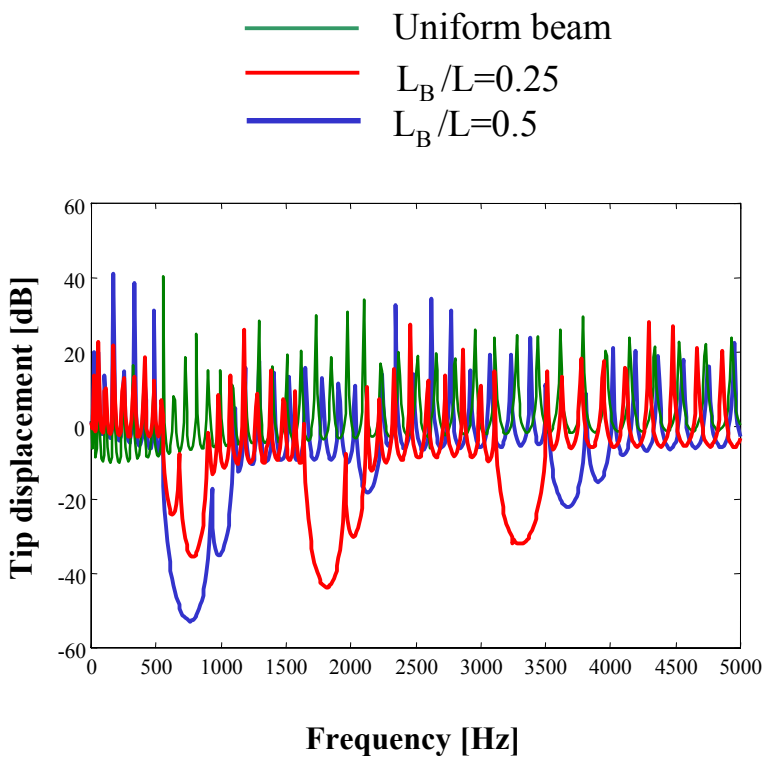


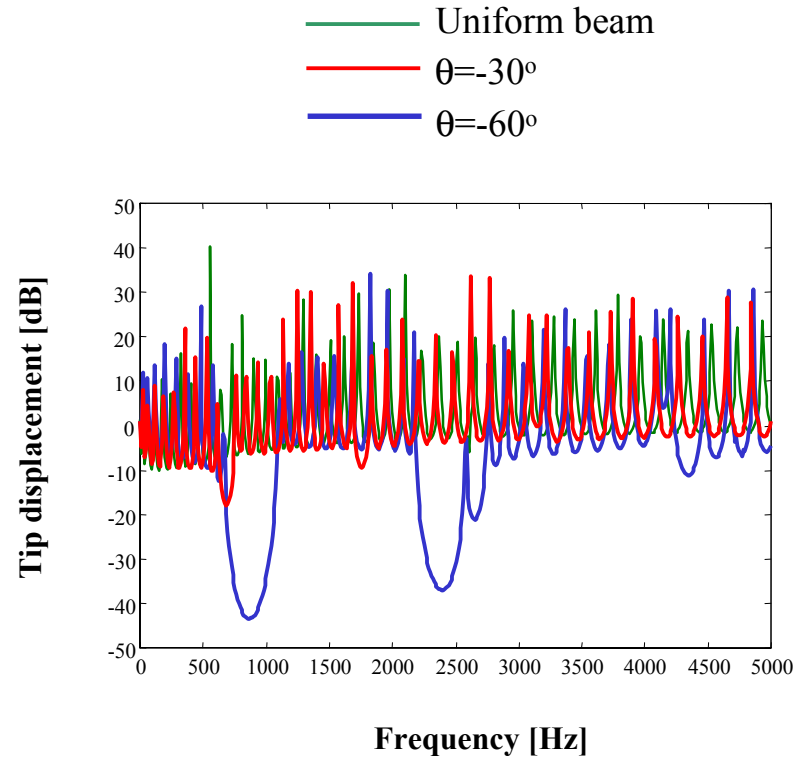



# Response of periodic sandwich beam: harmonic motion of the base



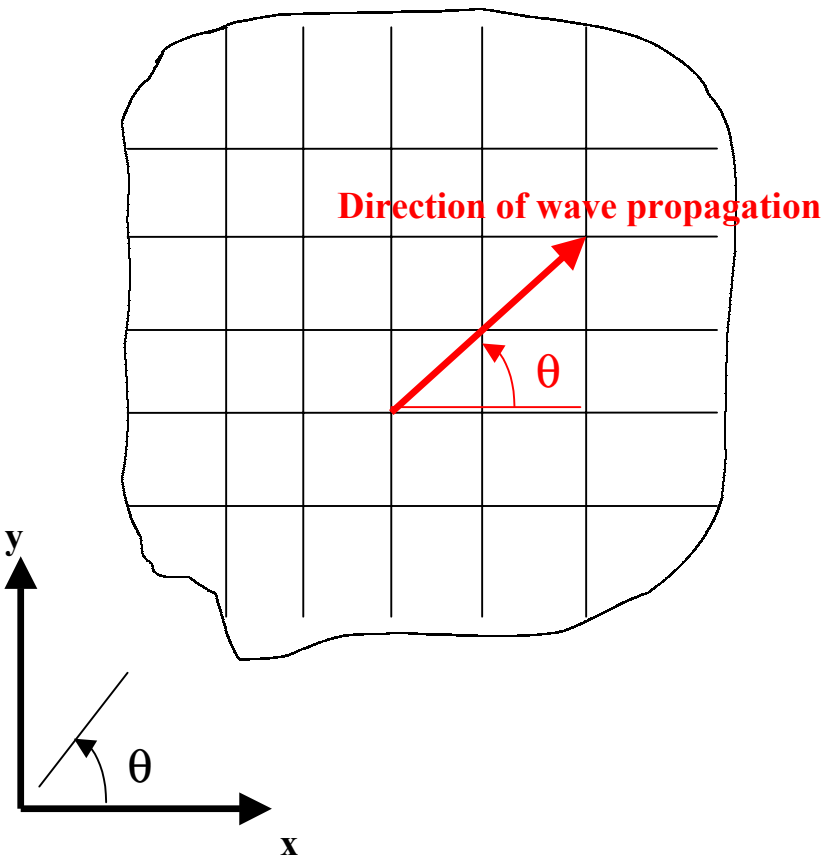



# CONTROL OF WAVE PROPAGATION IN SANDWICH PLATES WITH AUXETIC CORE

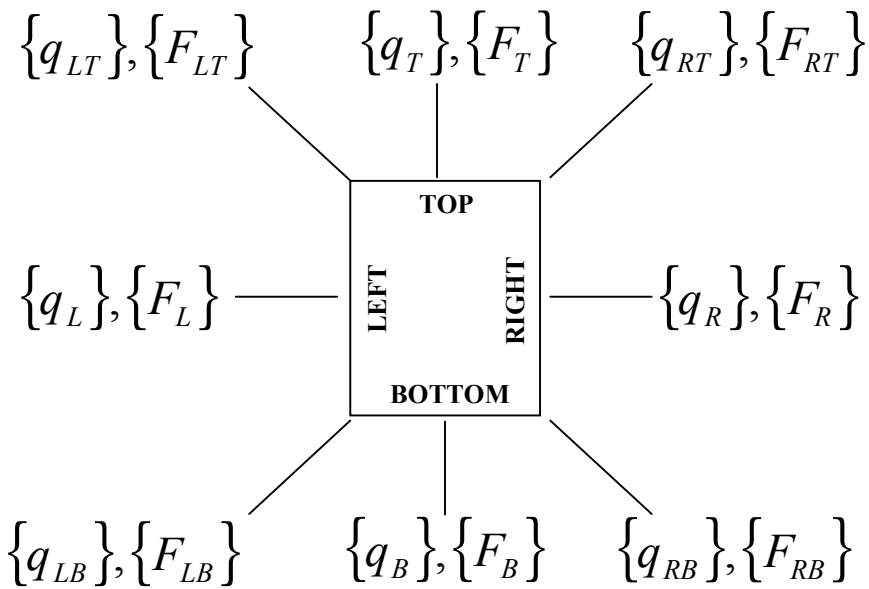
**(Work in progress)**

# Wave Propagation in 2-Dimensional Periodic Structures

**2-Dimensional periodic structure**




**Unit Cell**




$\{q_{ij}\}$ Vector of generalized displacements at interface *ij*

