Analysis and Design of Multi-Wave Dilectrometer (MWD) for Characterization of Planetary Subsurface Using Finite Element Method

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Introduction:

Hopper Mission to Moon/Mar

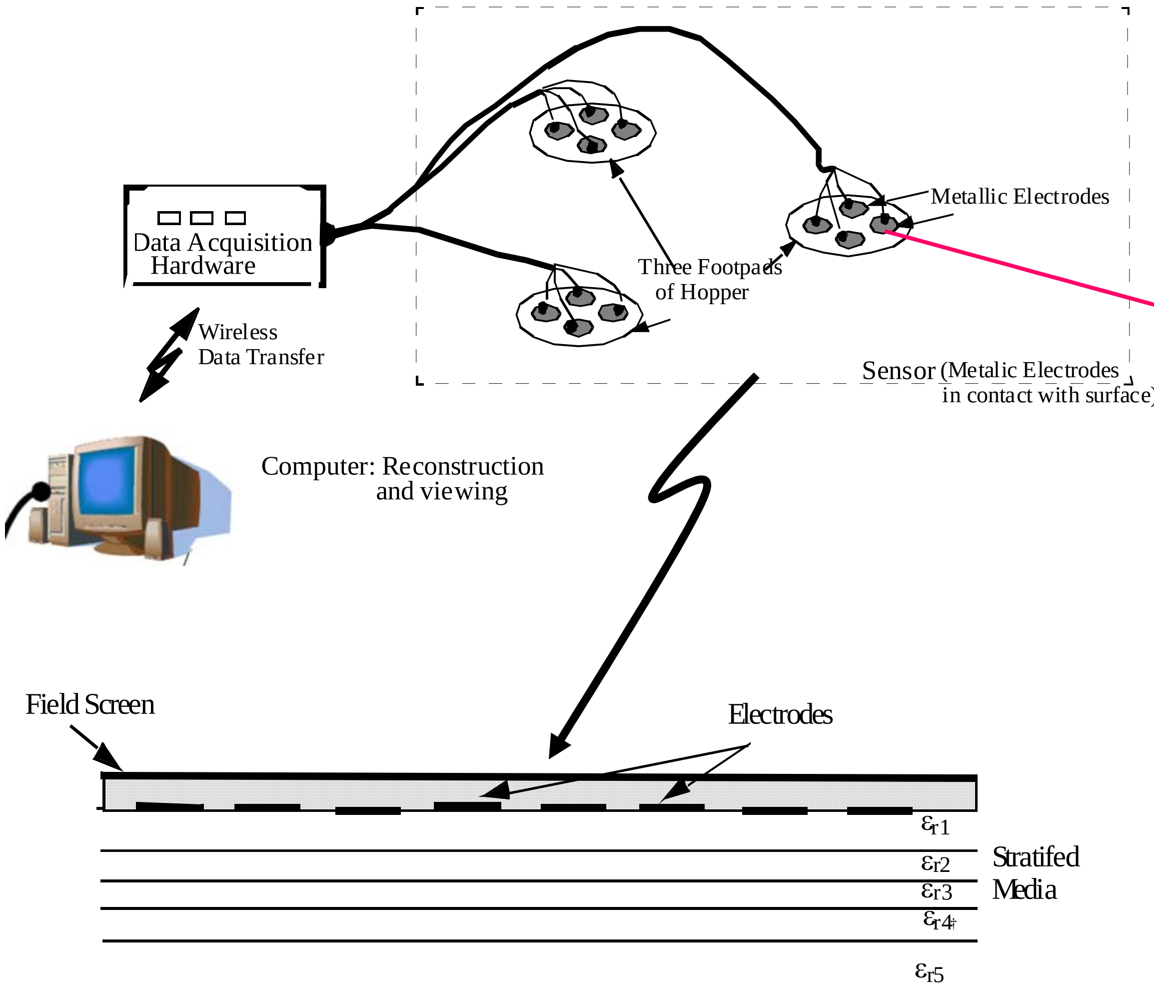
Rover Mission to Mar

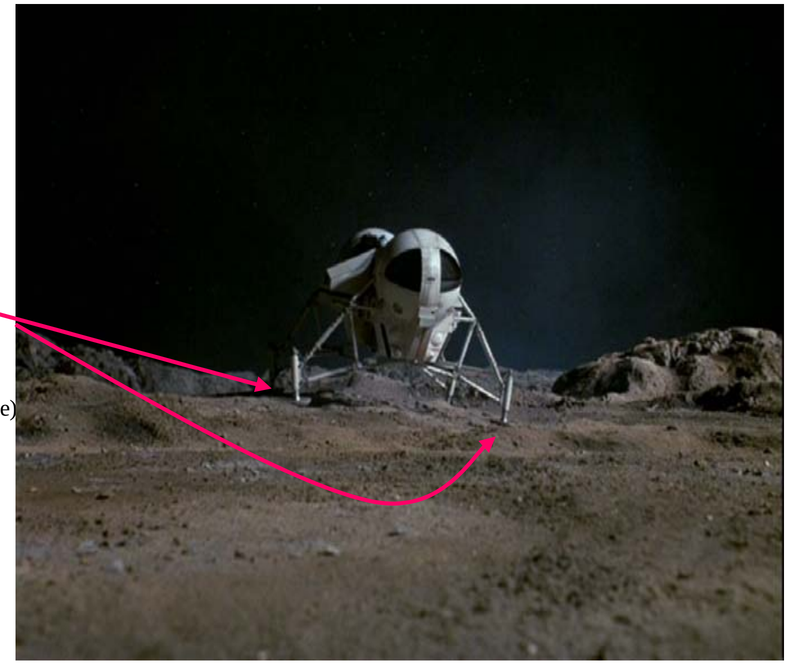
1. One of the scientific goals of these missions is to search for water and other essential materials for human survivability on these planets.
2. A simple, light weight, and low power instrument called Multi-Wave Dielectrometer (MWD) if integrated with Rover/Hopper will help in
   (a) characterizing electrical properties of subsurface Moon/Mar's soil,
   (b) understanding geological evolution of subsurface of these planets without digging .

In this presentation an attempt is made to demonstrate use of Finite Element Procedure to analyze and design proposed MWD instrument

Multi-Wave Dielectrometer

Schematic Diagram of MWD





Moon Hopper

Operational Principle of MWD

Data Acquisition

- Measure mutual capacitances

$$C_{ij}^{m}, i \,\&\, j = 1,2..N$$

between all combinations of electrodes.

Data Processing

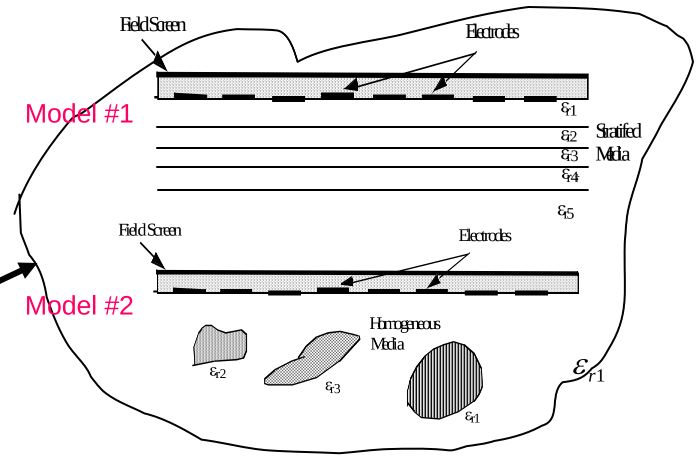
1. Assume a physical model for stratified medium
2. Using FEM modeling, mutual capacitances are estimated as a function of dielectric profile.

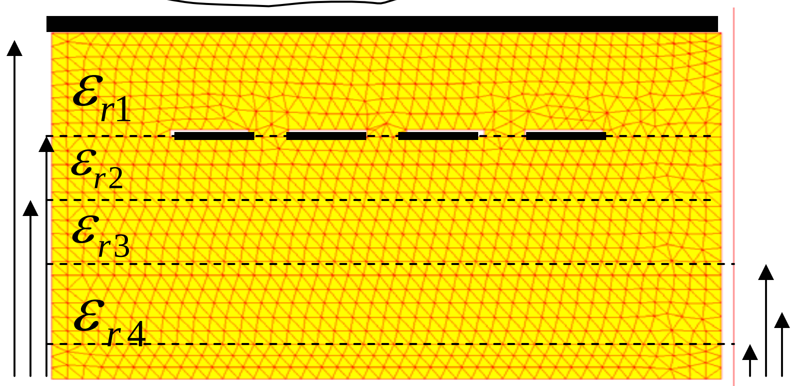
$$C_{ij}^{e}(\varepsilon_{r1},..),\quad i \;\&\; j = 1,2,3,4$$

3. Define error function as

$$f(\varepsilon_{r1},..) = \frac{\sum_{i=1}^{4}\sum_{j=1}^{4}\left(C_{ij}^{m} - C_{ij}^{e}(\varepsilon_{r1},..)\right)^{2}}{\sum_{i=1}^{4}\sum_{j=1}^{4}\left(C_{ij}^{m}\right)^{2}}$$

4. Using optimization procedure based on gradient based method, Genetic Algorithm (GA), or Neural Network (NN), error is minimized to arrive at a probable dielectric profile.





