



# Application of NASTRAN Sensitivity Coefficients To the Design of High-Performance Optical Systems

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# JWST Overview



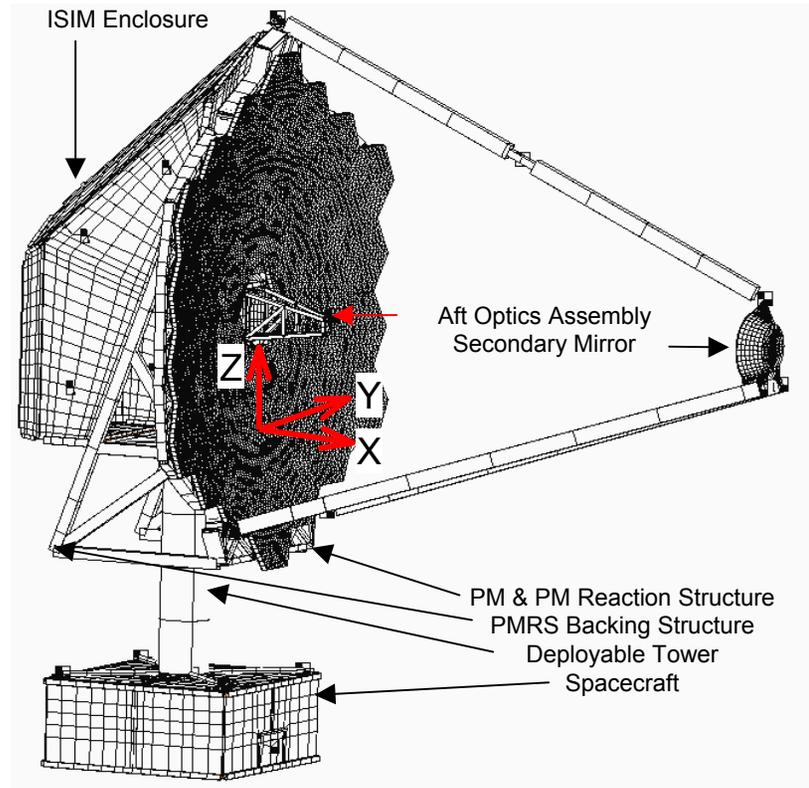
- The James Webb Space Telescope (JWST) is an orbiting infrared observatory that will take the place of the HST.
  - Mass: 5400 kg (12,000 lbs)
  - Diameter of primary mirror: ~6.5 m (21.3 ft)
  - Optical resolution: ~0.1 arc-seconds
  - Wavelength coverage: 0.6 - 28 microns
  - Orbit: L2 halo orbit (1.5 million km from Earth)
  - Operating Temperature: <50 K (-370 °F)
- Major primary mirror innovations
  - Lightweight optics
  - Folding segmented mirror
  - Cryogenic actuators & mirror control
- Above information courtesy of JWST project web page (<http://ngst.gsfc.nasa.gov>)



# NASTRAN Model



- Model logically consists of 6 sections
  - Spacecraft
  - Deployable Tower
  - PM Backplane
  - PM Segments
    - Center section
    - Two deployed wings
  - Secondary Mirror
  - Aft Optics Assembly





# Analysis Technique Objective



- Observatory designed to allow required level of performance for very long exposures
  - Low-expansion materials
  - Minimize thermal gradients
  - Wavefront sensing and control
  - Traditional STOP analysis examines basic design
- Sought a fast integrated modeling procedure
  - Achieve sufficiently accurate error estimates without either NASTRAN statics or ray-tracing runs
  - Allow performance of Monte Carlo simulations or consideration of large trade spaces



# Sensitivity Coefficients

- MSC/NASTRAN Optimization approximates response quantities by a truncated Taylor Series expansion
  - Sensitivity coefficient is the constant of the Taylor series linear term
  - Sensitivity is therefore the first step in optimization
- Finds partials of response quantities,  $r_i$ , wrt design variable,  $x_i$  by means of chain rule,  $\frac{\partial r_j}{\partial x_i} = \frac{\partial r_j}{\partial \{u\}} \cdot \frac{\partial \{u\}}{\partial x_i}$ 
  - First term is unity since we are studying displacements
  - Second term comes from static equilibrium equation,