$\{F_{ij}\}$ Vector of generalized forces at interface *ij*

# Wave Propagation in 2-Dimensional Periodic Structures

Cell's equation of motion (Finite Element Formulation):

$$\left(-\omega^2[M]+[K]\right)\{q\}=\{F\}$$

Relation between interface displacements (Bloch's theorem):

$$\{q_R\}=e^{\mu_x}\{q_L\};\qquad \{q_T\}=e^{\mu_y}\{q_B\};$$

$$\{q_{RB}\}=e^{\mu_y}\{q_{LB}\};\qquad \{q_{LT}\}=e^{\mu_y}\{q_{LB}\};\qquad \{q_{LB}\}=e^{\mu_x+\mu_y}\{q_{RT}\};$$

Relation between interface forces (Bloch's theorem and equilibrium conditions):

$$\{F_R\}=-e^{\mu_x}\{F_L\};\qquad \{F_T\}=-e^{\mu_y}\{F_B\};$$

$$\{F_{RB}\}=-e^{\mu_y}\{F_{LB}\};\qquad \{F_{LT}\}=-e^{\mu_y}\{F_{LB}\};\qquad \{F_{LB}\}=e^{\mu_x+\mu_y}\{F_{RT}\};$$

# Wave propagation in 2-Dimensional periodic structures

Cell's equation of motion is reduced as

$$\left(-\omega^2[M^*(\mu_x,\mu_y)]+[K^*(\mu_x,\mu_y)]\right)\begin{Bmatrix} q_{LB} \\ q_L \\ q_B \end{Bmatrix}=\{0\}$$

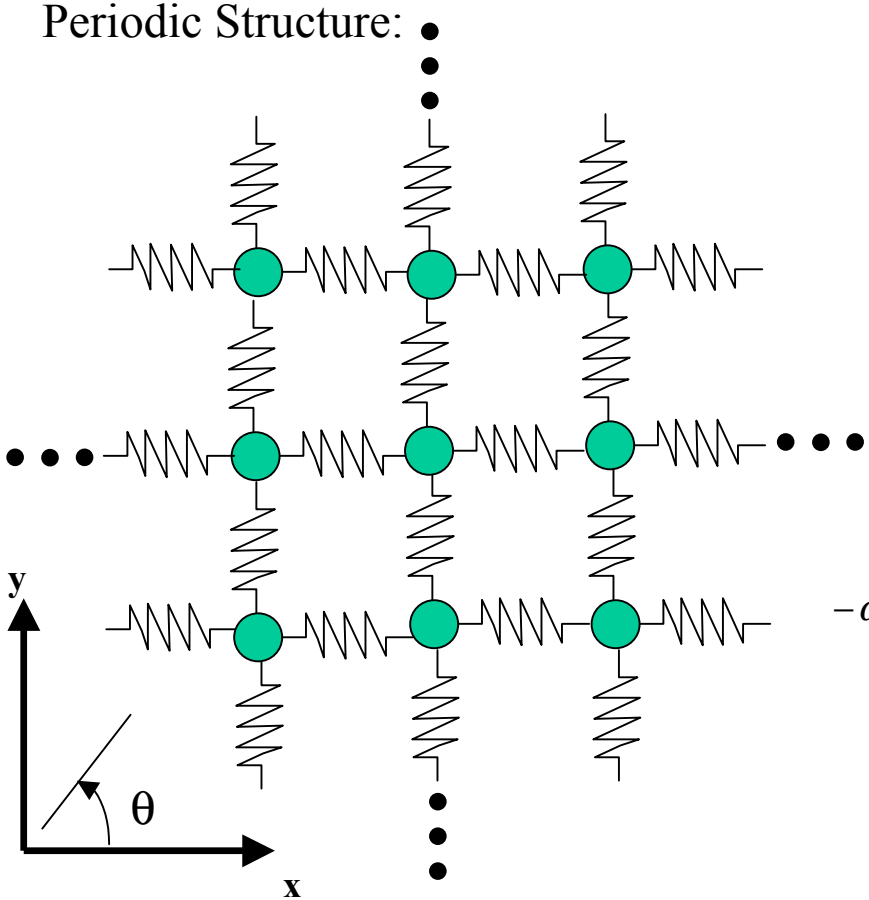
Eigenvalue problem whose solution provides for an assigned pair $\mu_x, \mu_y$ the corresponding value of frequency $\omega$

