Optomechanical Design and Analysis of Adaptive Optical Systems using FEA and Optical Design Software

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Integrated Optomechanical Analysis

Thermal Analysis

- Steady-State & Transient
- conduction
- convection
- radiation

Structural Analysis

- static & dynamic
- linear / nonlinear
- stress
- displacement
- shock & vibration
- thermo-elastic
- inertial
- buckling

Optical Analysis

- Interpolated
- Temperatures

Interface Programs
- Zernike Fitting
- Interpolation

Optical Performance Metrics

- Wavefront Analysis
- Point Spread Function
- Modulation Transfer Function
- Encircled Energy
Integrated OptoMechanical Analysis

Example Telescope: Must pass structural distortions to optical model for analysis
Zernike Polynomials

- Polynomial series with two real variables, \( r \) and \( \theta \)

\[
\Delta Z(r, \theta) = A_{00} + \sum_{n=2}^{\infty} A_{n0} R_n^0(r) + \sum_{n=1}^{\infty} \sum_{m=1}^{n} R_n^m [A_{nm} \cos(m\theta) + B_{nm} \sin(m\theta)]
\]

\[
R_n^m(r) = \sum_{s=0}^{\frac{n-m}{2}} (-1)^s \frac{(n-s)!}{s! \left(\frac{n+m}{2} - s\right)! \left(\frac{n-m}{2} - s\right)!} r^{(n-2s)}
\]

- Standard Zernike polynomials (See Born & Wolf, *Principles of Optics*)
  - use as many terms as required to represent the data

- Fringe Zernike polynomials are a subset of the Standard Zernikes
  - include higher-order symmetrical terms (\( r^{10} \) & \( r^{12} \)) that are more important to wavefront propagation; eliminates the higher-order azimuthal terms

\( r \) - dimensionless normalized radius
\( \theta \) - polar angle
\( A_{nm} \) & \( B_{nm} \) - polynomial coefficients
ZernikeSurfaces

Bias/Piston: 1

Tilt: \( r \cos(\theta) / r \sin(\theta) \)

Power/Defocus: \( 2r^2 - 1 \)

Pri-Astigmatism
\( 2r^2 \cos(2\theta) / 2r^2 \sin(2\theta) \)
Zernike Surfaces

Pri-Coma:
\[(3r^3-2r)\cos(\theta) / (3r^3-2r)\sin(\theta)\]

Pri-Trefoil:
\[r^3\cos(3\theta) / r^3\sin(3\theta)\]

Pri-Spherical:
\[6r^4-6r^2+1\]

Sec-Astigmatism:
\[(4r^4-3r^2)\cos(2\theta) / (4r^4-3r^2)\sin(2\theta)\]
Integrated OptoMechanical Analysis - *Current Technology*

- FEA code (Nastran) => surface deformations
- SigFit => Fit Zernikes to FEA data, output in Optics format
- Optics code (CodeV) => read Zernikes, calculate system optical response

**Disadvantages**
- requires optical engineer in the loop
- analysis process turnaround is slow
- can not use in FEA optimization loop
Why Adaptive Optical System

- Optical surfaces are deformed and moved based on measured or anticipated information to compensate for unwanted disturbances.
- Uses
  - Fabrication & assembly errors in deployable systems
  - Thermoelastic & humidity distortion
  - Atmospheric disturbance in ground based telescopes
  - Vibrations & dynamic disturbances
Adaptive Simulation Method - Conceptual

- Adaptive Performance Can Be Simulated With Finite Element Analysis
  - Generate two sets of deformation predictions
    - Uncorrected disturbances
    - Actuator influences
  - Solve for actuator inputs, \( x_1, x_2, x_3 \ldots x_n \), to minimize surface error, \( E \)

- If focus compensation exists elsewhere, terms like \( 2\rho^2 - 1 \) or \( \Delta R \) can be added as *augment* actuators
Adaptive Analysis - *Current Technology*

- FEA code => surface distortions
- FEA code => actuator influence functions
- SigFit => read FEA data, calculate actuator force to correct that surface
- Optics code => read SigFit data, calculate system response

- Disadvantages
  - Error correction for that single surface, *not* system response
  - Not correcting other optical surfaces effects
  - Can not combine multiple adaptive surfaces

- To correct system level effects, the system wavefront error must be related back to the adaptive optic as an equivalent surface distortion.
Integrated System Analysis - *New Technology*

- Optics code => system response sensitivity due to unit Zernikes at each surface
- FEA code => surface distortions of all surfaces
- FEA code => influence functions for all actuators (if adaptive)

- SigFit => calculate system response
  - fit Zernikes to FEA distortions of each surface
  - multiply by system sensitivities to get system response

- SigFit => calculate corrected system response (if adaptive)
  - fit Zernikes to FEA influence functions
  - calculate actuator forces to minimize system error