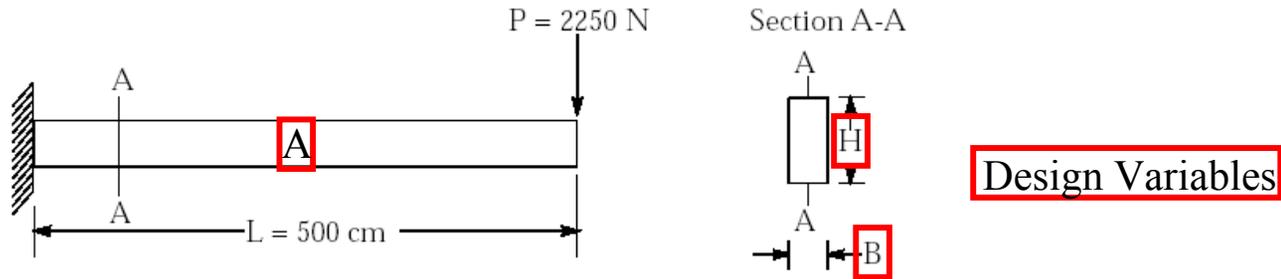
$$[\mathbf{K}] \cdot \frac{\partial u}{\partial x_i} = \frac{\partial P}{\partial x_i} \frac{\partial \mathbf{K}}{\partial x_i} \{u\}$$

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<sup>1</sup>: Equations, example from NASTRAN optimization documentation



# Sensitivity Analysis Example



- Sensitivity coefficients always computed for the design goal and constraints
  - Volume
    - $V = B H L$
    - $V \approx B_0 H_0 L + H_0 L \cdot \Delta B + B_0 L \cdot \Delta H + 0 \cdot \Delta A$
  - Bending stress
    - $\sigma_{\text{bend}} = Mc/I = 6PL/BH^2$
    - $\sigma_{\text{bend}} \approx 6PL/B_0 H_0^2 - 6PL/(B_0 H_0)^2 \cdot \Delta B - 12 PL/B_0 H_0^3 \cdot \Delta H + 0 \cdot \Delta A$
  - Limit on thermal deflection
    - $\delta_{\text{Therm}} = AL(T_2 - T_1)$
    - $\delta_{\text{Therm}} \approx A_0 L(T_2 - T_1) + 0 \cdot \Delta B + 0 \cdot \Delta H + L(T_2 - T_1) \cdot \Delta A$
    - Can not make CTE(T) a design variables



# Linear Optical Modeling

- Joe Howard creates optical sensitivity array
  - Introduces a small motion for each optical DOF
    - Performs a ray trace
    - Recovers the OPD at the exit pupil
  - 3-D array of size  $100 \times 100 \times 6n$ 
    - This work considered only rigid optics
    - Exported to MATLAB
- Disturbed optical performance calculated
  - Motion of each optical DOF generated in some way
  - Array elements multiplied by calculated motion
  - Scaled arrays summed
    - Generate a single  $100 \times 100$  OPD map
    - Mean despace and centroid-drift removed



# Material CTE Analysis

- Monte Carlo analysis used to establish probabilistic criteria for metering structure CTE
  - Determine critical parts of the metering structure
  - Create design variables for each selected area
  - Create relationships to the selected properties
  - Create design constraints of optical DOF
  - Select optimization via `DSAPRT(END=SENS)=ALL`
- Read coefficients into MATLAB
  - Calculate WFE using linear optics
  - Repeated these calculations thousands of times
  - Calculate mean and standard deviations of optical performance figure of merit (OPD)