$\mu_x, \mu_y$ : Propagation Constants along $x$ & $y$

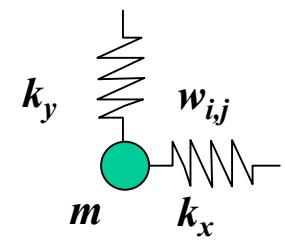
$\dfrac{\mu_x}{\mu_y} = \tan\theta$ : Direction of wave propagation

# Example: spring mass system

Periodic Structure:




Unit Cell:




**Equation of motion of mass:**

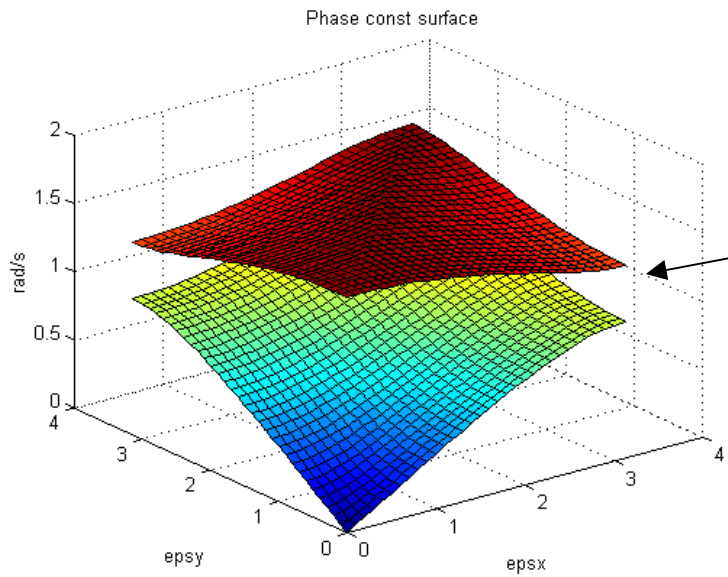
$$-\omega^2 m w_{i,j} + 2(k_x + k_y) w_{i,j} - k_x (w_{i,j-1} + w_{i,j+1}) - k_y (w_{i-1,j} + w_{i+1,j}) = 0$$