FEM Modeling

Finite Element Procedure for Estimation of Mutual Capacitances:

How are the capacitances defined?

Let Q_{11}, Q_{12}, Q_{13} and Q_{14} be the charges collected on electrodes #1,#2, #3, and #4 when a low frequency voltage V_1 is applied to electrode #1 and other electrodes are grounded. Then capacitances are defined as

$$C_{11} = \frac{Q_{11}}{V_1}, \quad C_{12} = \frac{Q_{12}}{V_1}, \quad C_{13} = \frac{Q_{13}}{V_1}, \quad C_{14} = \frac{Q_{14}}{V_1}$$

Other capacitances are similarly defined. In general capacitance C_{ij} is defined as $C_{ij} = \frac{Q_{ij}}{V_i}$

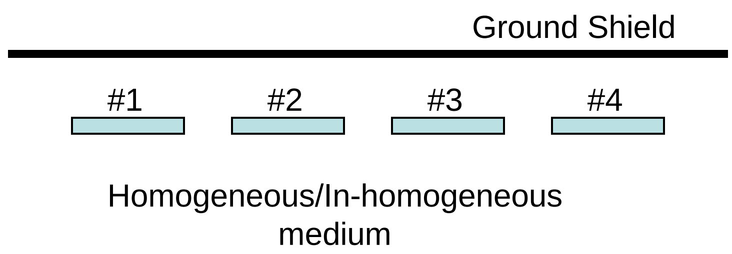
where the charge collected Q_{ij} is given by

$$Q_{ij} = -\oint_{c_j} \varepsilon(x, y) \nabla \phi_i \bullet d\vec{c}$$

$\phi_i(x, y)$ being electrical potential

Conclusion:

Mutual capacitances can be estimated by solving Laplace's equation $\nabla^2 \phi_i(x, y) = 0$ subjected to proper boundary condition dictated by the geometry of MWD instrument.



Solution of Laplace Equation Using FEM Method

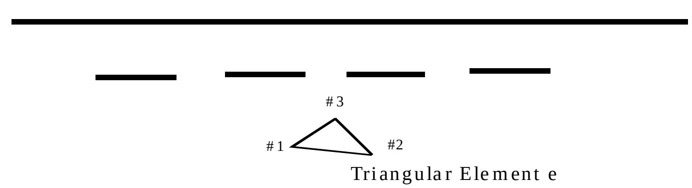
Laplace Equation: $$\nabla^2 \phi_i(x,y) = 0 \qquad (1)$$

Electric Field Distribution: $$\vec{E}(x,y) = -\nabla \phi_i(x,y) \qquad (2)$$

Electric energy stored in the structure:

$$W = \frac{1}{2} \int_S\!\!\int \varepsilon(x,y) \left|\vec{E}(x,y)\right|^2 ds \qquad (3)$$

For 2-D case: divide the region into triangles (as shown below).
If e_{e1}, e_{e2}, e_{e2} are the nodal voltages
then electric potential and field over
the triangle can be written as



$$\phi^e = \sum_{i=1}^{3} e_{ei} \alpha_i(x,y)$$

$$\vec{E}^e = -\sum_{i=1}^{3} e_{ei} \nabla \alpha_i$$

Electric energy stored over the triangle: $$W_e = \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \varepsilon_e e_{ei} \left\{ \int_{i^{th}}\!\!\int \nabla \alpha_i \bullet \nabla \alpha_j ds \right\} e_{ej}$$

Stored energy over the triangle:

$$W_e = \frac{1}{2}\begin{bmatrix} e_{e1} & e_{e2} & e_{e3} \end{bmatrix} \begin{bmatrix} C_{11}^e & C_{12}^e & C_{13}^e \\ C_{21}^e & C_{22}^e & C_{23}^e \\ C_{31}^e & C_{32}^e & C_{33}^e \end{bmatrix} \begin{bmatrix} e_{e1} \\ e_{e2} \\ e_{e3} \end{bmatrix}$$

where $C_{ij}^e = \varepsilon^e \iint_e \nabla\alpha_i \bullet \nabla\alpha_j ds$

If there are N triangles, total stored energy:

$$W = \frac{1}{2}\left[\begin{array}{l} \varepsilon_1\left\{C_{11}e_1^2 + C_{12}e_1e_2 .... + C_{1N}e_1e_N\right\} + \\ \varepsilon_2\left\{C_{21}e_2e_1 + C_{22}e_2^2 + ... + C_{2N}e_2e_N\right\} + \\ \varepsilon_3\left\{C_{31}e_3e_1 + C_{32}e_3e_2 + ... + C_{3N}e_3e_N\right\} + \\ .... \\ \varepsilon_N\left\{C_{N1}e_Ne_1 + C_{N2}e_Ne_2 + ... + C_{NN}e_N^2\right\} \end{array}\right.$$

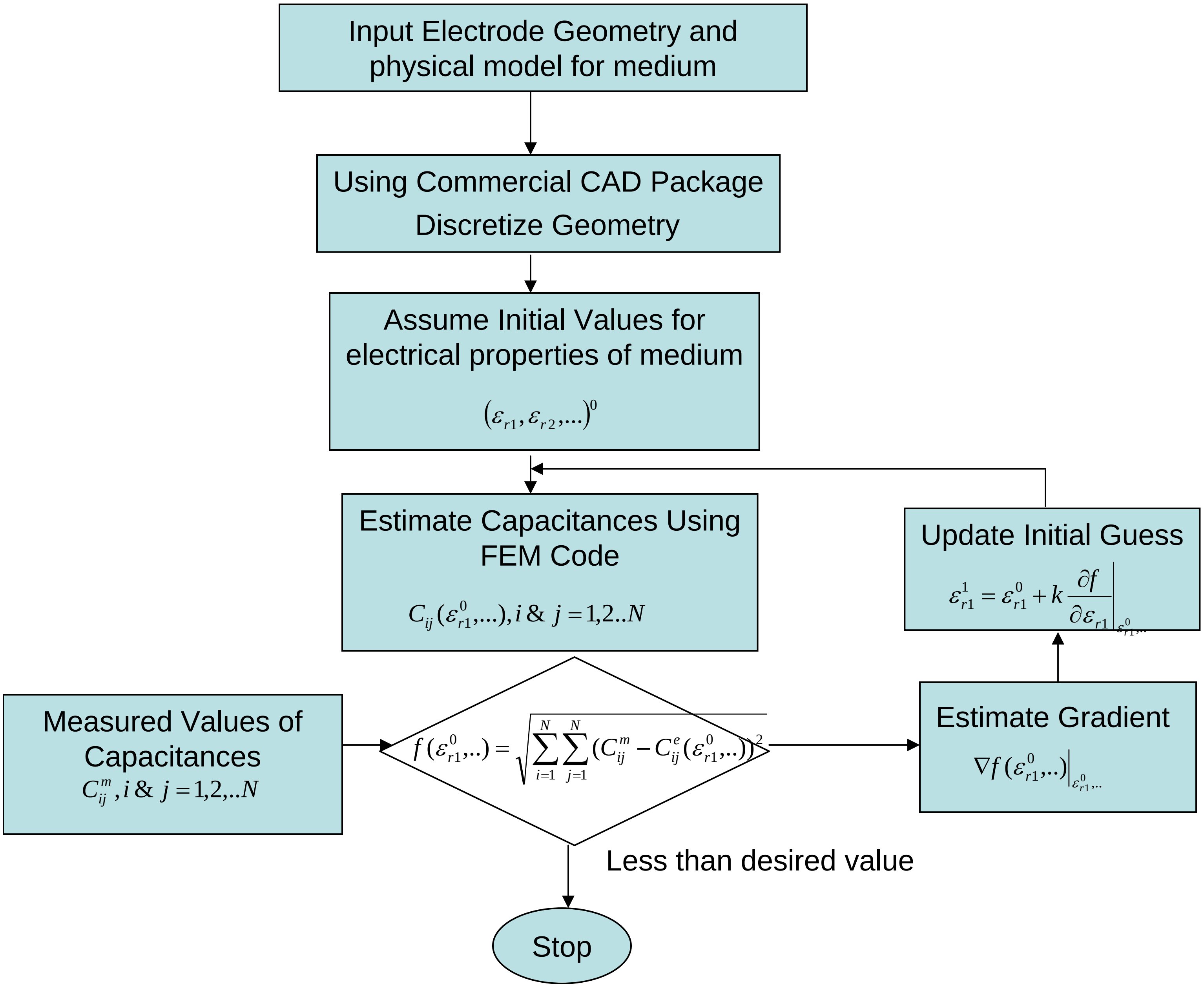
The nodal voltages are obtained by minimizing the total energy. This minimization yields following matrix equation which can be solved for nodal potentials.