# Analytical Judgments

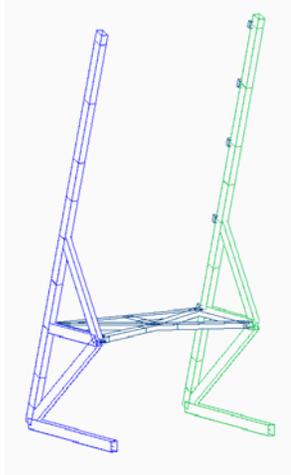
- Used contractor-delivered structural math models
  - Constrained kinematically
- Applied uniform  $\Delta T$  to entire model
  - Magnitude comparable to the slew temperature changes
  - Gradients from actual temperature sets would have complicated results interpretation
    - Highest coefficients would have come from locations experiencing the greatest temperature changes during the slew
    - Would have been slew-case dependent



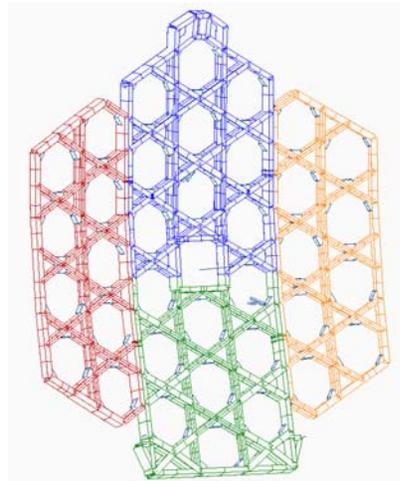
# Structural Details



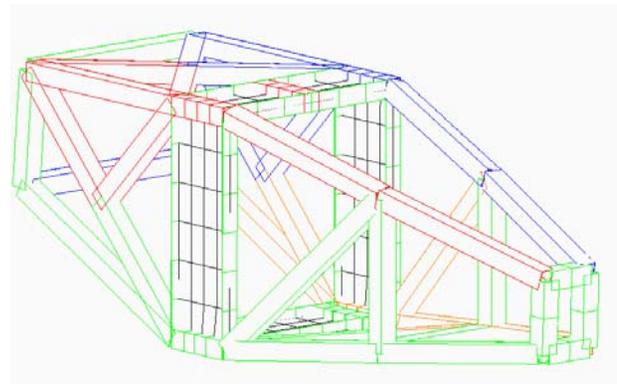
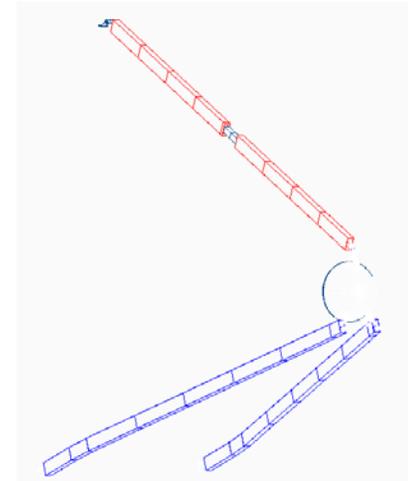
PM Support Structure



PM Backplane



Secondary Tower Assembly



Aft Optics Assembly



# Random CTE Generation

- Generated Gaussian random number
  - $\mu = 0, \sigma = 1$
  - Multiplied by the  $1 - \sigma$  value of CTE for particular trial
    - Representing variations in manufacturing process control
  - Filtered using a trial-specific bandpass
    - Repeat process if value exceeds limits
  - Add in nominal CTE value
    - Representing variations in OTE temp. from nominal T.
- Generated random number between  $-1$  and  $1$ 
  - Uniform distribution
  - Multiplied by a test uncertainty factor
  - Added to the value from previous step



