Geometrically Nonlinear Finite Element Analysis of a Composite Reflector

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FEMCI WORKSHOP

May 17 , 2001

Motivation

- Recognized a need for nonlinear FEA in our branch
- Interested in Element Design, Element Locking, and the effects of element distortion on solution accuracy
- Found a suitable project with the appropriate resources (interest, money, and time)

Objective

- Develop a finite element research model that can be used to study highly flexible structures undergoing geometrically nonlinear behavior
- Use the model to predict the structural response of a thin composite reflector

The Reflector

Problem Formulation Highlights

Approach

- Compute the deformation of a composite reflector with an areal density of 4 kg/m^2 under point loads using a geometrically nonlinear solid shell finite element model.
- Validate the computational results through comparison with the experimental data.

Problem Formulation Highlights

The Solid Shell Approach

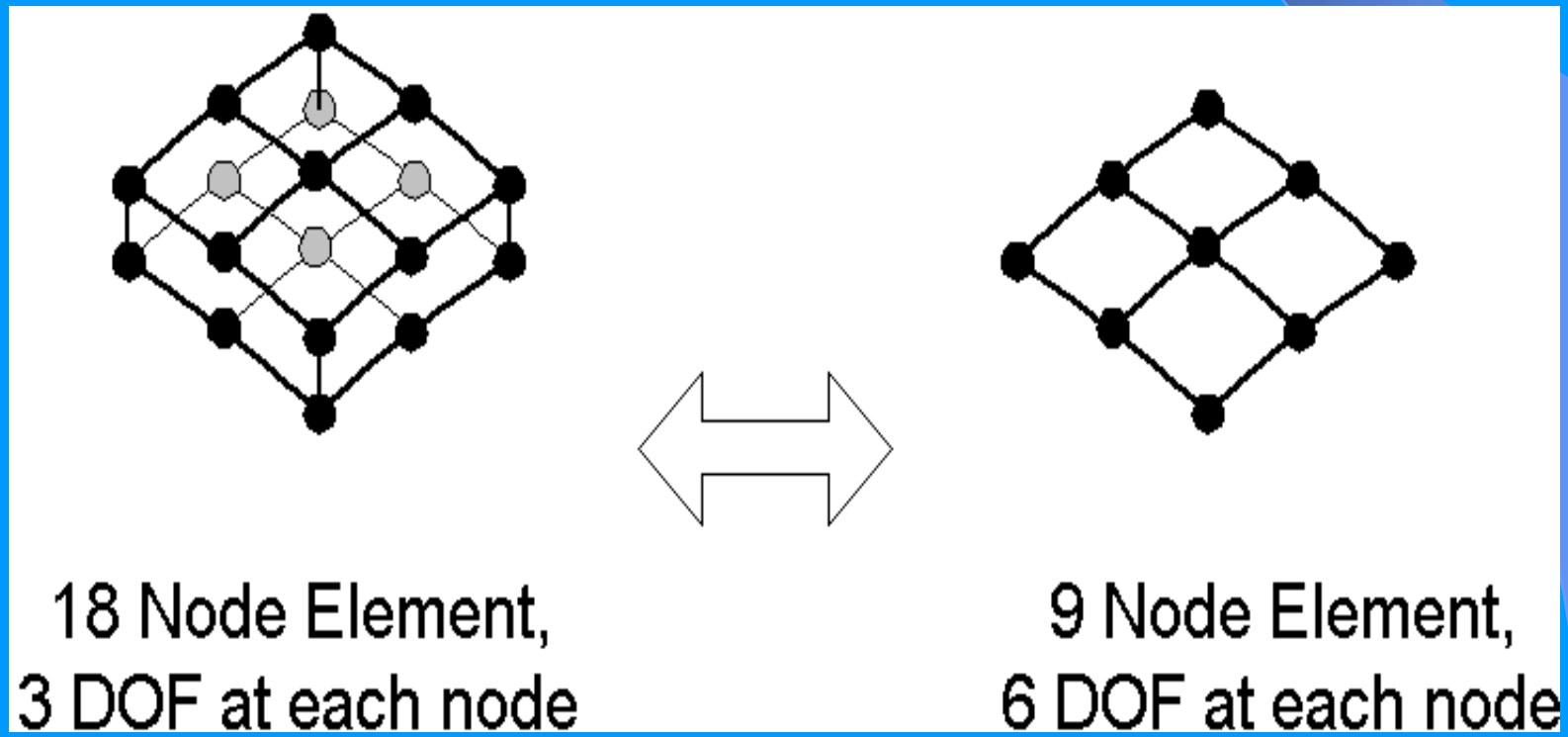
Elements

- 4-node plate elements (6 dof/node)
- 9-node shell elements (6 dof/node)
- 18-node shell elements (3-dof/node)

Solid Shell Formulation

All kinematic variables : vectors only

No rotational angle are used : easy connections between substructures



Problem Formulation Highlights

Element Locking: Zero-Energy (Spurious) Modes

- Plate and shell elements lock as the thicknesses decrease
- METHODS OF ELIMINATION
  - Reduced/ Selective Integration
  - Stabilization Matrix
  - Assumed Strain approach

Problem Formulation Highlights

Assumed Strain Function

$$\boldsymbol{\varepsilon} = \mathbf{P}(\xi, \eta)\boldsymbol{\alpha}$$

where

$$\boldsymbol{\alpha} = \mathbf{H}^{-1}\mathbf{G}\mathbf{q}_{\mathbf{e}}$$

and

$$\mathbf{H} = \int_{\mathbf{v}} \mathbf{P}^{\mathbf{T}}\mathbf{C}_{\mathbf{e}}\mathbf{P}\mathbf{dv}$$

$$\mathbf{G} = \int_{\mathbf{v}} \mathbf{P}^{\mathbf{T}}\mathbf{C}_{\mathbf{e}}\mathbf{B}\mathbf{dv}$$

Problem Formulation Highlights

Bubble Displacement

- Eliminates some forms of element locking
- Reduces the sensitivity of elements to distortion

Problem Formulation Highlights

Bubble Displacement Function

$$N_b = (1-\xi^2)(1-\eta^2) \quad \text{for } 4\text{-node quadrilateral element}$$

$$\mathrm{N_b} = \xi\eta(1-\xi^2)(1-\eta^2) \text{ for } 9\text{-node quadrilateral element}$$

Problem Formulation Highlights

Model Equations

HELLINGER-REISSNER FUNCTIONAL

$$\pi_R = \int_v (\varepsilon^T C_e \bar{\varepsilon} - \frac{1}{2}\varepsilon^T C_e \varepsilon) dv - W$$