$$\begin{bmatrix} \varepsilon_1 C_{11} & \frac{1}{2}(\varepsilon_1 C_{12} + \varepsilon_2 C_{21}) & . & \frac{1}{2}(\varepsilon_1 C_{1N} + \varepsilon_N C_{N1}) \\ \frac{1}{2}(\varepsilon_2 C_{21} + \varepsilon_1 C_{12}) & \varepsilon_2 C_{22} & . & \frac{1}{2}(\varepsilon_2 C_{2N} + \varepsilon_N C_{N2}) \\ . & . & . & . \\ \frac{1}{2}(\varepsilon_N C_{N1} + \varepsilon_1 C_{1N}) & \frac{1}{2}(\varepsilon_N C_{N2} + \varepsilon_N C_{2N}) & . & \varepsilon_N C_{NN} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ . \\ e_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Mutual Capacitance:

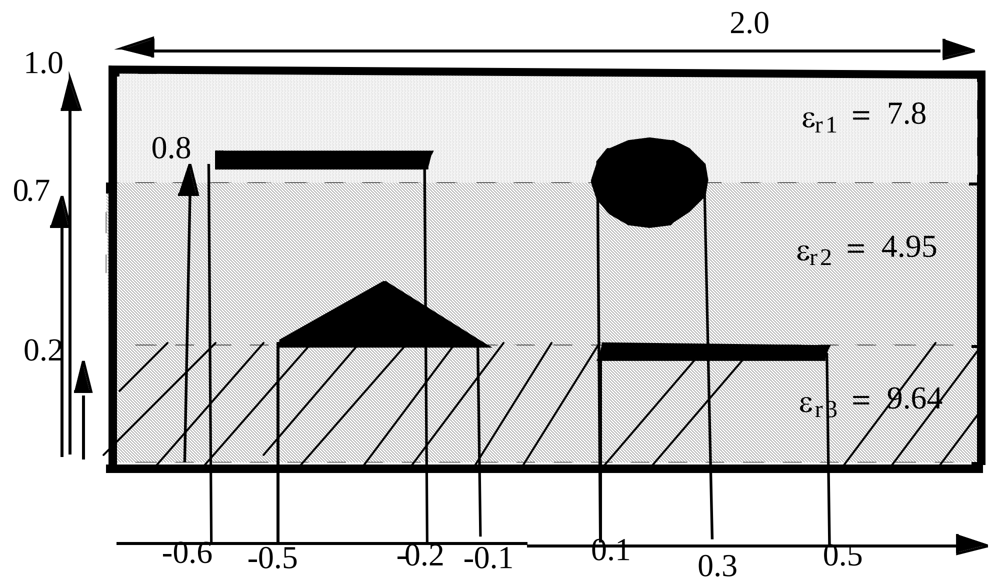
$$C_{ij} = \varepsilon_j \oint_{c_j} \nabla \phi_i . d\vec{c}$$

Flow Chart for Dielectric Profile Estimation



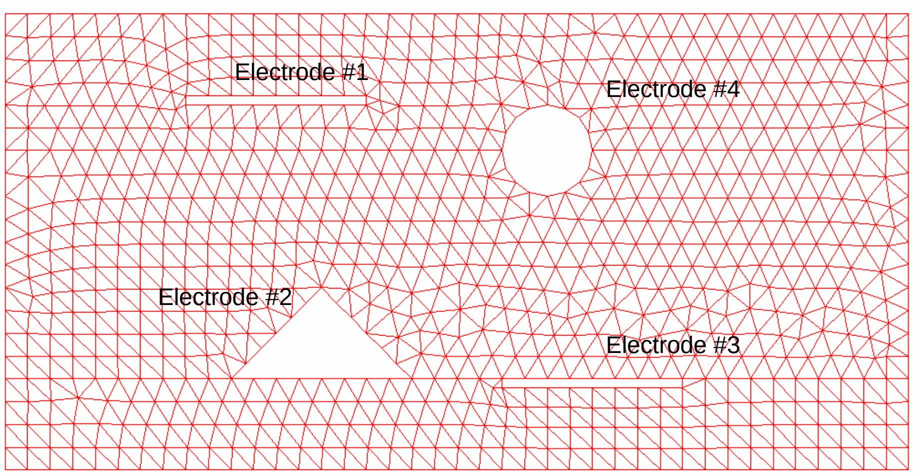
Numerical Validation:

Example 1:



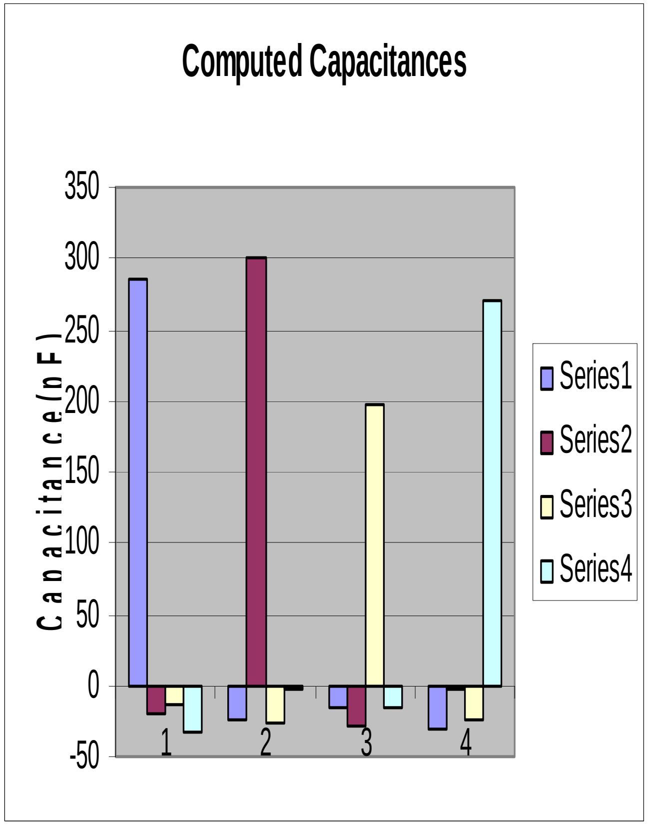
Geometrical Model

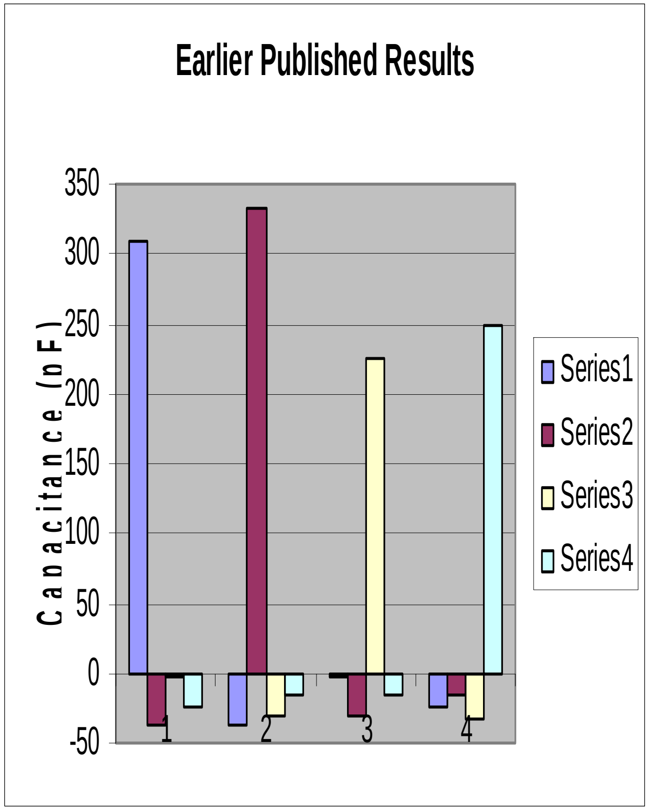
Thickness of Strips = 0.1
All dimensions in cm