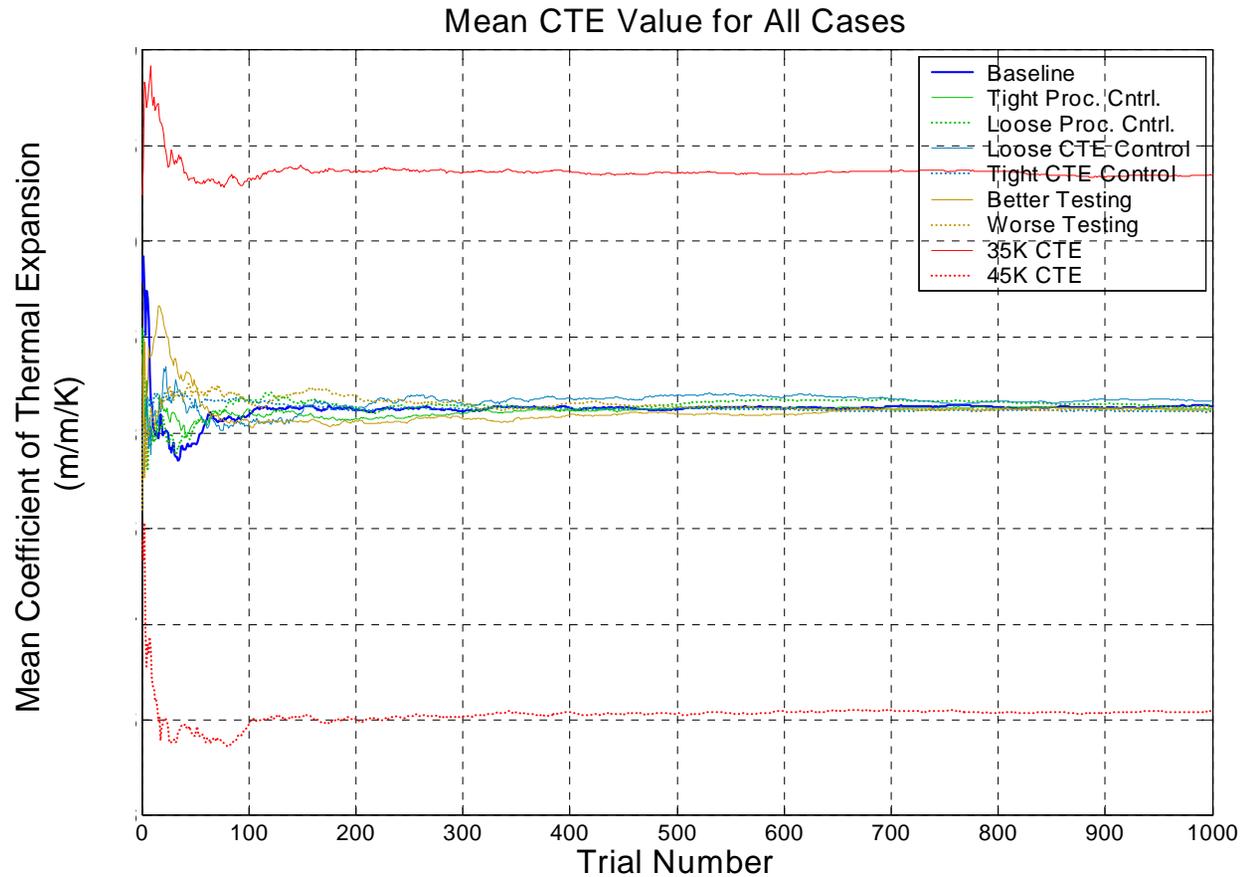
# Analysis Cases

Case	1 Stnd. Deviation (ppb/K)	Bandpass Filter (ppb/K)	Test Uncertainty (ppb/K)	Reject Rate (%)	Mean CTE (ppb/K)	Comments
1	$\sigma$	$\pm \omega$	$\pm \beta$	20	$\alpha$	Baseline Case
2	$.78\sigma$	$\pm \omega$	$\pm \beta$	10	$\alpha$	Narrow CTE Spread
3	$1.9\sigma$	$\pm \omega$	$\pm \beta$	50	$\alpha$	Wide CTE Spread
4	$\sigma$	$\pm .40\omega$	$\pm \beta$	61	$\alpha$	Tighter CTE control
5	$\sigma$	$\pm 2.0\omega$	$\pm \beta$	1.4	$\alpha$	Looser CTE control
6	$\sigma$	$\pm \omega$	$\pm .50\beta$	20	$\alpha$	Better CTE testing
7	$\sigma$	$\pm \omega$	$\pm 2.0\beta$	20	$\alpha$	Worse CTE testing
8	$\sigma$	$\pm \omega$	$\pm \beta$	20	$1.15\alpha$	35K Nominal Temp
9	$\sigma$	$\pm \omega$	$\pm \beta$	20	$.805\alpha$	45K Nominal Temp