EQUILIBRIUM

$$\int_v \delta \bar{\varepsilon}^T C_e \varepsilon \; dv - \delta W = 0$$

COMPATIBILITY

$$\int_v \delta \varepsilon^T C_e (\bar{\varepsilon} - \varepsilon) dv = 0$$

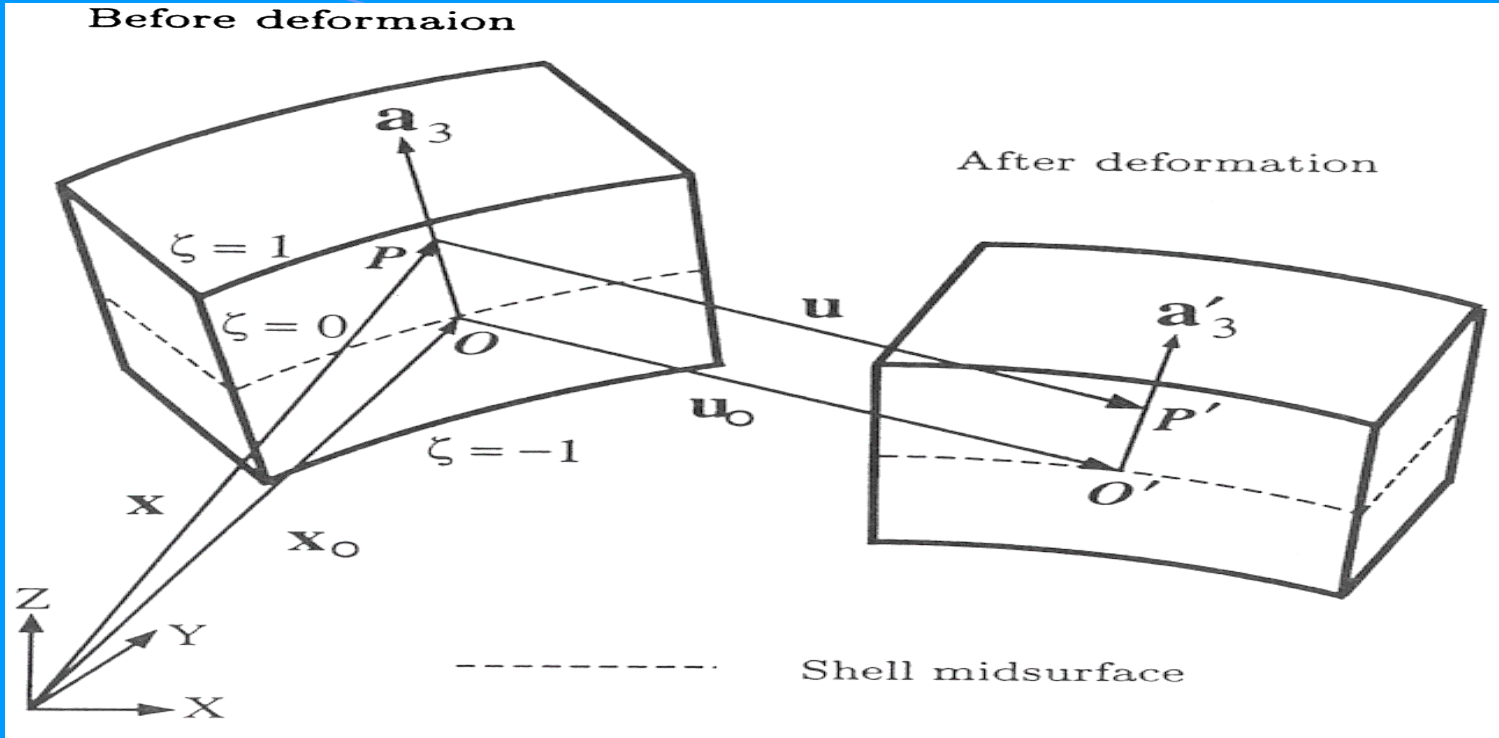
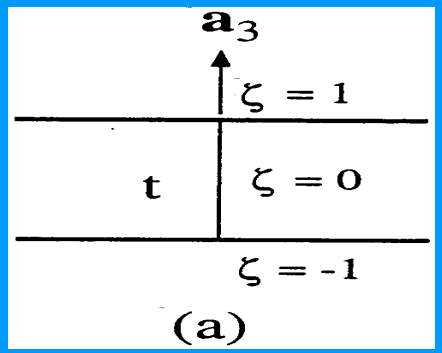
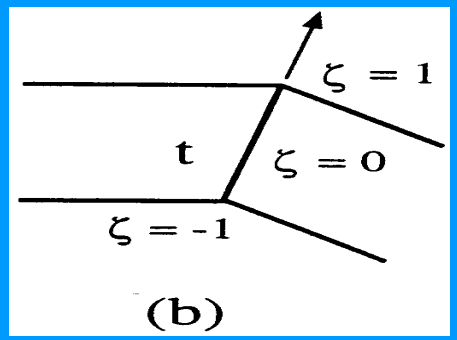


Figure 4 Kinematics of shell deformation





The a3 vectors

Problem Formulation Highlights

Finite Element Equations

GEOMETRY

$$x = \sum_{i=1}^{n} N_i(\xi,\eta)(x_0)_i + \varsigma \sum N_i(\xi,\eta)(\frac{t}{2}a_3)_i$$

ASSUMED DISPLACEMENT WITH BUBBLE FUNCTION

$$u = \sum_{i=1}^{n} N_i(\xi,\eta)(u_0)_i + \varsigma \sum_{i=1}^{n} N_i(\xi,\eta)(\frac{t}{2}u_z)_i + N_b(\xi,\eta)(u_{b0} + \varsigma u_{b_z})$$

Problem Formulation Highlights

Element Stiffness Matrix

$$K_e = G^T H^{-1} G$$

Problem Formulation Highlights

Nonlinear Equations

$$ {}^{(i)}\mathrm{K} \quad \Delta \mathrm{q} - (\mathrm{F}_{ext} - {}^{(i)}\mathrm{F}_{\mathrm{int}}) \approx 0 $$