FEM Model

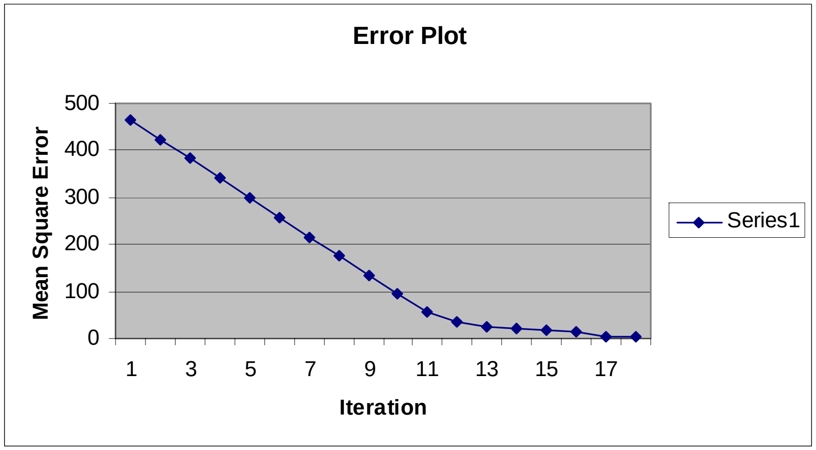
Comparison of computed values of capacitances with earlier published data forward problem

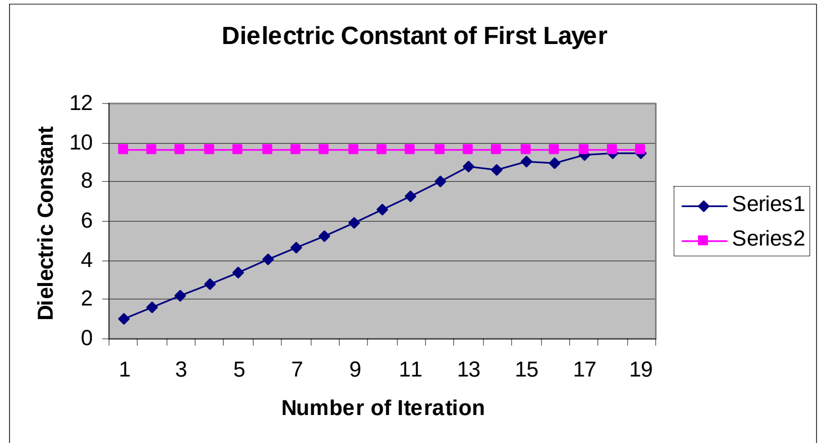




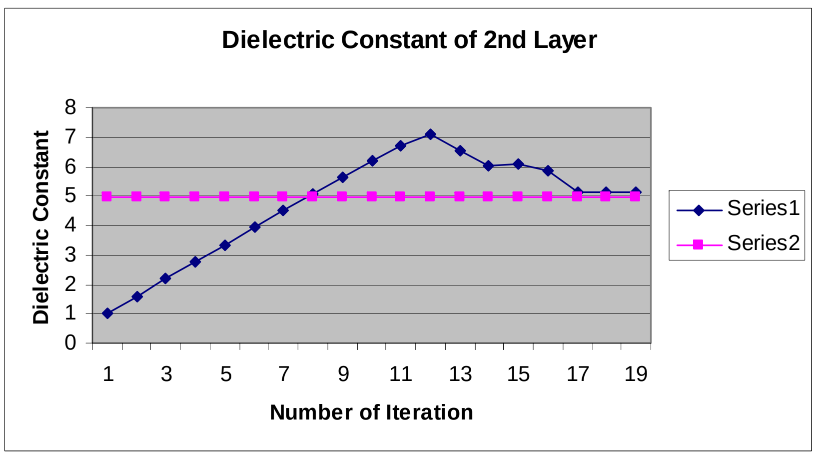
Inversion/Extraction Procedure:

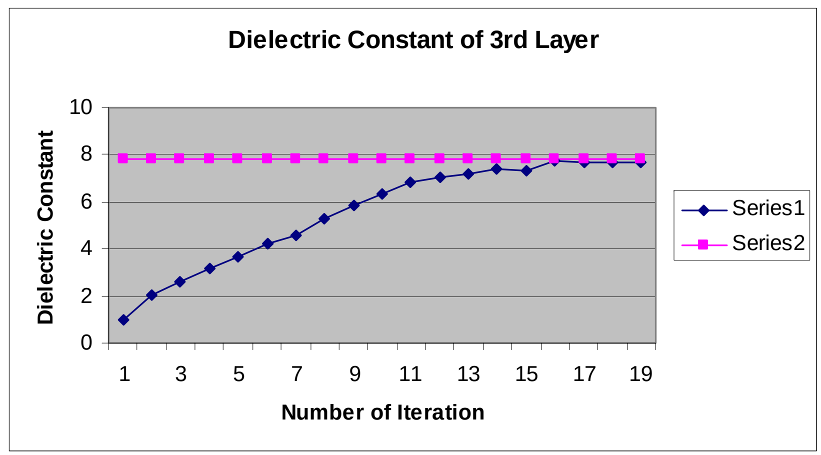
Can we extract a dielectric profile of medium if the capacitances between electrodes embedded in the medium are known?





