Motivation

- Recognized a need for nonlinear FEA in our branch
- Interested in Element Design, Element Locking, and the effects of element distortion on solution accuracy
- Found a suitable project with the appropriate resources (interest, money, and time)
Objective

- Develop a finite element research model that can be used to study highly flexible structures undergoing geometrically nonlinear behavior
- Use the model to predict the structural response of a thin composite reflector
The Reflector
Problem Formulation Highlights

Approach

- Compute the deformation of a composite reflector with an areal density of 4 kg/m² under point loads using a geometrically nonlinear solid shell finite element model.

- Validate the computational results through comparison with the experimental data.
Problem Formulation Highlights

The Solid Shell Approach

Elements

- 4-node plate elements (6 dof/node)
- 9-node shell elements (6 dof/node)
- 18-node shell elements (3-dof/node)
Solid Shell Formulation

All kinematic variables: vectors only

No rotational angle are used: easy connections between substructures

18 Node Element, 3 DOF at each node

9 Node Element, 6 DOF at each node
Problem Formulation Highlights

Element Locking: Zero-Energy (Spurious) Modes

- Plate and shell elements lock as the thicknesses decrease
- METHODS OF ELIMINATION
  - Reduced/ Selective Integration
  - Stabilization Matrix
  - Assumed Strain approach
Problem Formulation Highlights

Assumed Strain Function

\[ \varepsilon = P(\xi, \eta)\alpha \]

where

\[ \alpha = H^{-1}Gq_e \]

and

\[ H = \int_v P^T C_e P dv \]

\[ G = \int_v P^T C_e B dv \]
Problem Formulation Highlights

Bubble Displacement

- Eliminates some forms of element locking
- Reduces the sensitivity of elements to distortion
Problem Formulation Highlights

Bubble Displacement Function

\[ N_b = (1 - \xi^2)(1 - \eta^2) \text{ for 4 - node quadrilateral element} \]

\[ N_b = \xi \eta (1 - \xi^2)(1 - \eta^2) \text{ for 9 - node quadrilateral element} \]
Problem Formulation Highlights

Model Equations

HELLINGER-REISSNER FUNCTIONAL

\[ \pi_R = \int (\varepsilon^T C_e \bar{\varepsilon} - \frac{1}{2} \varepsilon^T C_e \varepsilon) dv - W \]

EQUILIBRIUM

\[ \int_v \delta \bar{\varepsilon}^T C_e \varepsilon \ dv - \delta W = 0 \]

COMPATIBILITY

\[ \int_v \delta \varepsilon^T C_e (\bar{\varepsilon} - \varepsilon) dv = 0 \]
Figure 4  Kinematics of shell deformation

(a)

(b)

The \( a_3 \) vectors
GEOMETRY

\[ x = \sum_{i=1}^{n} N_i(\xi, \eta)(x_0)_i + \zeta \sum_{i=1}^{n} N_i(\xi, \eta)(t a_3)_i \]

ASSUMED DISPLACEMENT WITH BUBBLE FUNCTION

\[ u = \sum_{i=1}^{n} N_i(\xi, \eta)(u_0)_i + \zeta \sum_{i=1}^{n} N_i(\xi, \eta)(t u_z)_i + N_b(\xi, \eta)(u_{b_0} + \zeta u_{b_z}) \]
Problem Formulation Highlights

Element Stiffness Matrix

\[ K_e = G^T H^{-1} G \]
Problem Formulation Highlights

Nonlinear Equations

\[
\begin{align*}
(i) \quad & K \Delta q - (F_{ext} - (i) F_{int}) \approx 0 \\
(i+1) q &= (i) q + \Delta q \\
\text{Converged if } & \frac{\|\Delta q\|}{\|q\|} \leq \varepsilon
\end{align*}
\]
Geometry Of the Reflector

\[ \frac{L}{t} \approx 3200 \]

A very thin structure
The modeled reflector (bottom view)
Material Properties and Lay-ups

- **M60J/954-3 Unitape**
  - $E_1$: 53 Msi (Tensile), 50 Msi (Compressive)
  - $E_2$, $E_3$: 0.95 Msi (Tensile), 0.91 Msi (Compressive)
  - $G_{12}$, $G_{13}$: 0.0681 Msi, $G_{23}$: 0.3185 Msi
  - $\theta_{12}$, $\theta_{13}$: 0.319, $\theta_{23}$: 0.46

- **Layups**
  - Dish shell: $(0/90/45/135)_s$ – 8 plies, 0.016" thick
  - Frame/Cap: $(0/60/-60)_s^2$ – 12 plies, 0.024" thick
Load Cases

- Three cases of point load applied on the reflector surface
Load direction

- The direction of load is fixed
- Always perpendicular to the original reflector surface
Boundary Conditions

Top View

A-A view

attachment points
Case 1 – Results

The graph shows the relationship between load (in pounds) and Z-displacement (in inches) for different cases.

- **Linear** case shows a linear increase in load with Z-displacement.
- **Test** results are represented by a smooth curve, indicating a more gradual increase.
- **Nonlinear** case exhibits a steeper increase in load compared to the linear case.

Load vs. Z-Displacement graph:
- **X-axis:** Z-Displacement [inches]
- **Y-axis:** Load [lbs]
- Legend: Test (blue line), Linear (pink triangles), Nonlinear (red circles)
Case 1 - Deformed Shape
Case 2 - Results

![Graph depicting load vs. Z displacement for linear and nonlinear cases.](image)
Case 2 - Deformed Shape
Case 3 - Results

![Graph showing load versus Z displacement for Test and Nonlinear scenarios. The graph includes a blue line for Test and a red dotted line for Nonlinear. The x-axis represents Z Displacement in inches, ranging from 0 to 0.5, and the y-axis represents Load in lbs, ranging from 0 to 8.]
Case 3 - Deformed Shape
Conclusion

The computed displacements are in reasonably good agreement with the experimental results.

The finite element software, based on the assumed strain solid shell formulation, can be used for analysis of highly flexible space structures such as a composite reflector.
Current and Future Work

- User friendly program: Combine with the existing pre or post-processing program (e.g., FEMAP)
- Geometrically nonlinear analysis under dynamic loading during launch
- Implementation of a triangular element for ease of mesh design and refinement
Why triangular elements?

For triangular elements, mesh refinement around the load point can be easily localized.