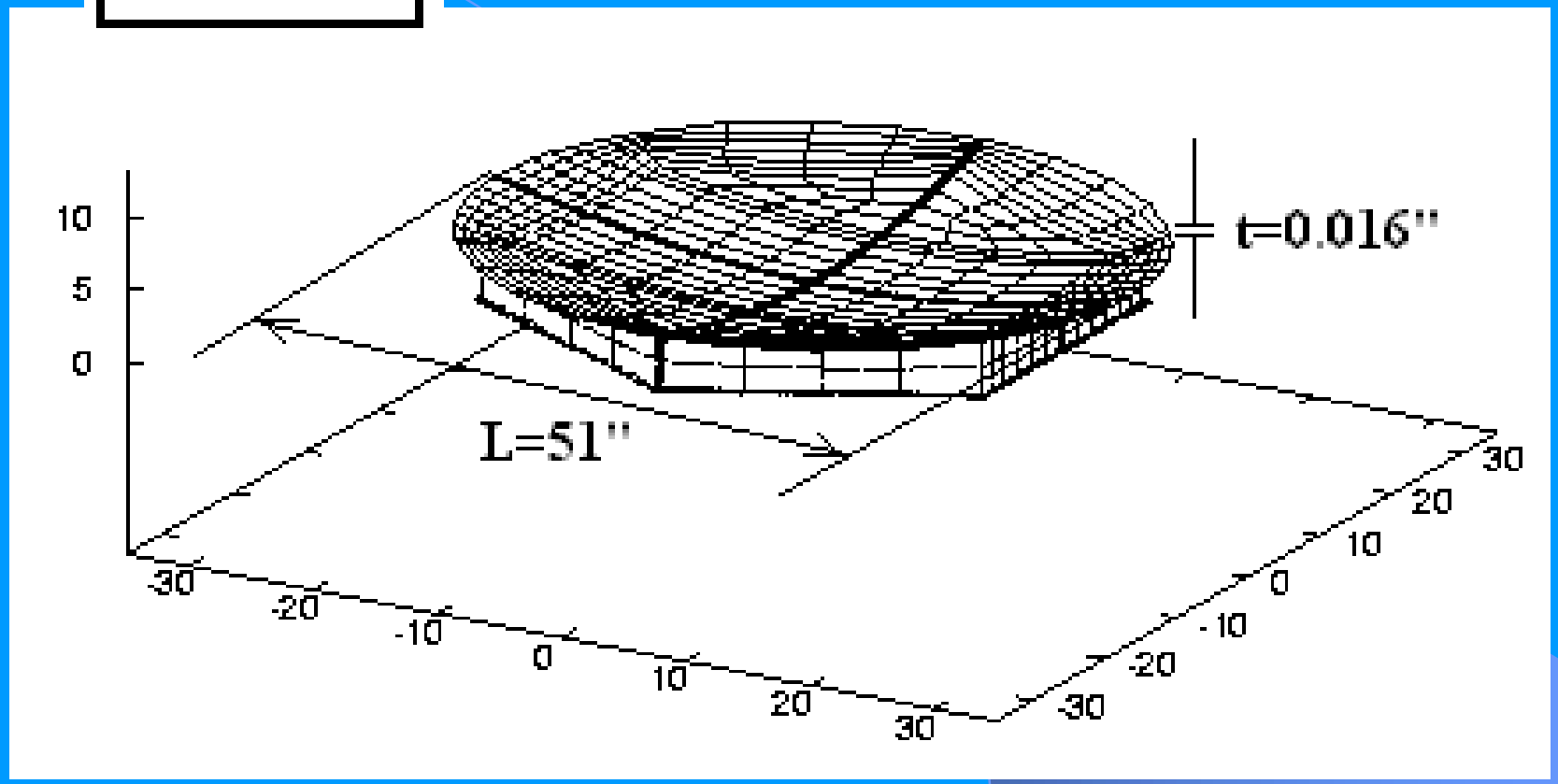
$$ {}^{(\mathrm{i}+1)}\mathrm{q} = {}^{(\mathrm{i})}\mathrm{q} + \Delta \mathrm{q} $$

$$ \textit{Converged} \quad \textit{if} \quad \frac{\|\Delta q\|}{\|{}^{(i+1)}q\|} \le \varepsilon $$