Estimation error = 1.4%

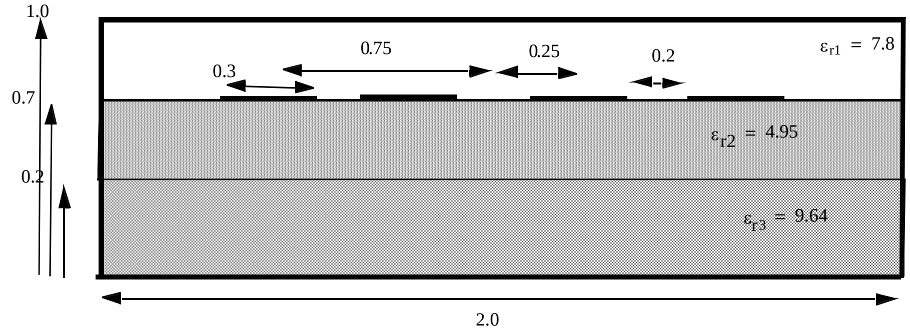


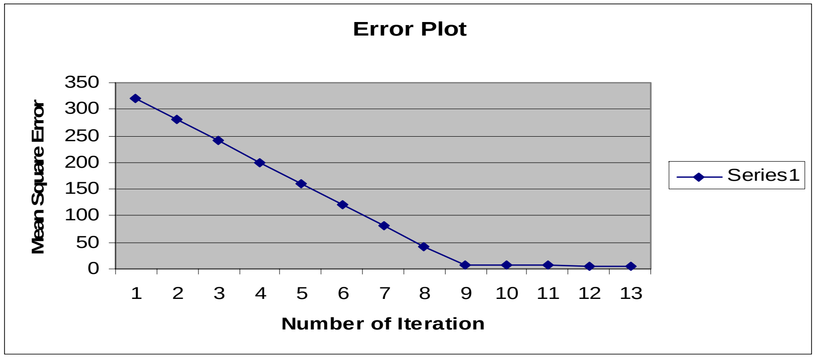


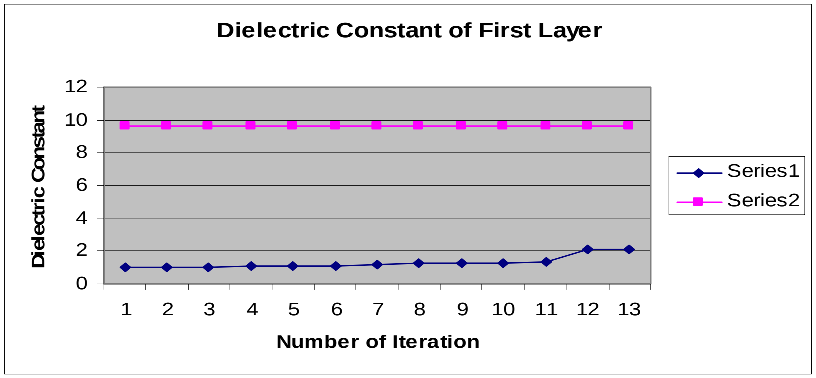
Estimation error = 2.9%

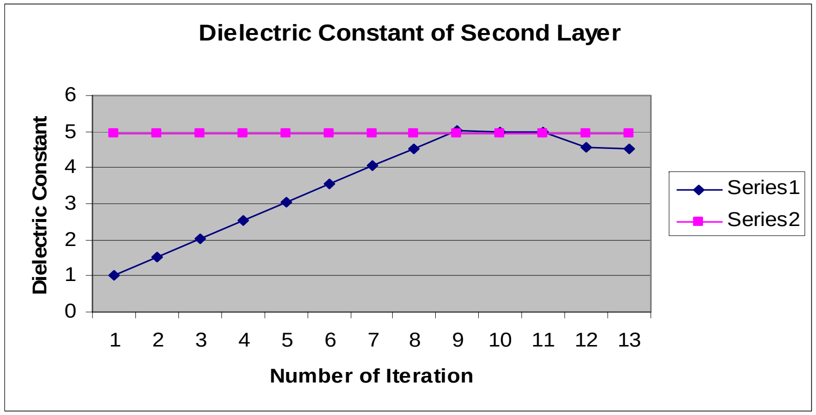
Estimation error = 1.5%

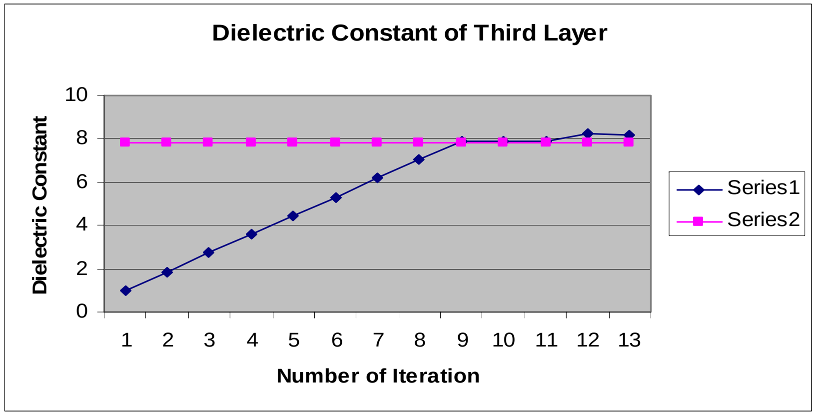
Example 2:

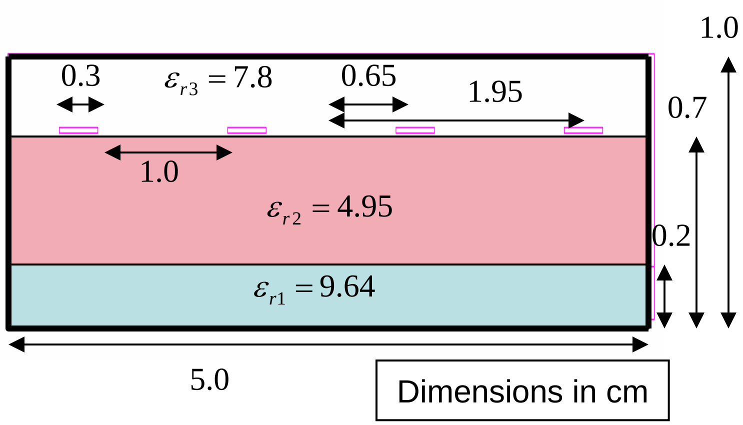

1.0
0.7
0.2
0.3
0.75
0.25
0.2
εr1 = 7.8
εr2 = 4.95
εr3 = 9.64
2.0


Error Plot
Mean Square Error
350
300
250
200
150
100
50
0
1 2 3 4 5 6 7 8 9 10 11 12 13
Number of Iteration
Series1

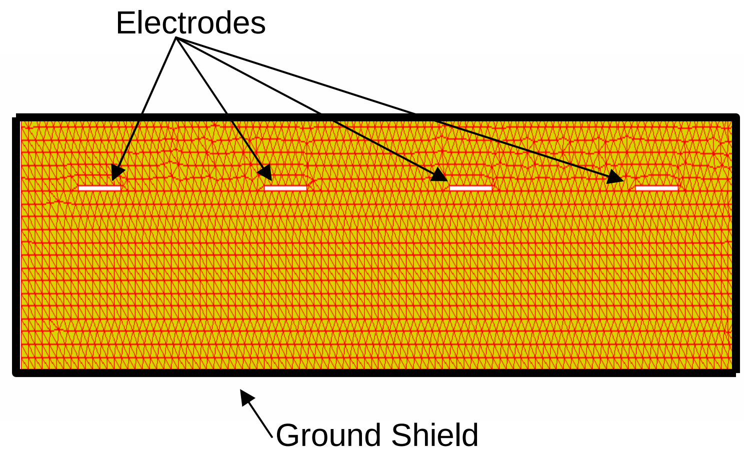

Dielectric Constant of First Layer
Dielectric Constant
12
10
8
6
4
2
0
1 2 3 4 5 6 7 8 9 10 11 12 13
Number of Iteration
Series1
Series2


Dielectric Constant of Second Layer
Dielectric Constant
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13
Number of Iteration
Series1
Series2

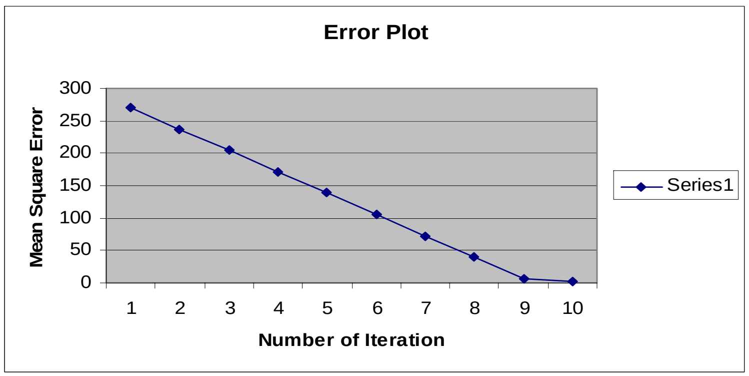

Dielectric Constant of Third Layer
Dielectric Constant
10
8
6
4
2
0
1 2 3 4 5 6 7 8 9 10 11 12 13
Number of Iteration
Series1
Series2