$w_{i,j}$ : off-plane displacement of mass $i,j$

# Example: spring mass system

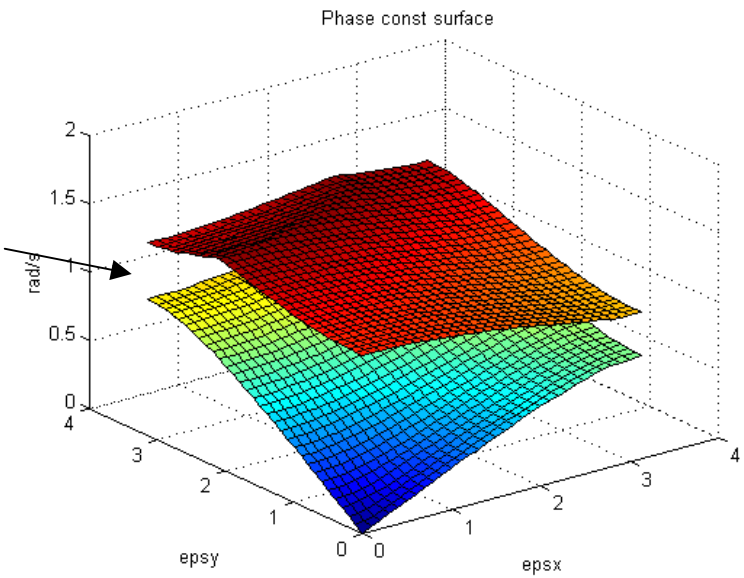
## PHASE CONSTANT ($\mu_x, \mu_y$) SURFACES