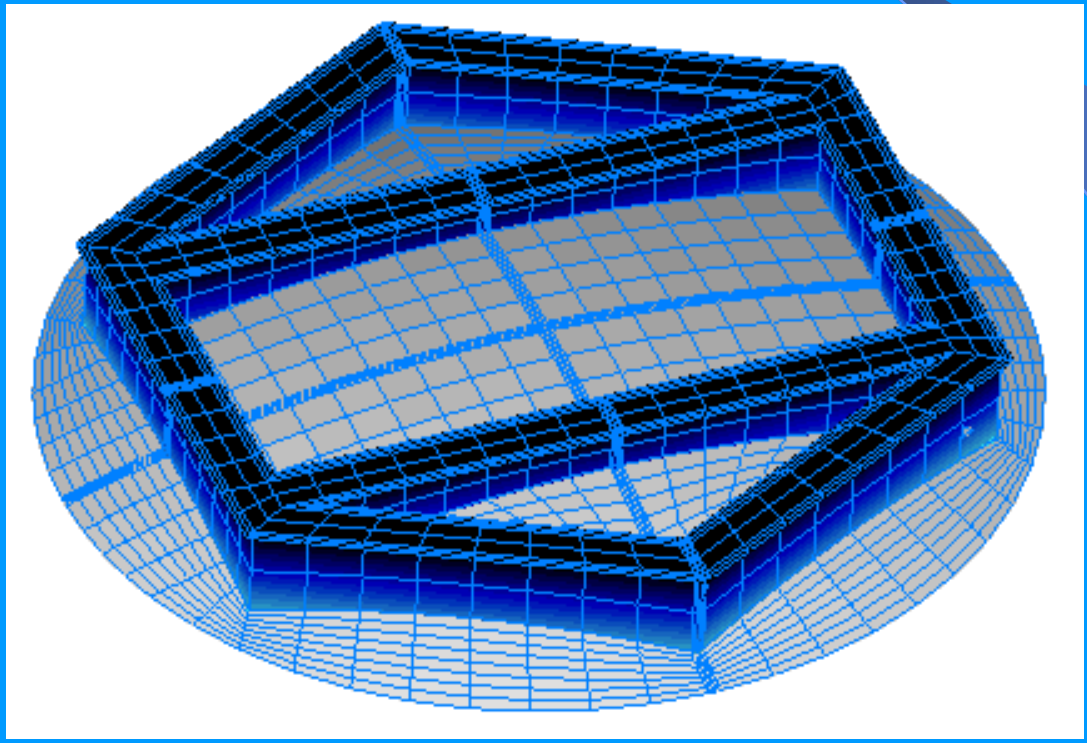
Geometry Of the Reflector

$$\frac{L}{t} \simeq 3200$$

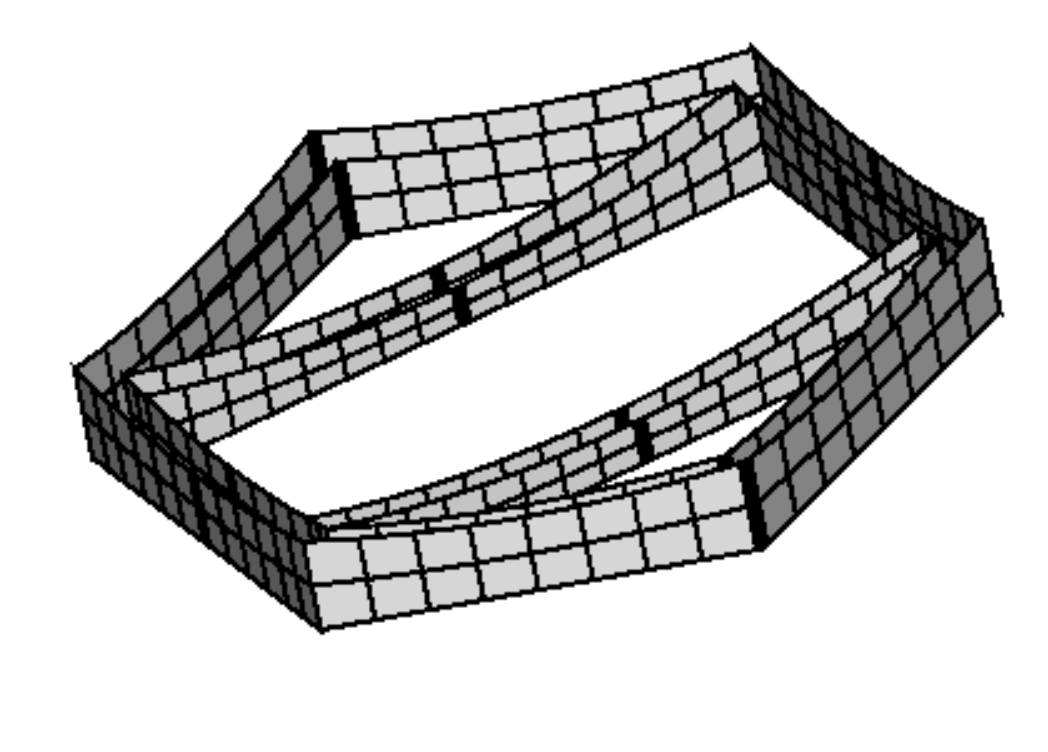
A very thin structure



The modeled reflector (bottom view)



Stiffener (Frame)



Material Properties and Lay-ups

- **M60J/954-3 Unitape**

  E_1: 53 Msi (Tensile), 50 Msi (Compressive)

  E_2, E_3: 0.95 Msi (Tensile).0.91 Msi (Compressive)