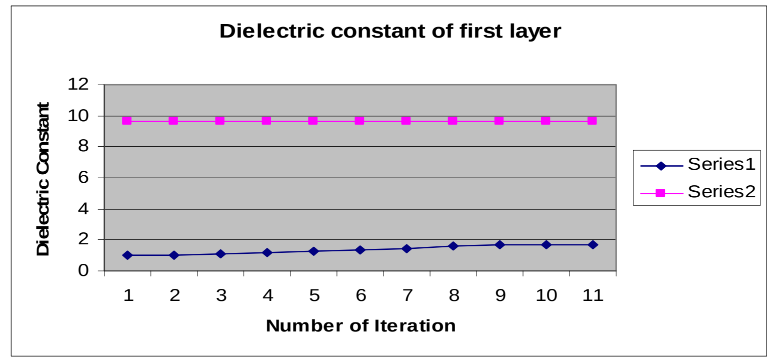


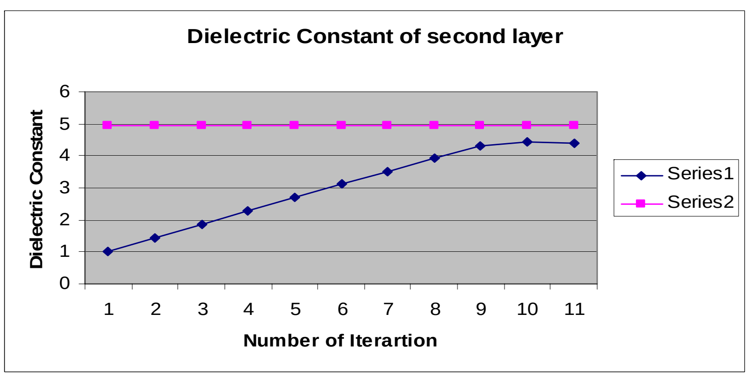
Electrode Sensor Geometry

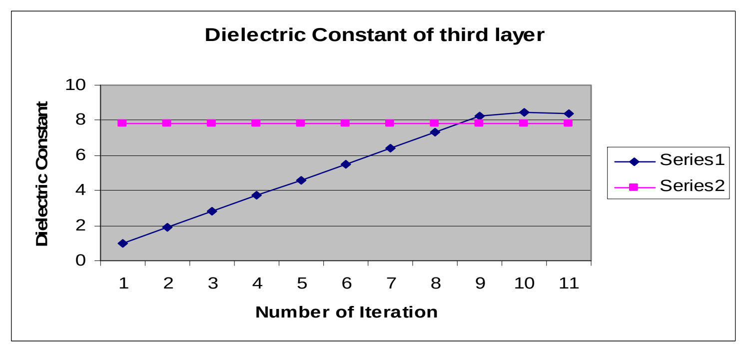


FEM Model

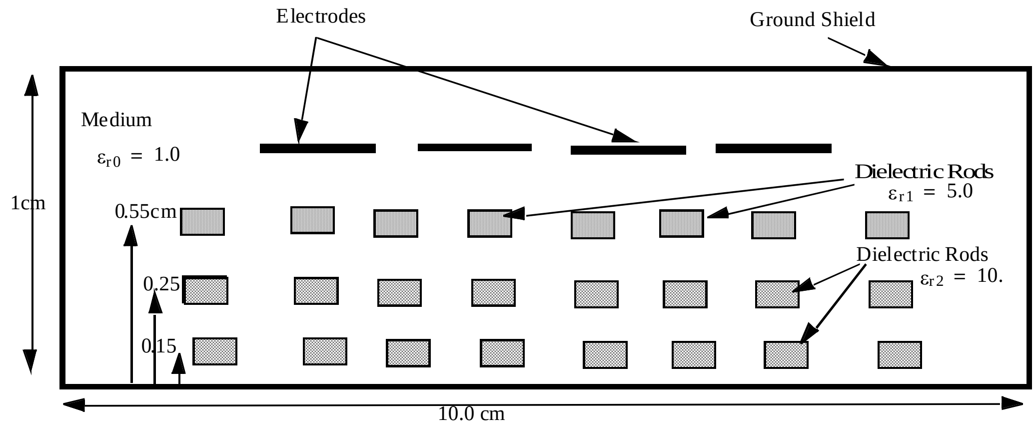




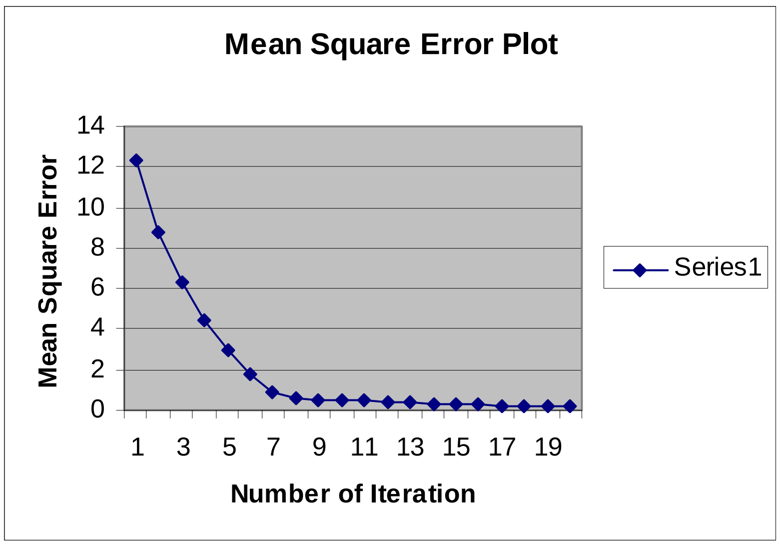




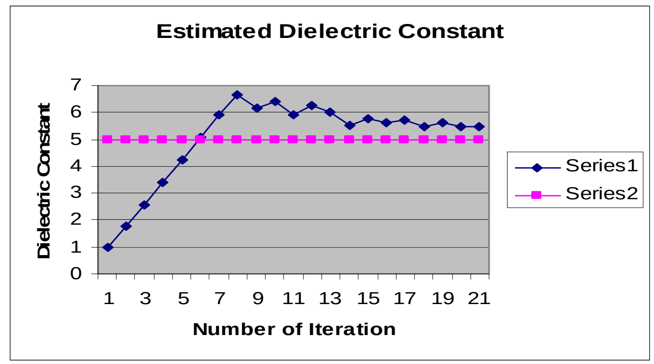
Example 3:



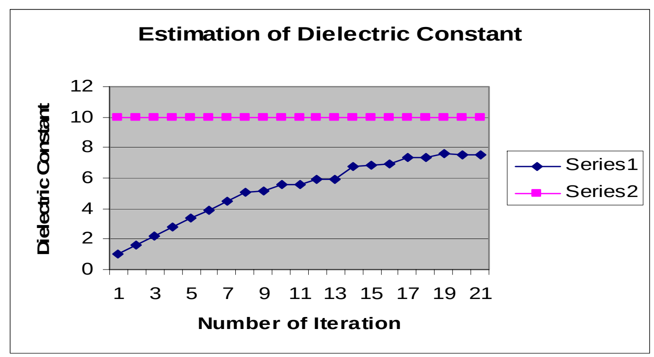
Electrode Width = 1.0 cm
Dielectric Rods = 0.5cm x 0.1 cm