- Advantage
  - speeds up analysis turn around
  - using system level performance generates superior designs
Integrated System Analysis - *New Technology*

- Optical surfaces: $n = 1$ to $S$  
  Number adaptive surfaces: $t = 1$ to $T$
- Zernike in/surface: $j = 1$ to $Z$  
  Zernike out/system: $k = 1$ to $Z$
- Load case number: $i = 1$ to $L$  
  Actuator number: $m = 1$ to $M$

- Sensitivity matrix = Zernike out ($k$) for Zernike in ($j$) at surface ($n$) = $S_{kj}^n$
- Disturbance fit = fit each load case ($i$) with Zernike ($j$) at surface ($n$) = $C_{ji}^n$
- Actuator influence = fit with Zernike ($j$) at surface ($t$) = $B_{jm}^t$
- System response = Zernike ($k$) at output location (0) for load case ($i$) = $Z_{ki}^0$
Integrated System Analysis - *New Technology*

- System level response = Zernikes at output (ie Exit Pupil)

\[ Z_{ki}^0 = \sum_{n}^{S} S_{kj}^n C_{ji}^n \]

Where \(S\) is the Zernike sensitivities from Code V

\(S_{kj}^n\) = matrix of size \((Z \times Z \times N)\)

and \(C\) is the Zernike fit to FEA deformations for each load case

\(C_{ji}^n\) = matrix of size \((Z \times L \times N)\)

Resulting \(Z^o\) is reported along with Surface RMS and Peak-Valley

Output a visualization file showing net response at output location
Integrated System Analysis - Adaptive - *New Technology*

- System level response at Output location due to Actuators

\[
U_{km} = \sum_{t}^{T} S_{kj} \cdot B_{jm}^t
\]

Where B is the Zernike fit to Actuator influence functions

\( B_{jm}^t = \text{matrix of size } (Z \times M \times T) \)

Define system level error \( E \) as

\[
E = \sum_{k}^{Z} w_k \left( Z_{ki} - \sum_{m}^{M} U_{km} A_m \right)^2
\]
Minimize System Error with respect to Actuator forces

\[ \frac{dE}{dA_q} = \sum_{k} Z_k 2 \left( Z_{ki} - \sum_{m} U_{km} A_m \right) U_{kq} = 0 \]

Solve resulting linear system for A

\[ [H] \{ A \} = \{ F \} \]

\[ H_{qm} = \sum_{k} w_k U_{qk} U_{km} \]

\[ F_q = -\sum_{k} w_k Z_k U_{kq} \]
Integrated System Analysis - Example

Example: Telescope

Finite Element Model

Optical Model
Integrated System Analysis - Example

Adaptive PM (*9 force actuators in red, 3 displacement actuators in blue*)
Integrated System Analysis - Example

- Load Case: 1g along optical axis
- Added $5\lambda$ of astigmatism on SM (represents a thermal distortion)

- PM sits on 3 points (displacement actuators)
  - 1g distortion = $4.62\lambda$ RMS
    - mostly trefoil = $12.5\lambda$

- SM sits on 3 edge points (with $5\lambda$ astigmatism added)
  - 1g distortion = $2.18\lambda$ RMS
    - trefoil = $2.0\lambda$
    - added astigmatism = $5.0\lambda$

- Note: Surface distortions have a doubling effect on reflected wavefront error
Integrated System Analysis - Example

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<tr>
<th>No Correction</th>
<th>PM Correction</th>
<th>Sys Correction</th>
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<tr>
<td>PM RMS= 4.62\lambda</td>
<td>PM RMS= 0.11\lambda</td>
<td>Sys RMS= 2.18\lambda</td>
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<td>Sys RMS= 11.25\lambda</td>
<td>Sys RMS= 4.31\lambda</td>
<td>Sys RMS= 0.23\lambda</td>
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</table>
Integrated System Analysis - Example

• Correcting PM disturbance only
  – Adaptive PM reduced PM error
  – Did not correct SM error, so SM effects still in System error

• Correcting System response
  – Adaptive PM corrected PM error and the SM error
  – Resulting System error greatly reduced
### Integrated System Analysis - Example: Compare Sys Response with CodeV

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Summary

• SigFit’s new System Level Analysis allows more rapid turn around of analyses
  – Optics engineer needed up front to get sensitivities
• Design and analysis under control of structural engineer
  – Can optimize on system level response
  – Reduces the need to budget each optic separately
• Improves and simplifies system level analyses
  – Can correct multiple surfaces’ effects with single adaptive optic
  – Can combine multiple adaptive optics to correct system response
  – More accurate & useful than correcting a single surface’s effect
• User features
  – Visualization plots of System Level Response
• Future development
  – Add System Level Response to SigFit dynamics
  – Add System Level Response to SigFit optimization equations for Nastran
References