**$k_y / k_x = 1$**




**STOP BANDS**

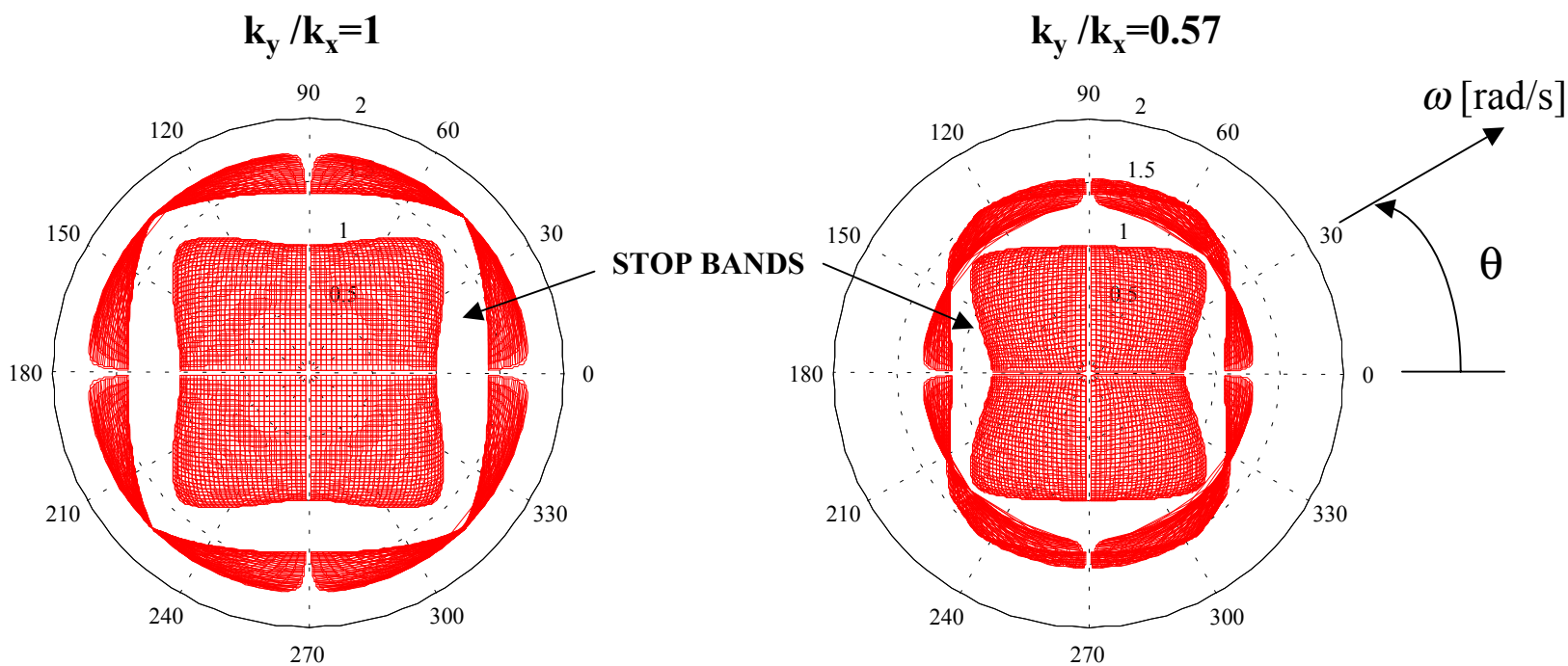
**$k_y / k_x = 0.57$**



# Example: spring mass system

## DIRECTIVITY PLOTS

Location of stop bands vs. direction of wave propagation

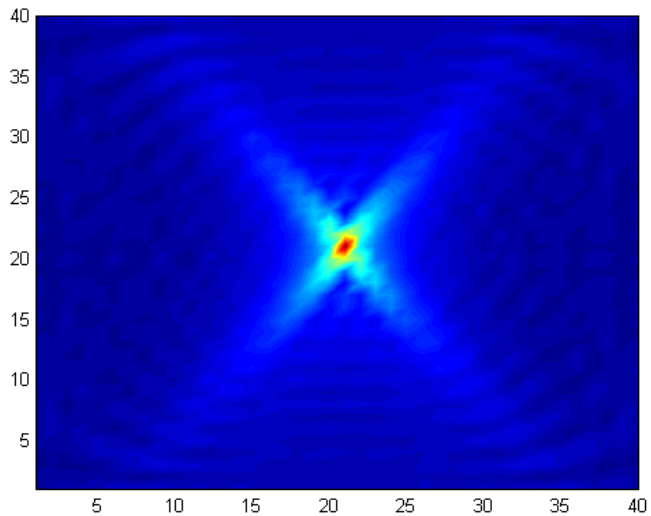


# Example: spring mass system

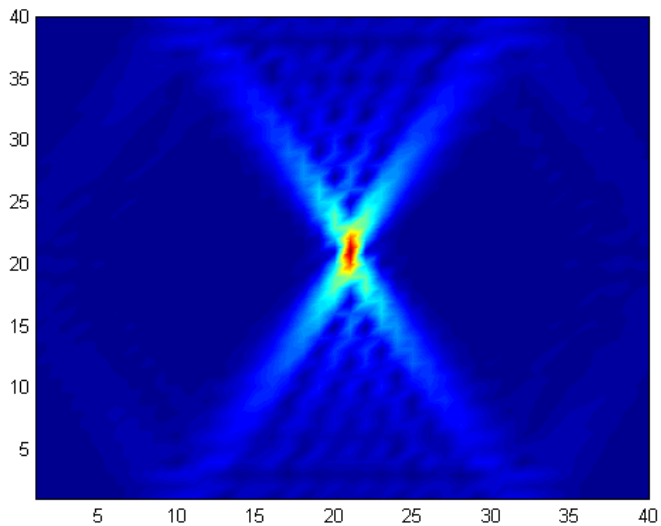
Response to harmonic point load for a 40*40 grid

$k_y / k_x = 0.57$,
Load applied in (21,21)