- Cases should be considered as four pairs to be compared to the baseline

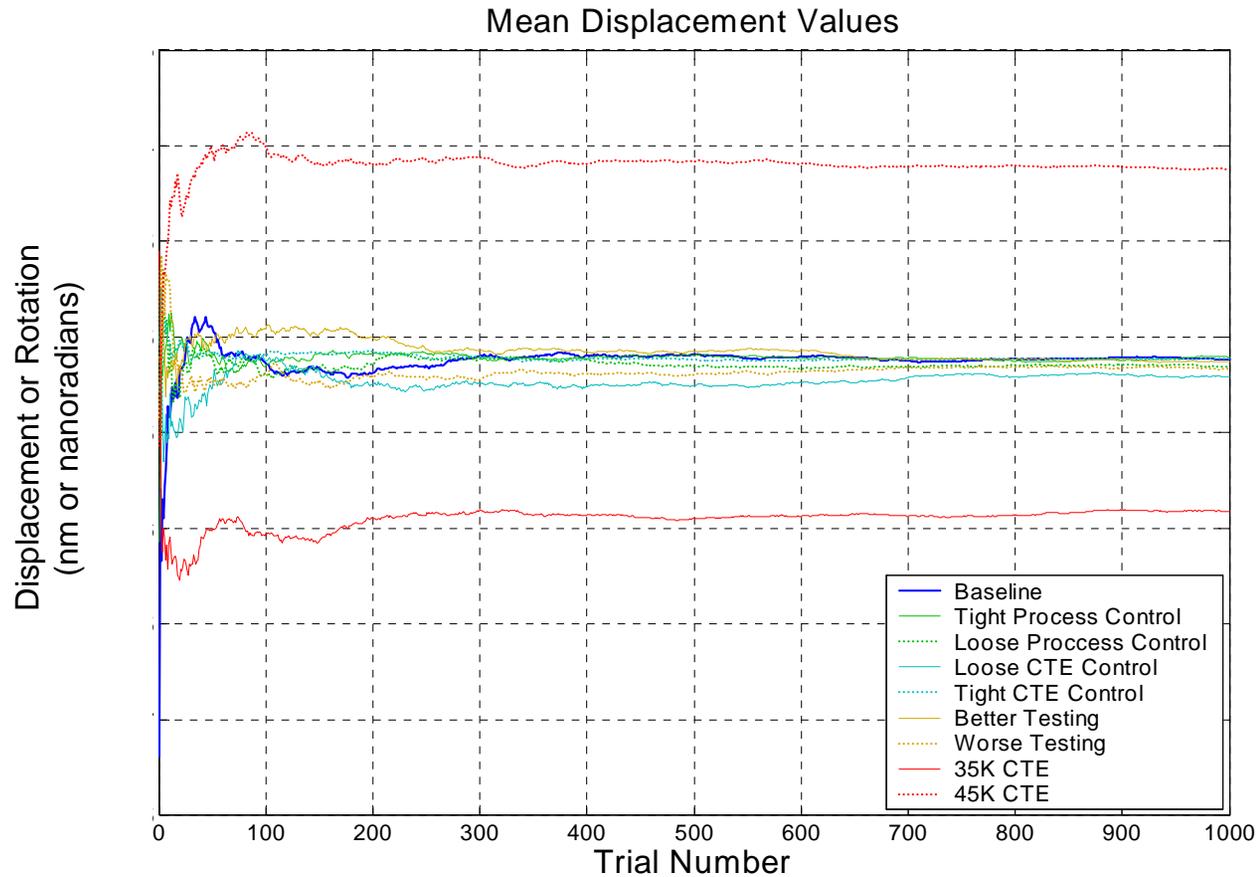


# Mean CTE Convergence (All Trials)





# Mean Displacement Convergence (All Trials)

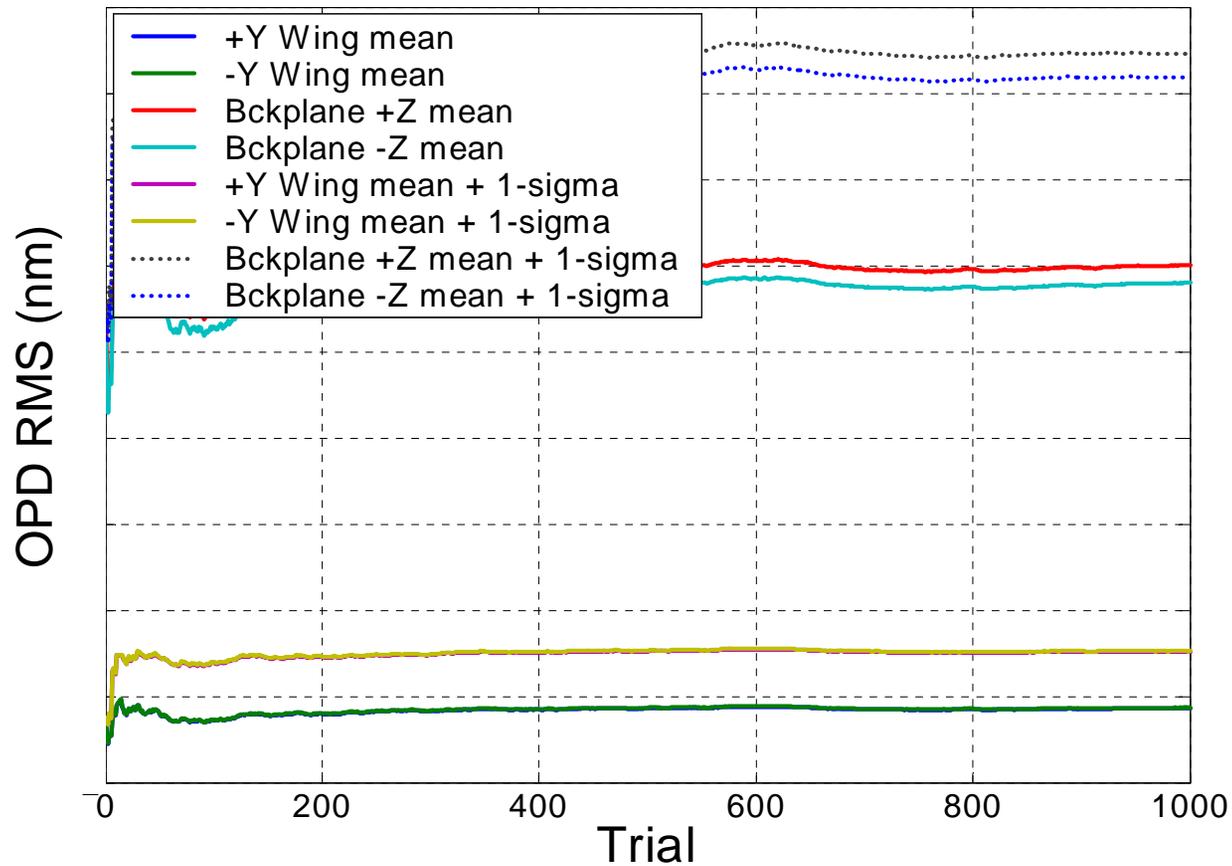




# Design Variable Error Contributions (Nominal Case)



## PMRS Variables OPD Stats





# Conclusions



- Detailed multi-disciplinary analyses can be augmented with sensitivity coefficient-based analysis
- NASTRAN models can generate linear estimations of the effect of changes in structural parameters
- Optical analysis tools can create similar linear models of wavefront error
- Eliminates thousands of NASTRAN model runs and equal numbers of ray-tracing runs