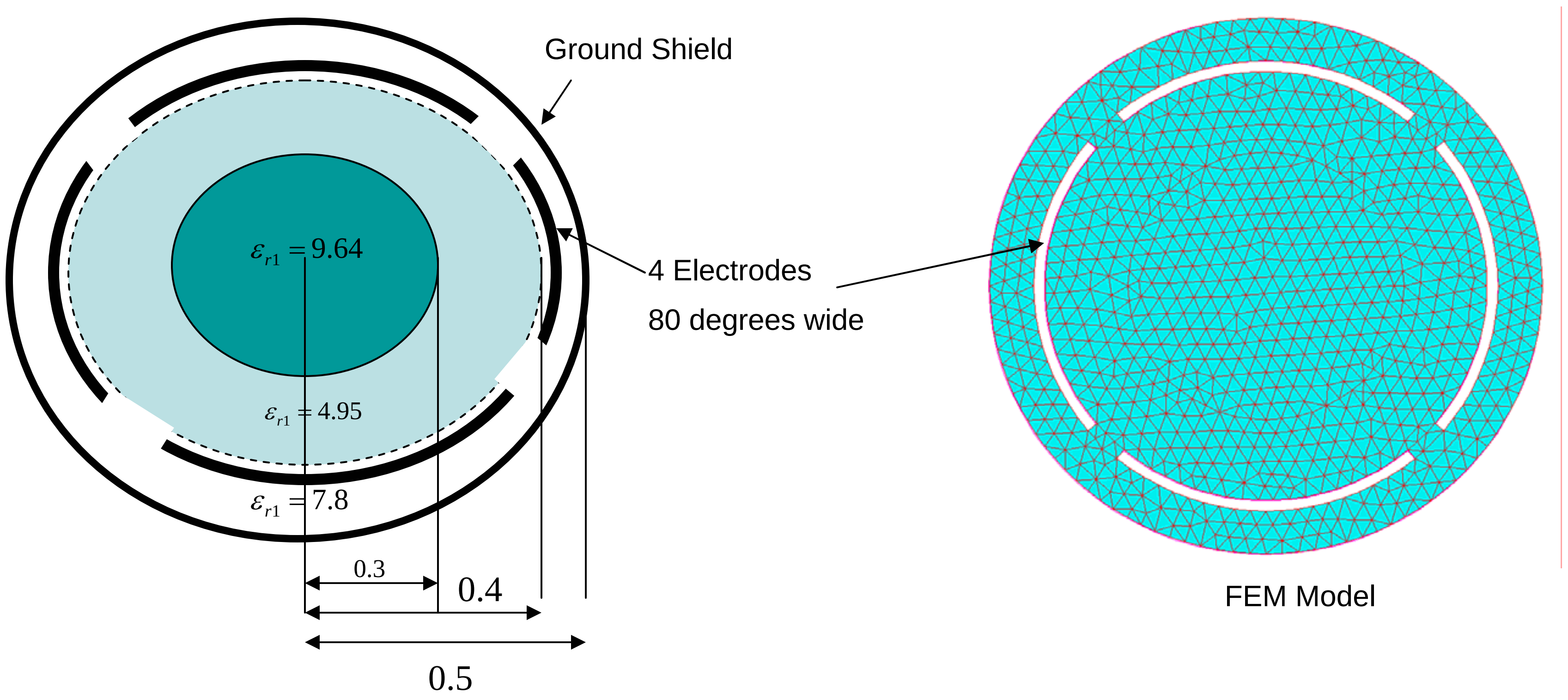


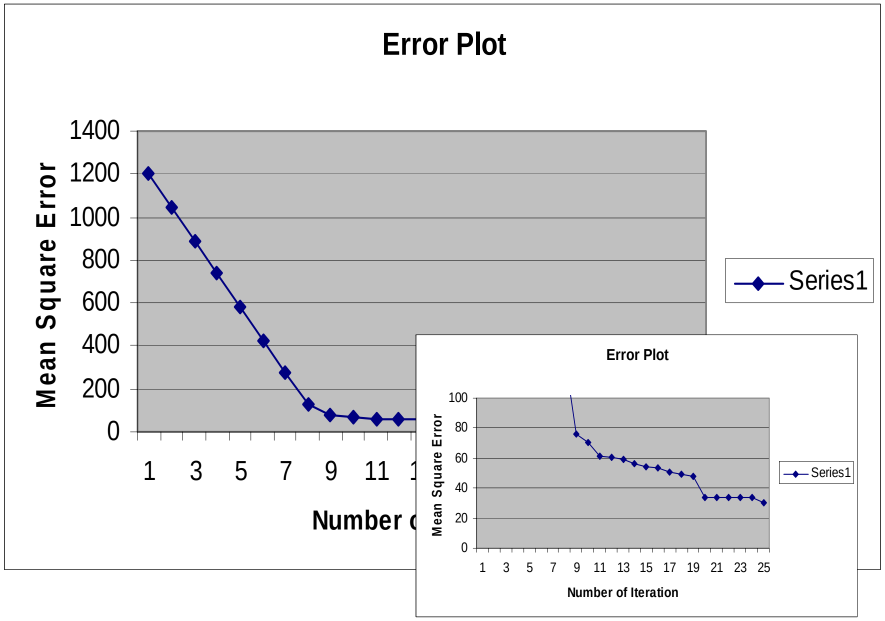
Percentage Error
In estimation= 9.5%

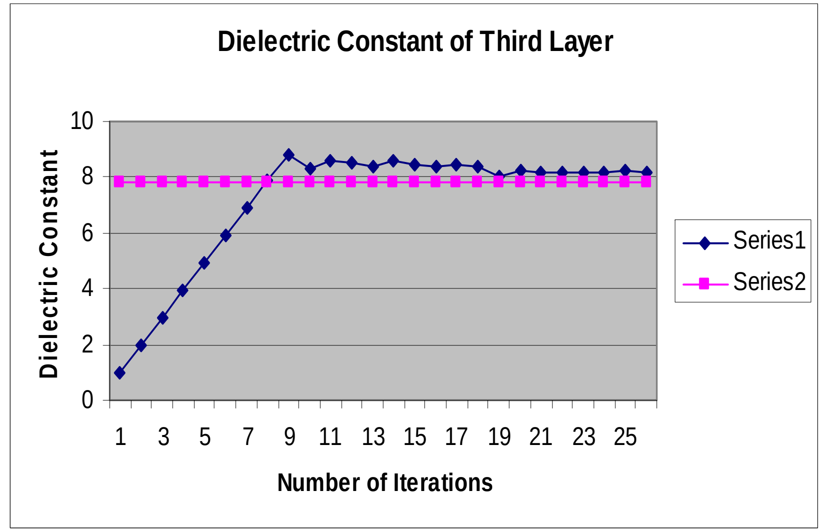


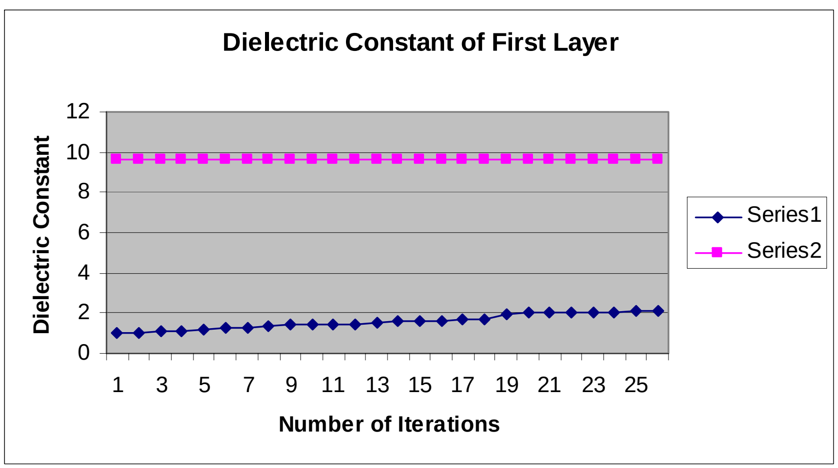
Percentage Error
In estimation= 24%

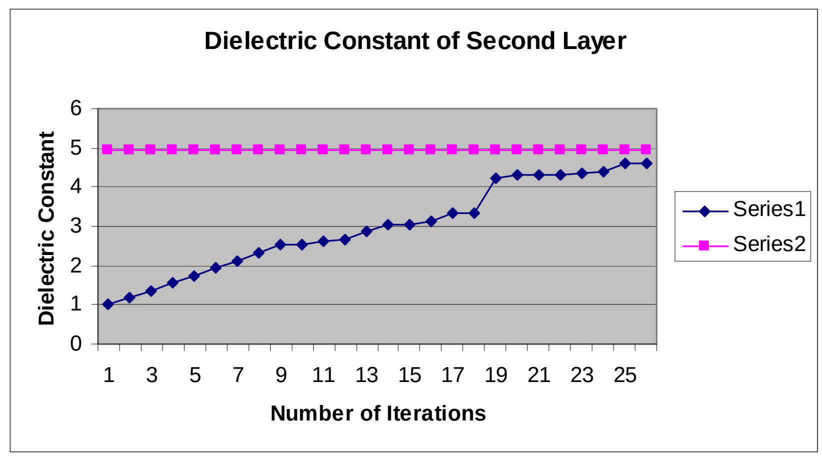



Ground Shield
$\varepsilon_{r1} = 9.64$
4 Electrodes
80 degrees wide
$\varepsilon_{r1} = 4.95$
$\varepsilon_{r1} = 7.8$
0.3
0.4
0.5
FEM Model


Error Plot
Mean Square Error
Series1
Error Plot
Mean Square Error
Number of Iteration
Series1

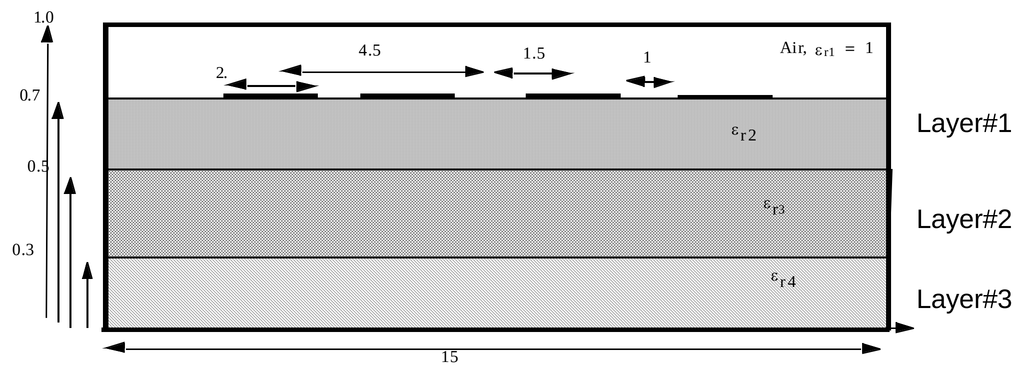

Dielectric Constant of Third Layer
Dielectric Constant
Number of Iterations
Series1
Series2

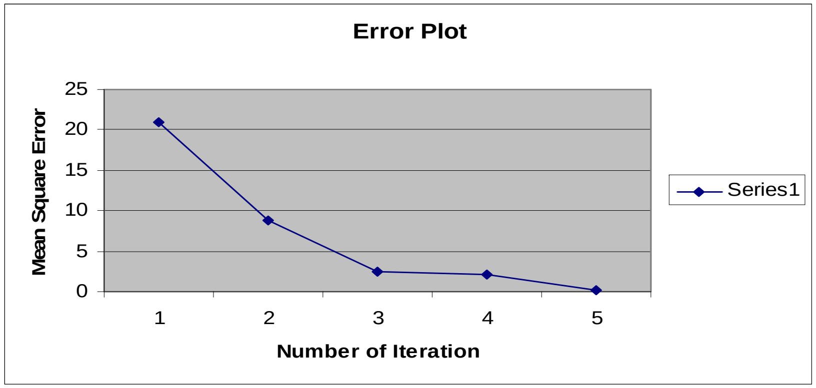

Dielectric Constant of First Layer
Dielectric Constant
Number of Iterations
Series1
Series2