$\omega$=1.033 rad/s

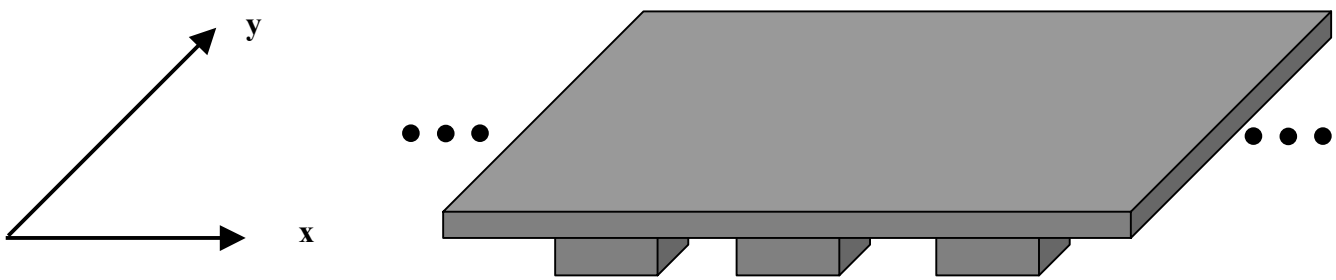



$\omega$=1.181 rad/s

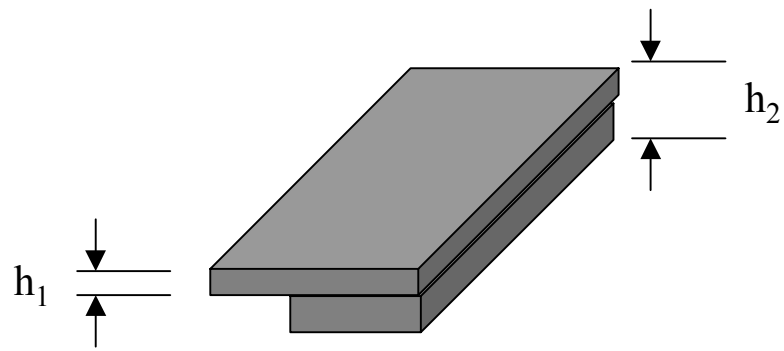


# Example: periodic stiffened plate

Stiffened Periodic Plate



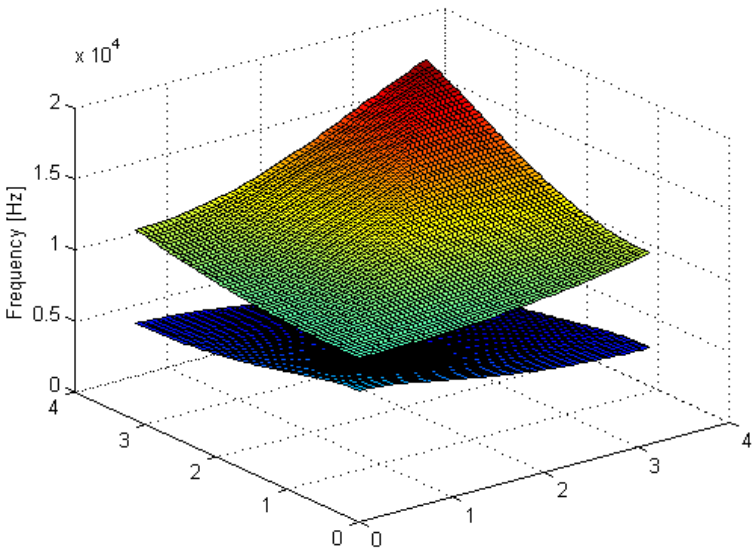

Unit cell



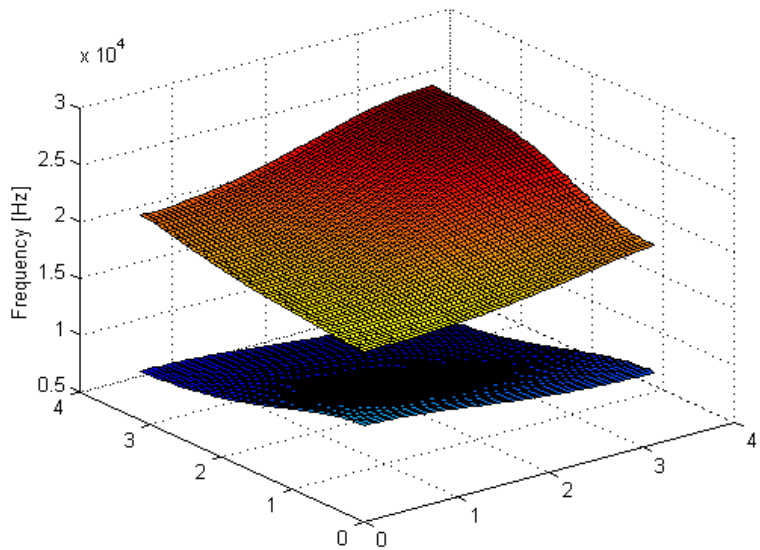
# Example: stiffened plate

## PHASE CONSTANT ( $\mu_x, \mu_y$) SURFACES

$h_2 / h_1 = 1$