  G_{12}, G_{13}: 0.0681 Msi, G_{23}: 0.3185 Msi

  $?_{12}$, $?_{13}$:0.319, $?_{23}$: 0.46

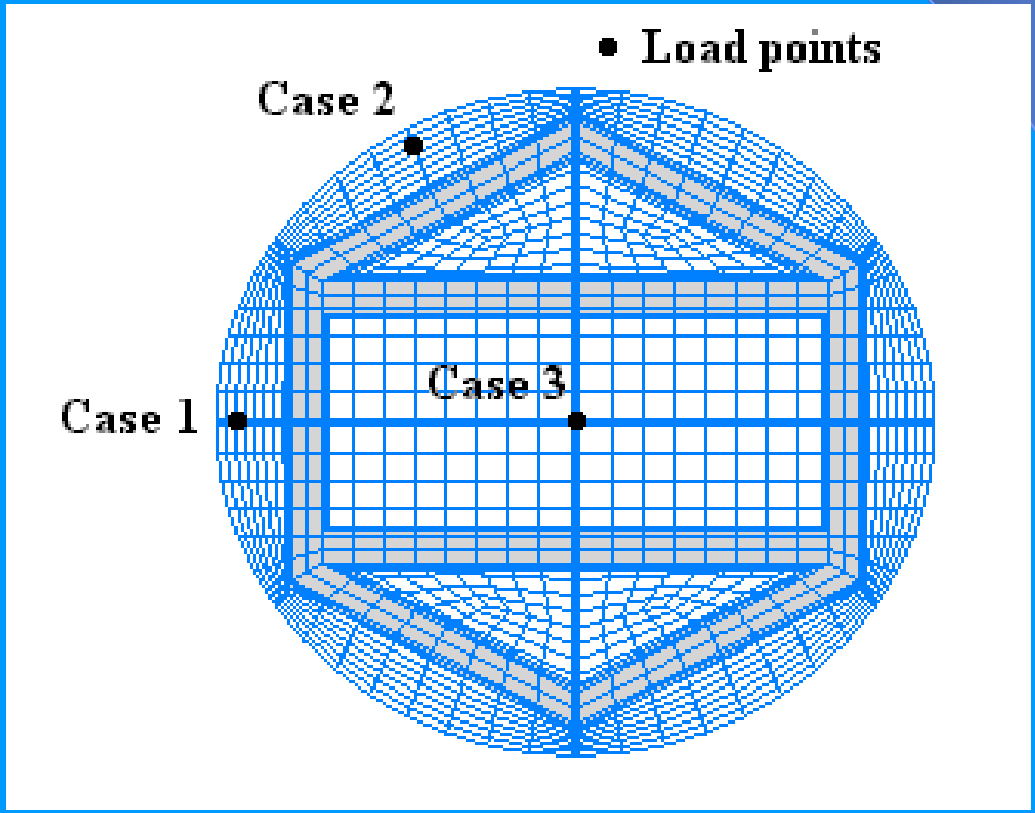
- **Layups**

  Dish shell: $(0/90/45/135)_s$ – 8 plies, 0.016" thick

  Frame/Cap: $(0/60/-60)_{s2}$ – 12 plies, 0.024" thick

Load Cases

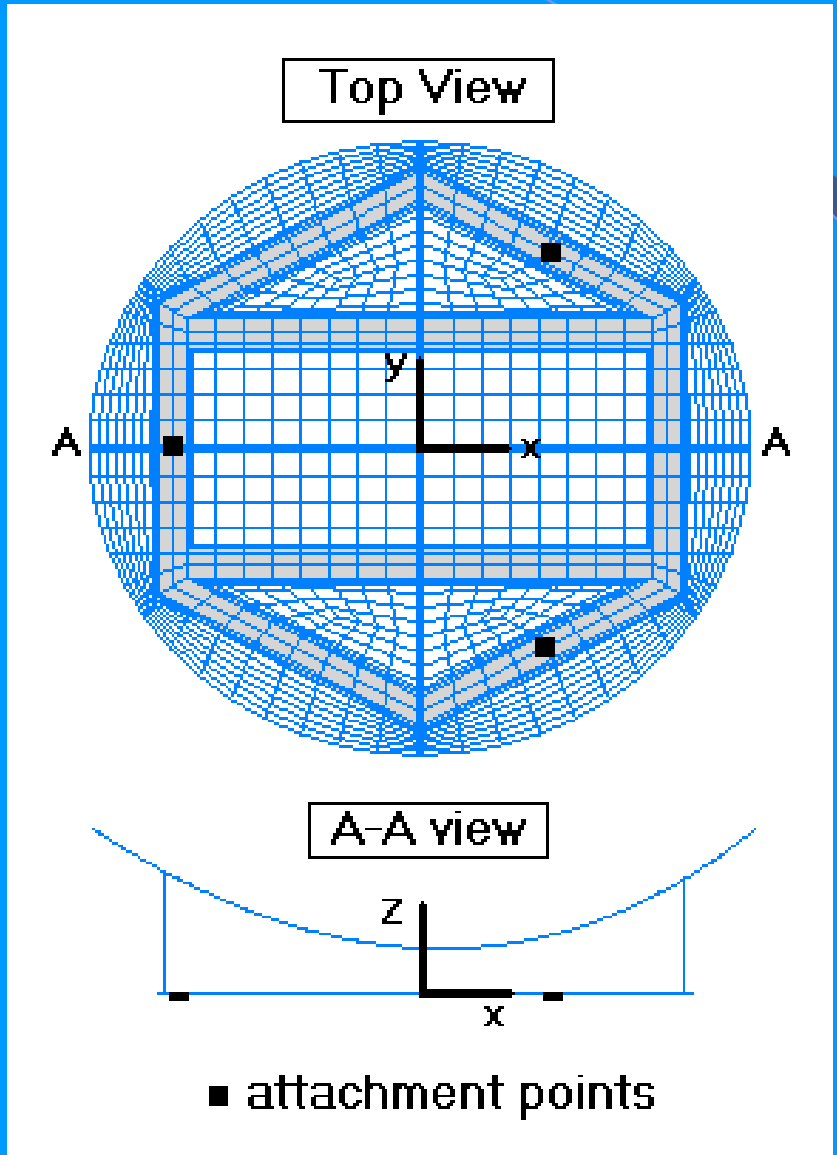
- Three cases of point load applied on the reflector surface



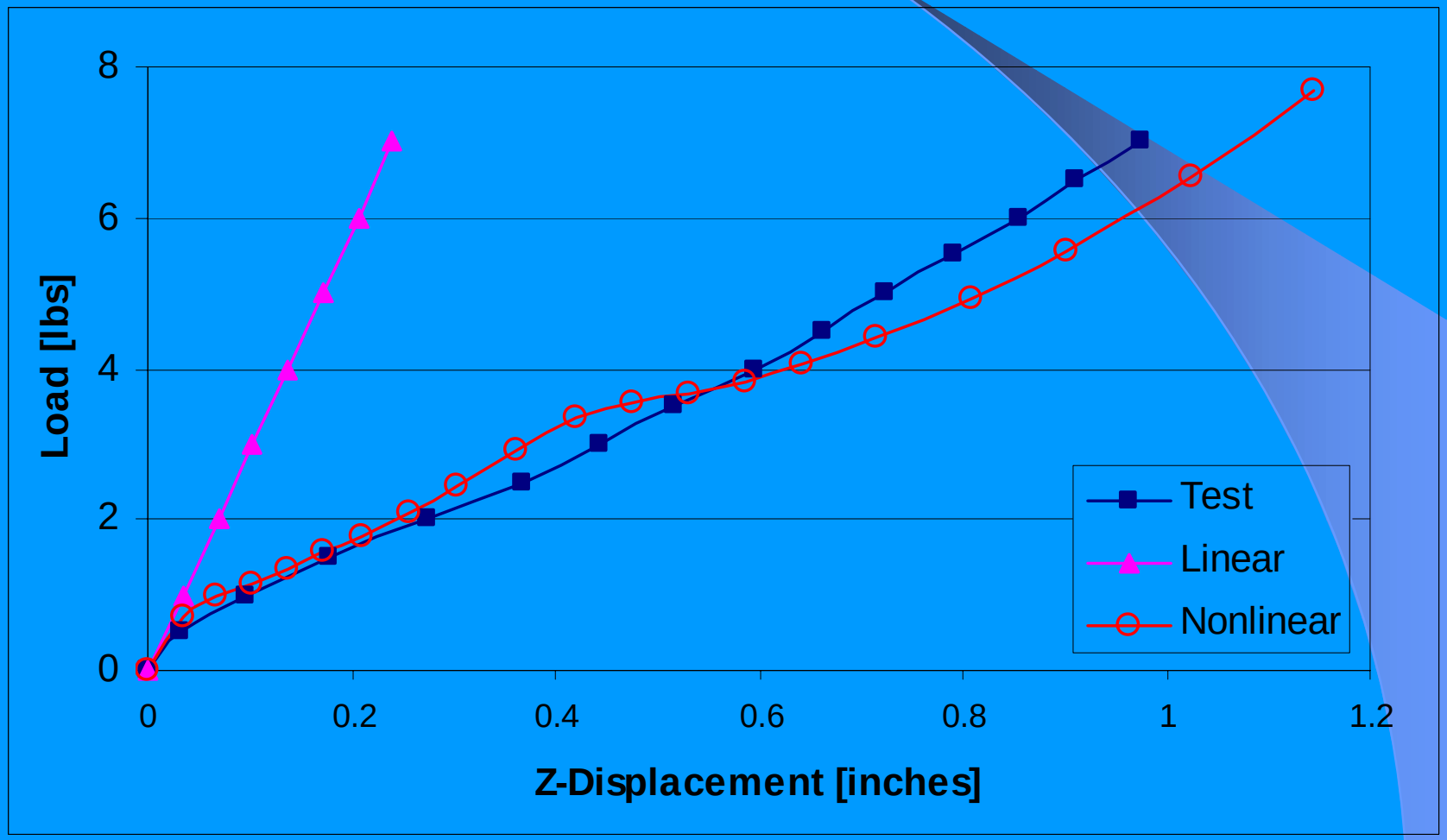
Load direction

- The direction of load is fixed
- Always perpendicular to the original reflector surface