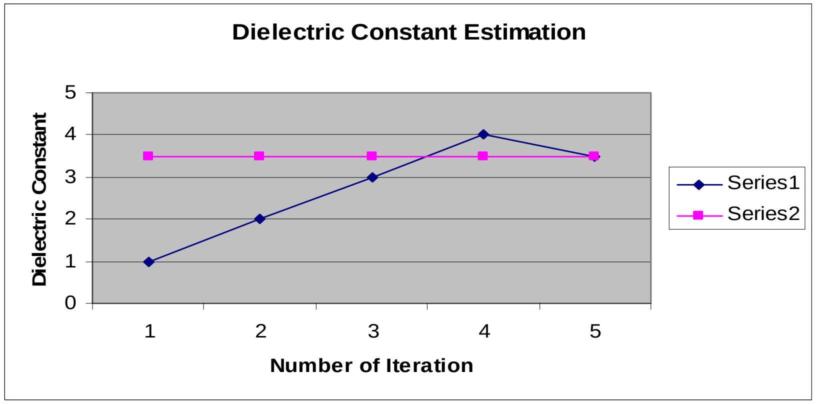

Dielectric Constant of Second Layer
Dielectric Constant
Number of Iterations
Series1
Series2

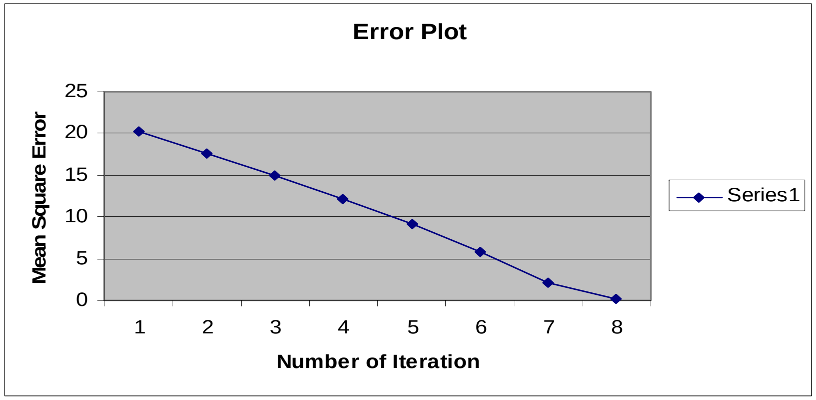
Example 4:

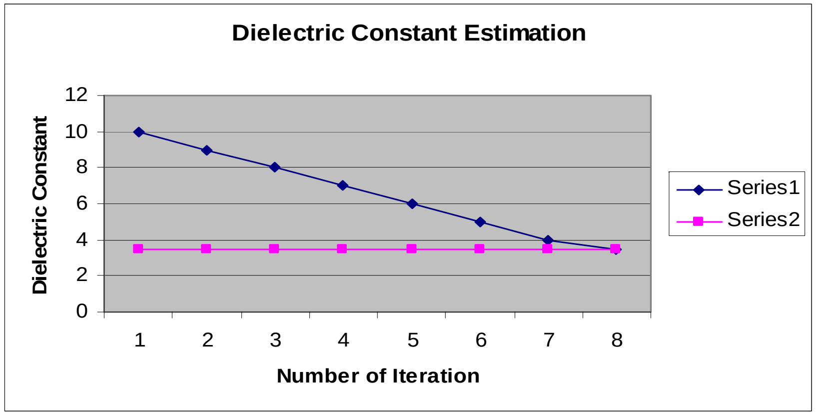
1. Assume that dielectric constants of layer #1,#3, and #4 are known (=1.0)
2. With er2 = 3.5 generate mutual capacitance data
3. Use the procedure to estimate dielectric constant of layer #2



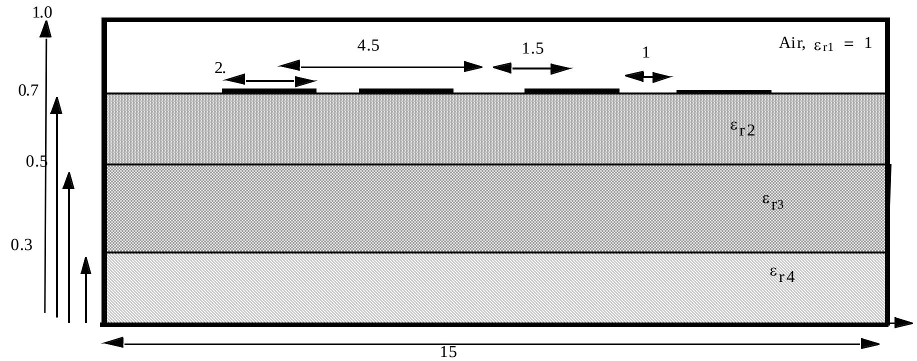


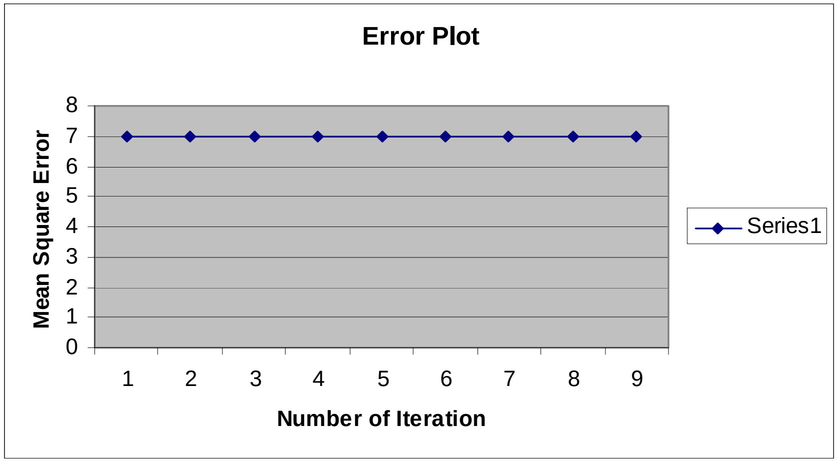


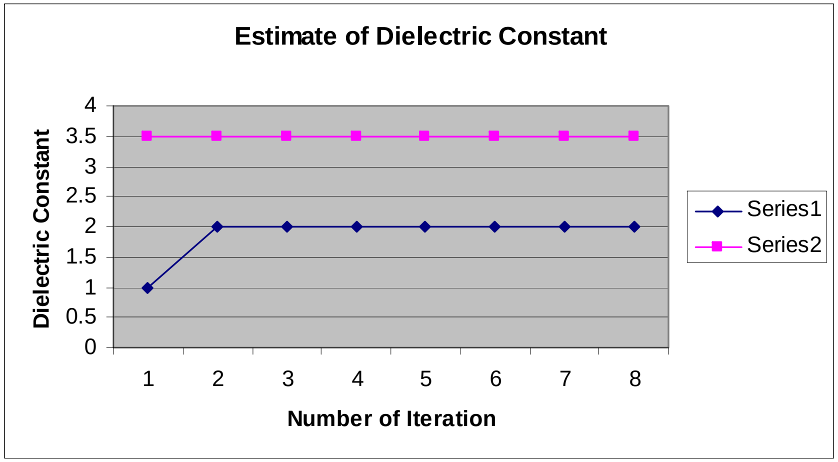




1. Assume that dielectric constants of layer #1,#2, and #4 are known (=1.0)
2. With er3 = 3.5 generate mutual capacitance data
3. Use the procedure to estimate dielectric constant of layer #3







Conclusions:

1. 2-Dimensional Finite Element Method has been successfully employed to analyze and design multi-wave dielectrometer to estimate dielectric constants of inhomogeneous medium.

2. When the electrodes are evenly distributed in a medium under test the multi-wave dielectrometer predicts the medium constants with accuracy greater than 97%

3. When the electrodes are distributed on a plane surface, the region closer to the electrodes are predicted with more accuracy compared to the regions away from the electrodes.

Future Work:

1. In the present formulation dielectrometer boundary was enclosed in a closed metallic ground shield. However, in a real situation, ground shielding is only at the top. We would like to modify our formulation to take into account actual geometry of MWD

2. The real MWD is a 3-Dimenional device. Our ultimate goal is to develop 3-D FEM model to analyze these devices.