$h_2 / h_1 = 2$

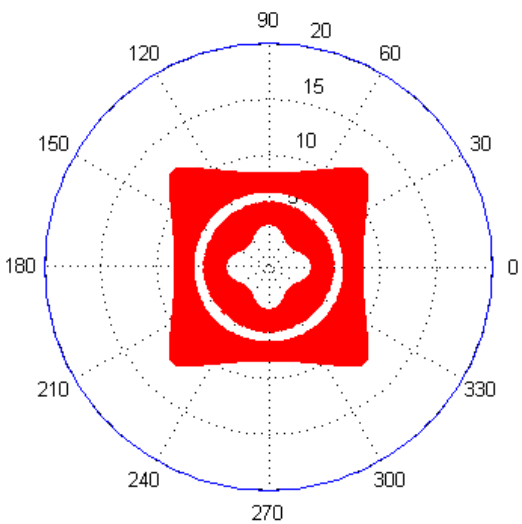


# Example: stiffened plate

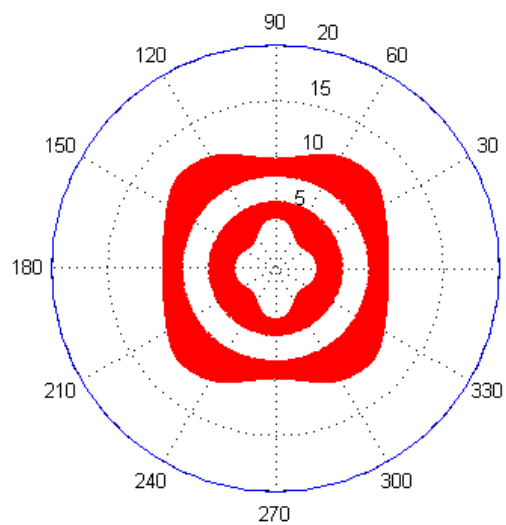
## DIRECTIVITY PLOTS

Location of stop bands vs. direction of wave propagation

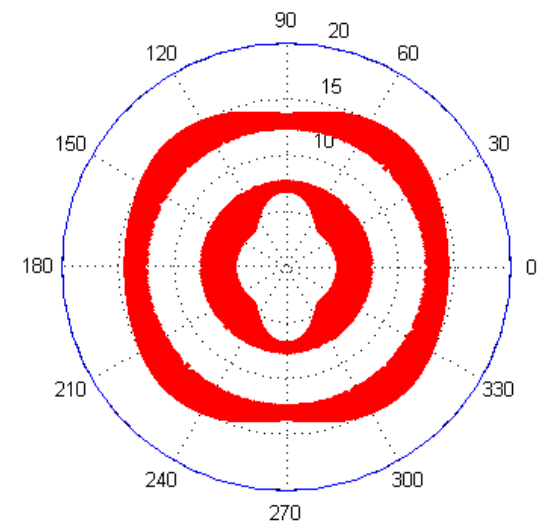
$h_2/h_1=1$




$h_2/h_1=2$




$h_2/h_1=4$



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