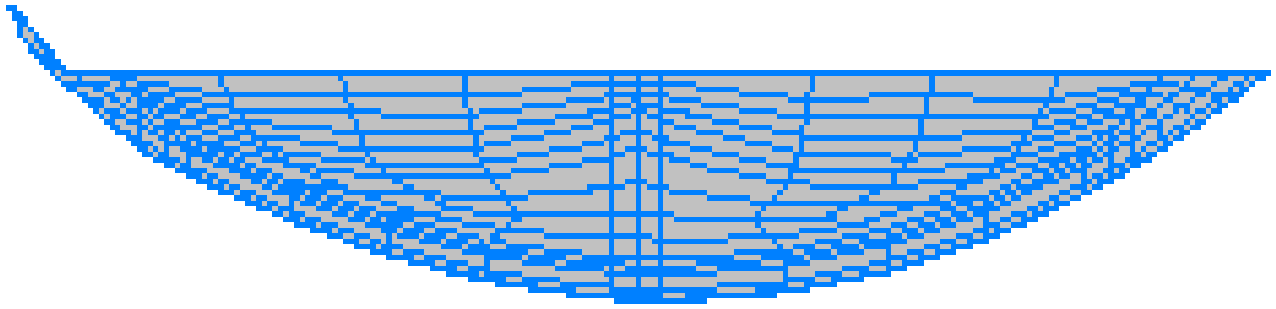
Boundary Conditions



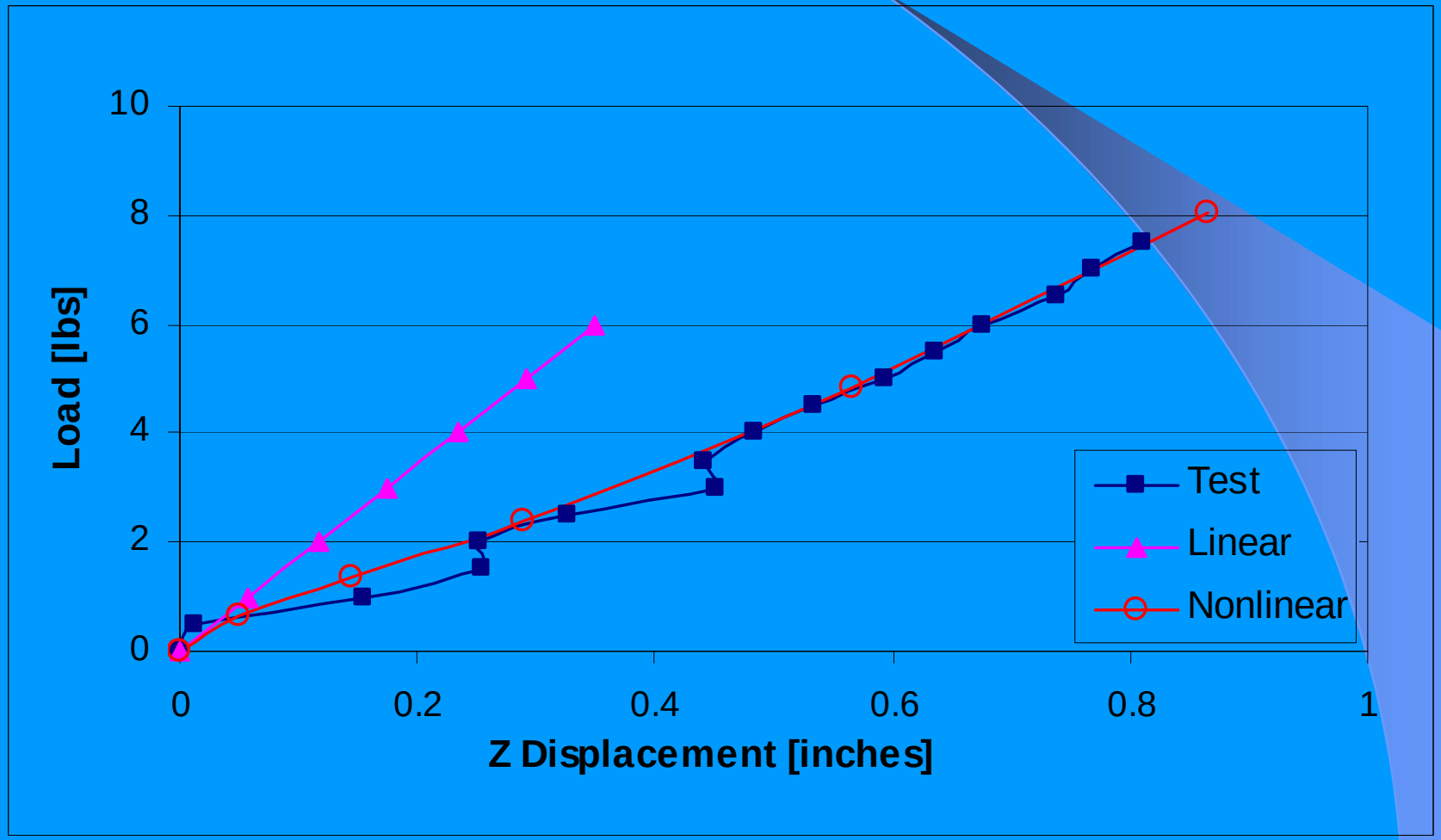
Case 1 –Results



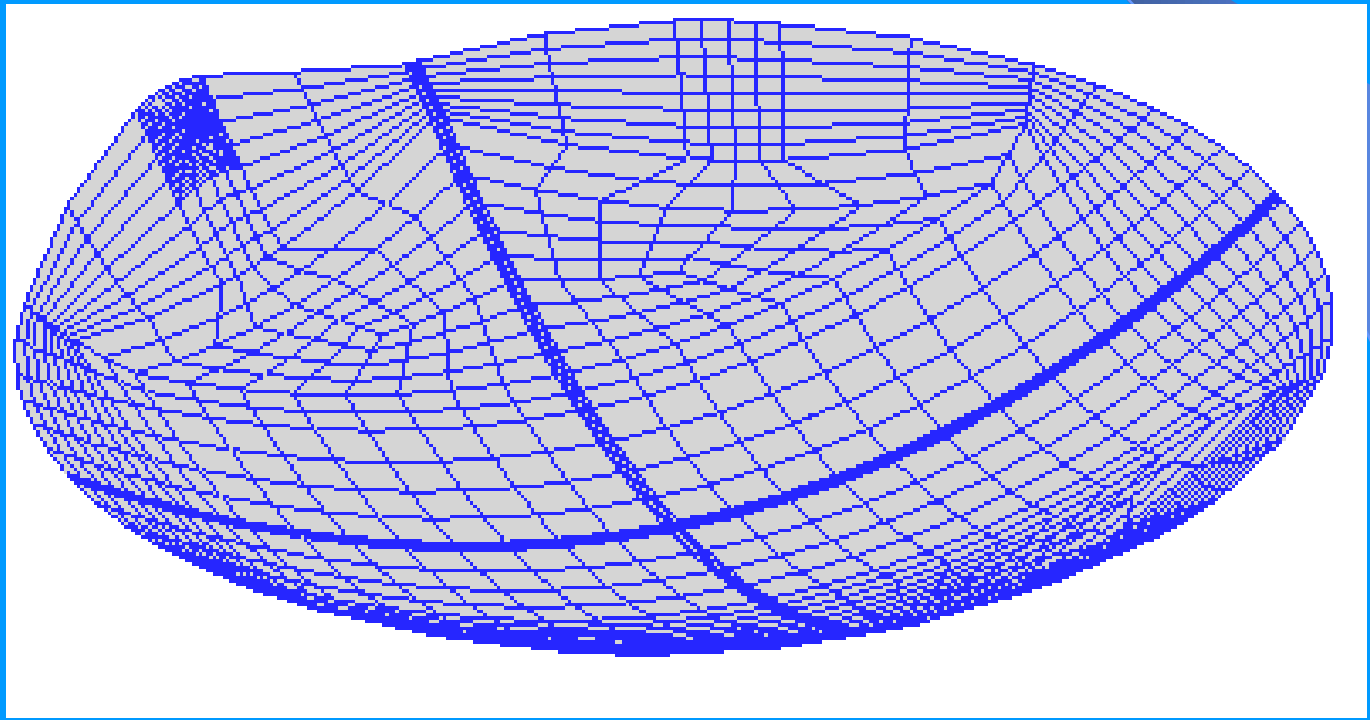
Case 1 - Deformed Shape



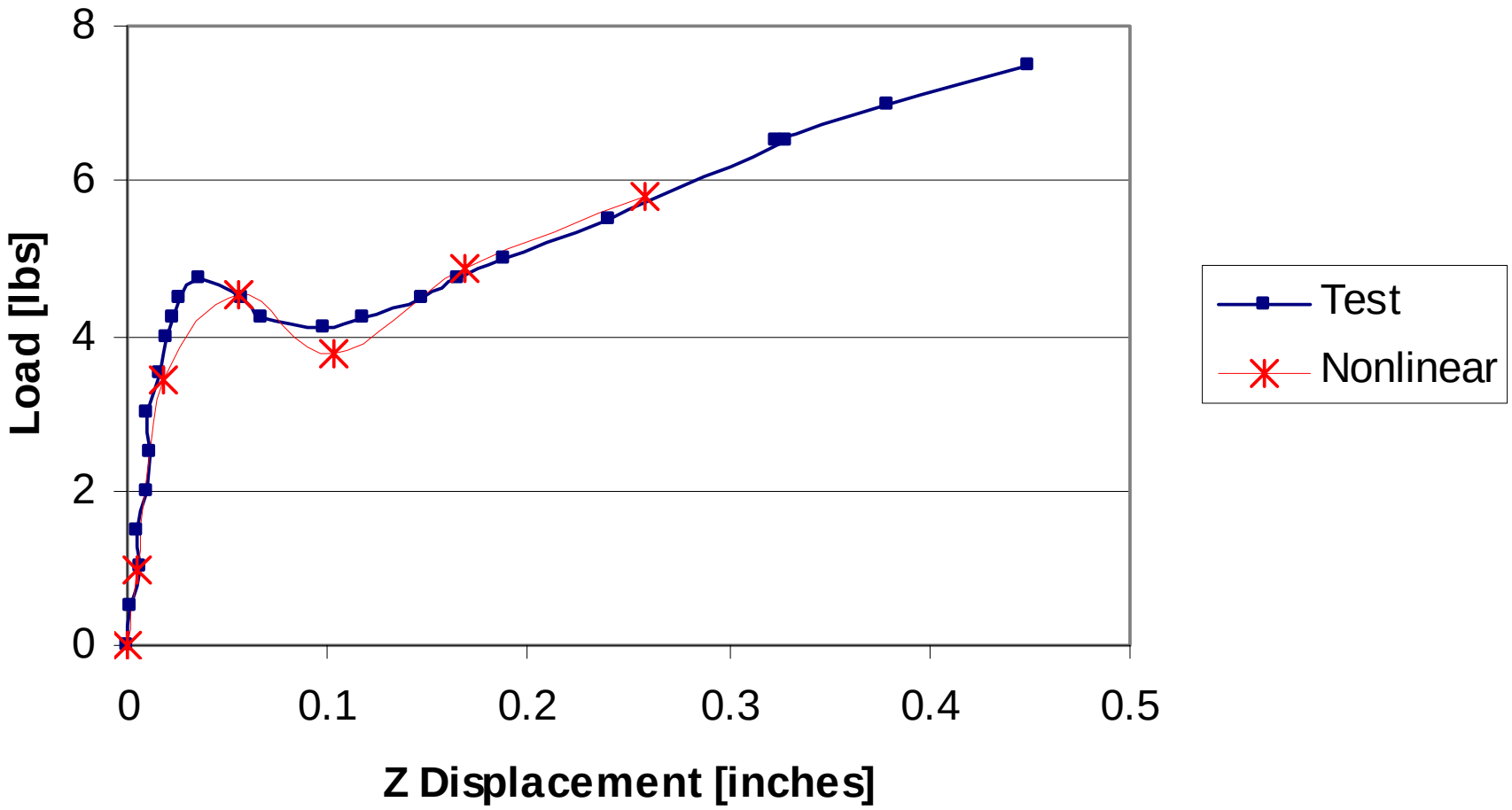
Case 2 - Results



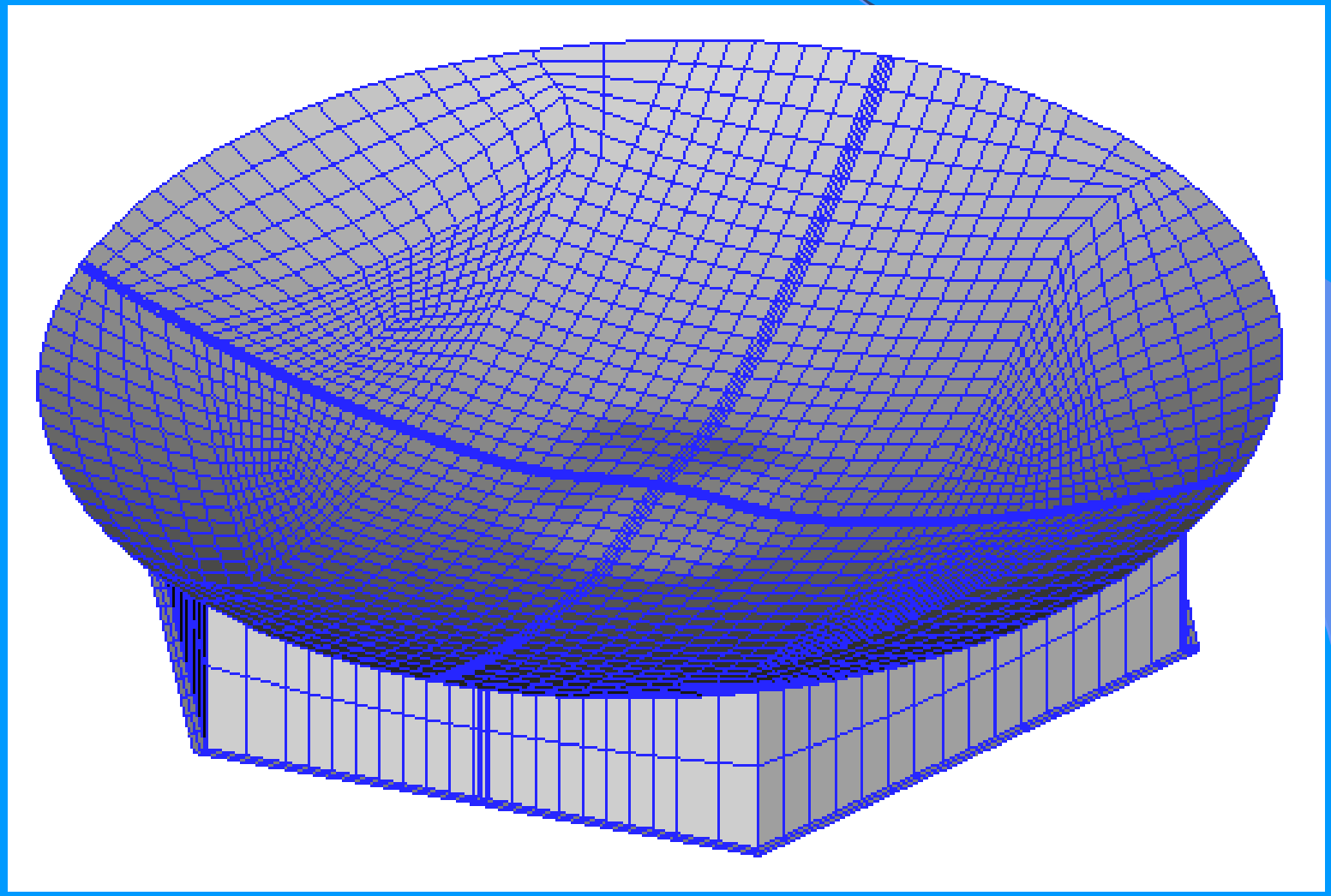
Case 2 - Deformed Shape



Case 3 - Results



Case 3 - Deformed Shape



Conclusion

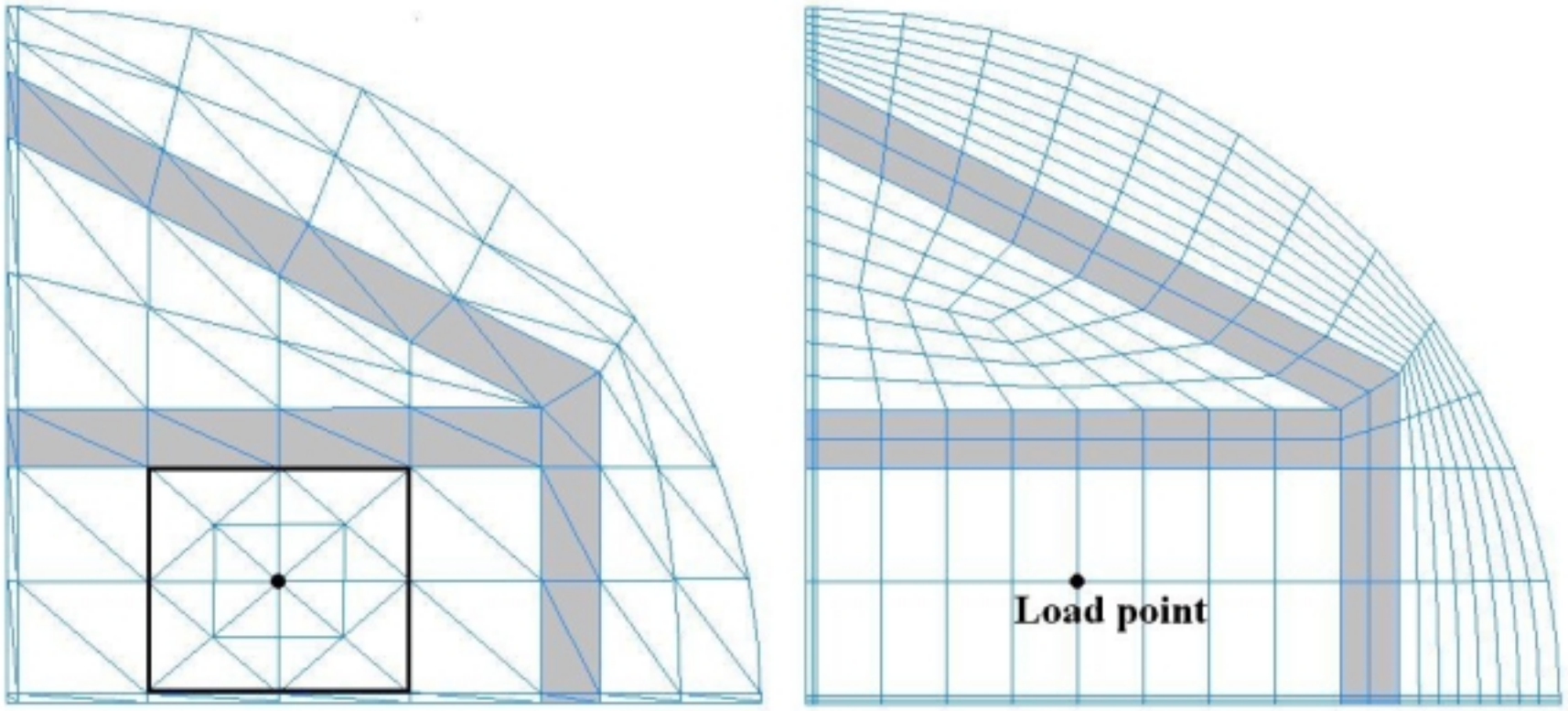
The computed displacements are in reasonably good agreement with the experimental results.

The finite element software, based on the assumed strain solid shell formulation, can be used for analysis of highly flexible space structures such as a composite reflector.

Current and Future Work

- User friendly program: Combine with the existing pre or post-processing program (e.g., FEMAP)
- Geometrically nonlinear analysis under dynamic loading during launch
- Implementation of a triangular element for ease of mesh design and refinement

Why triangular elements ?



For triangular elements, mesh refinement around the load point can be easily localized